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Research Report 2016-11

November, 2016

Operations Research and Control Systems
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Modelling the interaction between flooding events and economic growth

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Abstract

Recently socio-hydrology models have been proposed to analyse the interplay of community risk-coping culture, flooding damage and economic growth. These models descriptively explain the feedbacks between socio-economic development and natural disasters such as floods. Complementary to these descriptive models, we develop a dynamic optimization model, where the inter-temporal decision of an economic agent interacts with the hydrological system. We assume a standard macro-economic growth model where agents derive utility from consumption and output depends on physical capital that can be accumulated through investment. To this framework we add the occurrence of flooding events which will destroy part of the capital.
We identify two specific periodic long term solutions and denote them rich and poor economies. Whereas rich economies can afford to invest in flood defence and therefore avoid flood damage and develop high living standards, poor economies prefer consumption instead of investing in flood defence capital

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and end up facing flood damages every time the water level rises like e.g. the Mekong delta. Nevertheless, they manage to sustain at least a low level of physical capital. We identify optimal investment strategies and compare simulations with more frequent, more intense and stochastic high water level events.

Keywords: flood, socio-hydrology, dynamic optimization, investment strategy

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- We study the feedback between floods and economic growth within a stylized macro-economic model.
- We apply dynamic optimization to determine the optimal mix of investment into physical and flood defence capital.
- We identify two investment strategies in flood defence capital depending on the initial endowment of physical capital.
- We investigate how the optimal investment strategy between physical and defence capital will change depending on the frequency and amplitude of the high water level events.

1. Introduction

Since the beginning of time, people have settled close to rivers and this is still the case nowadays. Rivers enable ways of transport, supply water for industry and agriculture and enhance the quality of living due to lively nature and beautiful scenery. However, living close to rivers also involves the risk of flooding, one of the most devastating natural threats on Earth (Ohl and Tapsell, 2000), whose impact has increased over the past decades in many regions of the world (Dankers et al., 2014; Hall et al., 2014). In order to avoid flood damage, societies have developed projects involving structural defences (e.g., dams. levees, retention basins) and non-structural measures (e.g., land-planning, insurance, forecasting, see e.g. Kundzewicz (2002)). These investments are costly, but may avoid damage in the future. This is an interesting dynamic trade-off structure which we aim to analyse in a stylized socio-hydrological model that is embedded in a macroeconomic set up. To account for the dynamic nature of optimal investment strategies, we apply dynamic optimization methods.

Floods and their consequences have been studied with different model approaches: Recent Integrated Assessment Models (IAM) aim to understand the interaction of society and floods (Merz et al., 2014) in a broad context. Climate change leads to more and bigger floods in certain regions Milly et al. (2002/01/31/print). Such models typically do not account for the impact of changes in the environment on economic growth (Estrada et al., 2015). The aim of Agent Based Models (ABM) such as Dawson et al. (2011), Safarzyska et al. (2013) and Li et al. (2015) is to understand the impact of floods on individual behaviour. ABMs can provide a qualitative analysis of the consequences of floods on different levels: the individual/micro-level, the aggregated economy/macro-level and the firm level/meso-level. Complementary Input-Output-Models (Koks et al., 2014; Hasegawa Ryoji) provide a quantitative cost-benefit-analysis of case studies. Okuyama analysed these model frameworks as well as computational equilibrium models for disasters. A dynamic spatial computable general equilibrium model based on the dynamic structure of a Ramsey growth model was developed by Nakajima et al. (2014) to numerically measure flood damage costs. It displays the dynamic tradeoff between the costs today and future savings, investments and consumption. Besides simulation modeling approaches, optimization models have been developed to calculate optimal dike heights (Eijgenraam, 2006; Brekelmans et al., 2012; Chahim et al., 2012). Larger stochastic programming models in water resource management and flood management (Li et al., 2007; Liu et al., 2014; Kleywegt et al., 2002; Needham et al., 2000) only allow optimal solutions for discrete variables and finite time horizon. Moreover, most of these models are linear, have only one control variable, either none or linear constraints and are therefore quite different to the proposed economic growth model in our paper.

While existing models on flood management have focused on the analysis at the firm level (e.g. Chahim et al. (2013) and Eijgenraam et al. (2014), who apply impulse control models for optimal dike heightening within an economic cost-benefit decision problem to minimize the sum of the investment and expected damage cost), our model aims to include flood dynamics into a macroeconomic growth model.

So far, floods have been rarely analysed in a macroeconomic model of economic growth considering not only direct and indirect damage costs, but also loss of future potential economic growth through dynamic consumption and investment decisions.

In environmental economics this approach is quite common. Economic growth models have been applied to study, e.g., the effect of climate change on long run economic growth (Xepapadeas et al., 2005). More formally, these models commonly postulate that pollution causes economic losses via a damage function that is positively related to an increasing temperature caused by pollution. (Rezai et al., 2014; Millner and Dietz, 2015; Morisugi and Mutoh, 2012; Zemel, 2015). Pollution itself is commonly modeled via the flow or stock of emissions. Indeed, emissions and investment in emission abatement have strong analogies to extreme water events (floods, droughts) and investment in abatement (flood defence capital, reservoirs), respectively. It therefore seems an obvious choice to apply this modelling framework also in the context of flood modelling. Similar to the increase in the temperature that underlies the economic damage in climate change models, the water level underlies the occurrence of floodings and hence the economic damage.

There is a new research line, socio-hydrology, that deals with such coupled systems. The main thrust of socio-hydrology is to add a new perspective to former models and studies in hydrology by coupling dynamics of human populations, economic growth and general resource availability (Sivapalan et al., 2012; Levy et al., 2016). Socio-hydrology aims at understanding emergent patterns and paradoxes that result from long-term co-evolution of non-linearly coupled human-water systems. Elshafei et al. (2014) and Sivapalan and Blöschl (2015) developed prototype frameworks for socio-hydrology models. Di Baldassarre et al. (2013) and Viglione et al. (2014) developed a socio-hydrology model to explain the feedbacks between settlements close to rivers and flooding events. Di Baldassarre et al. (2015) use the model to capture processes such as the levee effect (e.g., Montz and Tobin (2008)) and the adaptation effect (Penning-Rowsell, 1996; IPCC, 2012; Mechler and Bouwer, 2014), which traditional flood risk models do not include. Pande et al. (2014) were one of the first who added a water related problem to a standard economic model of finitely lived agents, the so called overlapping-generations model (OLG).

In this paper, we build a macro-economic model in the context of floods and use a dynamic optimization model which is a different perspective from the more common descriptive models, simulations and scenario analyses. This is where we regard our model to add to the literature. More specifically, while there exist economic growth models that include the feedback between the environment and economic output, our novel contribution is to add an exogenous time varying water level function and study the resulting optimal path of consumption and investment. Mathematically this poses the challenge that we have to solve a non-autonomous optimization model.

Our model uses the model of Di Baldassarre et al. (2013) and Viglione et al. (2014) as a starting point. Their simulations show that building high levees leads to fewer flooding events with higher impacts which may slow down economic growth. Protecting a settlement by levees can, however, increase the damage to downstream settlement due to the loss of flood retention volume. Furthermore, building levees or any other defence capital will lower flooding probability and may therefore increase the willingness of citizens to build close to the river. If water levels rise higher than the crest of the levees, the physical capital next to the river is destroyed. Since there is a higher physical capital stock next to the river, the flood hits even harder on the economy.

Based on their model set up we build an economic model to analyse the tradeoffs and feedbacks associated with settlements close to rivers. We assume two types of capital: physical capital and defence capital. Decision makers can invest in physical capital, such as machines, buildings and infrastructure. On the other hand, investments in defence capital can avoid the actual damage of floods and have thereby a positive influence on output. Total output of the economy consequently depends on both capital stocks.

We apply a periodic non-autonomous exogenous function to represent the water level. The periodic water function is introduced in Grames et al. (2015). Even though the assumption of non-stochastic flood occurrence is a strong one, we believe that useful insights on the system can still be obtained. Alternatively, we can interpret our water function as approximation of past flood events. Assuming the periodic non-autonomous exogenous function for flood occurrence allows us to solve the dynamic optimization problem, for which we further develop the solution method of Moser et al. (2014) where a similar mathematical problem in the context of renewable energy has been solved.

Including a non-autonomous exogenous deterministic function into a dynamic decision framework over an infinite time horizon requires already quite sophisticated methodologies of optimization and a highly challenging numerical approach. If we would model the water level function stochastically, the long run optimization problem could neither be solved analytically nor numerically. Recent research in that field of stochastic optimization is using much simpler objective and state functions (Nisio, 2015) without such strong nonlinearities as they exist in our model. Climate models include uncertainty in the timing of events (Tsur and Withagen, 2013), where the hazard rate of the event can depend on e.g. a stock of pollution of greenhouse gases (Zemel, 2015). Our exogenous water level function does not depend on any state variable, so the solution method applied in e.g. Zemel (2015) cannot be transferred to our model. Moreover, climate change models with an exogenous hazard rate capture only one random event (Zeeuw and Zemel, 2012), whereas floodings in our model are recurrent random events over an infinite time horizon. Hence the model structure of stochastic climate models and our flood model is fundamentally different. However, in order to investigate the sensitivity of our results to the stochasticity of floods, we also present simulations of our model assuming a stochastic water level function like e.g. Viglione et al. (2014).

The aim of this paper is to understand the mechanisms behind investment decisions in the context of flood risk prevention. For this purpose we choose a stylized macro-economic model to investigate the optimal investment strategy between flood protection measures and physical capital to enable economic growth.

The remainder of the paper is organized as follows. The following section provides an introduction to the feedbacks between society and floods and outlines the model framework and its equations. In a first step we present various simulations of our model and show the sensitivity of the resulting dynamics on the investment strategy chosen. To determine the optimal investment strategy between physical and defence capital taking into account the dynamic feedback between the economic and hydrological system we next apply the tools of dynamic optimization. We also show the sensitivity of the model dynamics on the initial endowment of the economy. In particular, the optimal investment strategies will be determined by the state of the economy. Furthermore, we investigate how the optimal investment strategy will change depending on the frequency and amplitude of the high level water events and whether a more efficient flood defence capital may foster economic growth. Last, we embed the optimal solutions in a stochastic simulation run. The paper concludes by discussing our scenarios in the context of flooding in various regions of the world.

2. Modelling the interaction between flood events and economic growth

2.1. Feedbacks between society and floods

Floods affect settlements close to rivers by destroying existing capital. Societies have developed different approaches to prevent or mitigate the damage. Building dikes, levees or flood control basins may prevent flood waters entering the settlements. Warning systems to assist in evacuations and settling further away from the river (Viglione et al., 2014) may also be regarded as mitigation measures.

In our model we represent all flood prevention technologies by one variable and name it *defence capital*. Similarly we model the physical capital stock — which represents machines, buildings, infrastructure — by one variable named *physical capital*. We assume that a flood causes damage of physical capital if the water level exceeds a specific threshold of the defence capital. The society chooses how much it invests into defence capital and therefore influences the occurrence of floodings.

The physical capital stock is used to produce economic output. Aggregate output in an economy can be used for consumption and investments in either physical or defence capital stock. We assume that the decision of the optimal share of output used for consumption and investment is taken by a social planner. This means we abstract from a market framework where factor renumerations such as interest rates on capital or wages for labour input would determine the optimal allocation of output between consumption and the two types of investment.

We assume a closed economy, which implies that all of the produced output will be used, and no further trade with other communities is possible.

Fig. 1 displays the dynamics of the model. Economic output $Y(t)$ depends on the amount of physical capital $k_y(t)$. The output can be either consumed $c(t)$ or invested in physical $i_y(t)$ or defence capital $i_d(t)$. The society chooses the level of consumption and the amount of investment into physical and defence capital in order to maximize utility. The defence capital can prevent the damage $d(W(t), k_d(t))$ caused by flooding events. The occurrence of flooding events depends on the water level $W(t)$. In case of flooding, both capital stocks are damaged.

2.2. Model equations

To model the aforementioned interaction between society and flood events we first define the utility function of the social planner and its choice variables. Next, we determine how output is produced in the economy and explain the dynamics of physical and defence capital which constitute the dynamic constraints

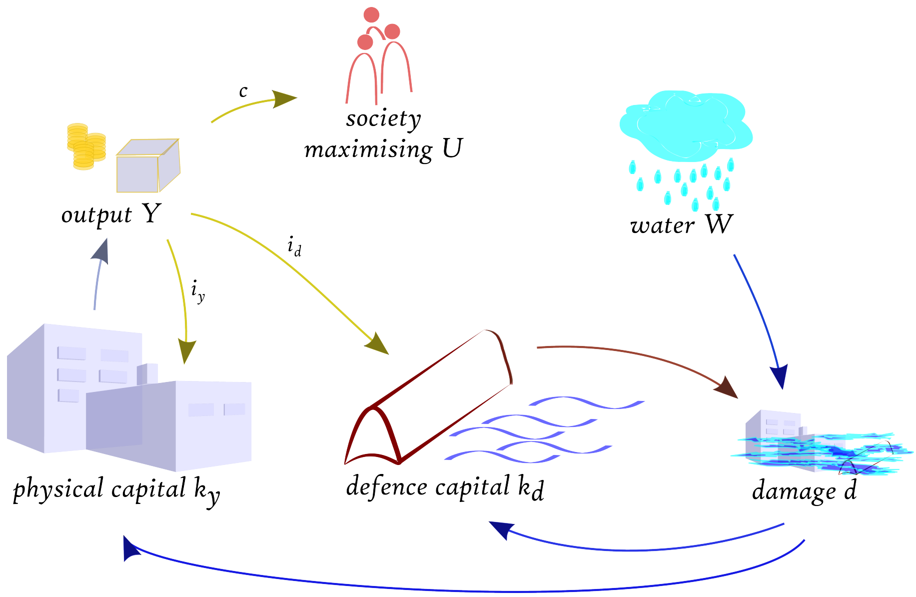

Figure 1: Overview of the dynamics within the presented model. The society chooses to consume ($c(t)$) or invest ($i_y(t)$ and $i_d(t)$) the economic output $Y(t)$ into the capital stocks.

for the optimization problem of the social planner. To model the water level we introduce an exogenous periodic function over time. Together with the level of defence capital, the water level will then determine the extent of the damage.

210 *2.2.1. Utility function*

The objective of the social planner is to maximize the discounted stream of aggregate utility $U(c(t)) = \ln(c(t))$ which depends positively on the consumption level $c(t)$:

$$\max_{\{c(t), i_d(t)\}} \int_0^\infty e^{-\rho t} U(c(t)) dt \tag{1}$$

where ρ denotes the time preference and indicates to which extent the social planner prefers utility of consumption today compared to utility of consumption tomorrow. Consumption $c(t)$ and investment in defence capital $i_d(t)$ are control variables [1] to be chosen optimally to maximize equation (1), given the level of output and dynamic constraints of physical and defence capital as stated below. More specifically, the dynamic optimization of the social planner guarantees that any decision taken today also incorporates the feedback on the future evolution of the system.

Since at every time period consumption together with investment in physical and defence capital is bounded by the available output, the choice of two variables implies the optimal choice of the third variable (investment in physical capital in our case).

2.2.2. Economic output

Output $Y(t)$ is given by a Cobb Douglas-production function

$$Y(t) = Ak_y(t)^{\alpha} \tag{2}$$

that depends on the physical capital stock $k_y(t)$ and an exogenous level of technology A. The production input factor labor is normalized to one. $\alpha \in [0, 1]$ denotes the elasticity of the production input factor capital.

Output can be used for consumption $c(t)$ as well as for investment in physical capital $i_y(t)$ and investment in defence capital $i_d(t)$. Since output is given in [$] and the unit of the defence capital is [m] we need to transform investment in defence capital i_d given in [m] into costs $Q(i_d(t)) = \theta_0\big(\theta_1 i_d(t) + \theta_2 i_d(t)^2\big)$ given in [$]. The parameters θ_i weight the linear and quadratic parts of the costs and are calculated according to Slijkhuis et al. (1997) and Bedford et al. (2008).

The overall budget constraint for the social planner is therefore given as:

$$Y(t) = c(t) + i_y(t) + Q(i_d(t)) \tag{3}$$

[1] In a less technical setting we refer to the control variables as decision variables.

2.2.3. State dynamics

Following the standard Ramsey model we write the dynamic constraints by the following two state equations for physical and defence capital:

$$\dot{k}_y(t) = i_y(t) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t) \tag{4}$$

$$\dot{k}_d(t) = i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t) \tag{5}$$

Each capital stock can be augmented by investments i_y and respectively i_d and depreciates by a constant rate δ_y, respectively δ_d. Moreover, flood damage $d(k_d(t), W(t))$ decreases both capital stocks.[2] The flood damage rate $d(k_d(t), W(t))$ is in the interval [0,1]. We allow for the fact that the damage may be different for physical and defence capital by introducing the parameter κ_d in equation (5).

2.2.4. Damage function

Flood damage and flood recovery are complex and discussed in various papers (Di Baldassarre et al., 2015; Merz et al., 2014). Our model constitutes a stylized model with the focus to analytically study and understand the basic feedbacks and mechanisms between society and hydrology. Therefore we assume a damage function $d(k_d(t), W(t))$ analogous to Viglione et al. (2014) and a recovery rate based on the economic capital, the technology and the optimal consumption behavior. Since the recovery is endogenous in our optimization framework, we can describe the optimal consumption and investment behavior given an exogenous forcing of the water level $W(t)$.

The amount of damage is related to the flood intensity $W_{eff}(W(t), k_d(t)) = W(t) + \xi_d k_d(t)$ which is a function of the water level $W(t)$ and the additional amount of water $\xi_d k_d(t)$. This additional amount of water occurs due to existing defence capital $k_d(t)$ such as levees: Levees at one place protect this area from flooding, but increase water levels further down the river due to loss of flood plain retention (Di Baldassarre et al., 2013).

[2] Rezai et al. (2014) model similar dynamics for pollution.

If the flood intensity $W_{eff}(W(t), k_d(t)) = W(t) + \xi_d k_d(t)$ exceeds the flood defence capital $k_d(t)$ and the levees spill over, a damage of the overall capital stock occurs. The higher the effective water level $W_{eff}(W(t), k_d(t))$, the higher the direct damage of the flooding (Jonkman et al., 2008). The damage rate $d(k_d(t), W(t)) \in [0, 1]$ gives the relative damage of the capital stocks. Beyond $k_d(t)$, the damage of the flood is proportional to the effective water level of the flood W_{eff} and, also, to the flood duration, which is the time interval when $W_{eff}(W(t), k_d(t)) > k_d(t)$ holds. This assumption reflects the common situation that structural damage is related to the water level, while damage to industry production and stocks is related to the duration of the inundation. The damage rate is then represented as follows.

$$d(k_d(t), W(t)) = \begin{cases} 1 - \exp\left(-W_{eff}(t)\right) & \text{if } W_{eff}(W(t), k_d(t)) > k_d(t) \\ 0 & \text{else} \end{cases} \tag{6}$$

For ease of obtaining a numerical solution of the optimization model, we approximate the damage function (6) with a continuous function. Still, damage ($d(k_d(t), W(t)) > \epsilon$ with a positive ϵ close to zero) only occurs if $W_{eff}(W(t), k_d(t)) > k_d(t)$. We choose the signum-approximation function and base it on the following four assumptions: First, the minimum value is 0 for the water level $W \leq 0$. Second, if $W_{eff}(W(t), k_d(t)) = W + \xi_d k_d > k_d$ and $W \to \infty$ we reach the maximum value 1. Third, the inflection point is at $W + \xi_d k_d = k_d$. Fourth, the gradient at the inflection point is chosen such as to approach infinity to approximate the jump between 0 and the relative damage $d > 0$ in equation (6). Furthermore, we add a multiplicative term $(1 - \frac{1}{1+W(t)^\eta})$ that is increasing in the water level $W(t)$ and bounded by the interval [0,1]. This term ensures that the damage is higher for a more intense flooding.

$$d(k_d(t), W(t)) = \frac{1}{2}\left(\tau_3 + \frac{\tau_2 + W(t) - (1 - \xi_d)k_d(t)}{\sqrt{(W(t) - (1 - \xi_d)k_d(t))^2 + \tau_1}}\right)\left(1 - \frac{1}{1 + W(t)^\eta}\right) \tag{7}$$

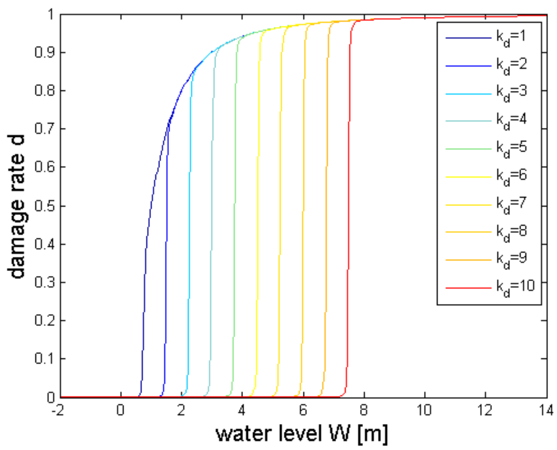

Figure 2: Form of the damage rate as a function of the water level $W(t)$ for various levels of the defence capital $k_d(t)$ and for $\xi_d = 0.5$. Both the water level and the defence capital are given in meters.

The coefficients τ_i adjust the accuracy of the approximation of (6) with (7), for the calculations we used $\tau_1 = 0.001$, $\tau_2 = 0$ and $\tau_3 = 1$.
Fig. 2 shows the damage rate with respect to the water level $W(t)$ for different values of defence capital stock $k_d(t)$. If the defence capital is higher than the water level, the damage is closer to zero (no damage) until the inflection point $W(t) = (1-\xi_d)k_d(t)$ given in equation (6) and then close to one (total damage).

2.2.5. Water function

The water level $W(t)$ [m] is approximated with a continuous function (Viglione et al. (2014) uses a discrete time series for flood events) to allow an analytical solution of the model. A similar function was developed by Langer (2014) and explained in Grames et al. (2015). The parameter κ_s determines the maximum level of water to be reached during a flood and κ_m controls the frequency of flood events.

$$W(t) = \frac{1}{2}\sum_{\kappa=1}^{\kappa_s} \cos(\kappa_m \kappa t) \tag{8}$$

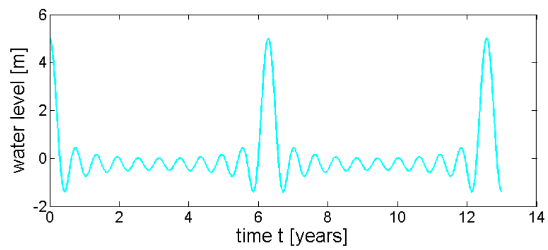

Figure 3: The periodic water level function gives quite frequent flood events. In brackets we display the units for the time and the water level itself.

The water function is shown in Fig.3. The water level is 0 when the river is bankfull and therefore the function (8) can be negative. Negative water levels $W(t) < 0$ are simply treated like $W(t) = 0$, since the water level only affects $d(k_d(t), W(t))$, and $d(k_d(t), 0) = d(k_d(t), w_-)$ holds for any $w_- < 0$.

2.2.6. Model summary

In summary, our model is represented by the following set of equations, where we have substituted $i_y(t)$ from equation (3) into equation (4):

$$\max_{\{c(t)\in[0,Y(t)],i_d(t)\in[0,Y(t)-c(t)]\}} \int_0^\infty e^{-\rho t} U(c(t))dt \tag{9a}$$

s.t.

$$\dot{k}_y(t) = Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y(t) - \delta_y k_y(t) \tag{9b}$$

$$\dot{k}_d(t) = i_d(t) - \kappa_d d(k_d(t), W(t))k_d(t) - \delta_d k_d(t) \tag{9c}$$

$$U(c(t)) = \ln(c(t)) \tag{9d}$$

$$Q(i_d(t)) = \theta_0\big(\theta_1 i_d(t) + \theta_2 i_d(t)^2\big) \tag{9e}$$

$$W(t) = \frac{1}{2}\sum_{\kappa=1}^{\kappa_s} \cos(\kappa_m \kappa t) \tag{9f}$$

$$d(k_d(t), W(t)) = \frac{1}{2}\left(\tau_3 + \frac{\tau_2 + W(t) - (1-\xi_d)k_d(t)}{\sqrt{(W(t) - (1-\xi_d)k_d(t))^2 + \tau_1}}\right)\left(1 - \frac{1}{1 + W(t)^\eta}\right) \tag{9g}$$

The variables and parameters are shown in Table 1 and in Table 2. We chose them based on existing literature and to replicate the stylized facts discussed in the introduction.

Table 1: Variables of the model and their units of measurement

Decision variable	Interpretation	Unit	init.value, function
c	Consumption	10^9 \$	
i_y	Investment in k_y	10^9 \$	
i_d	Increase in k_d after investment of $Q(i_d)$	m	
Endogenous variable	**Interpretation**	**Unit**	
Y	Output	10^9\$	
k_y	physical capital	10^9 \$	6.5, 5
k_d	Defence capital	m	2
d	Damage rate	1/year	
W_{eff}	Effective water level	m	$W(t) + \xi_d k_d(t)$
Q	Costs for defence capital	\$	$Q(\tau_0, \tau_1, \tau_2, i_d(t))$
Exogenous variable	**Interpretation**	**Unit**	
W	Water level	m	periodic

3. Results

3.1. Simulation

To gain a better understanding of the model dynamics we start with numerical simulations of the uncontrolled system where the dynamics of the control variables are exogenously given. Assuming perfect consumption smoothing, we postulate $c(t)$ to be constant over time. Investment into physical and defence capital, $i_y(t)$ and $i_d(t)$ are, therefore, functions of the exogenous consumption level and the aggregate economic output $Y(t)$. To determine the specific investment in either one of the capital stocks we propose two alternative settings.

Table 2: Parameters of the model and their units of measurement

Parameter	Interpretation	Unit	Base case	Case study
A	Technology	[ ]	2.3	
α	Output elasticity of physical capital	[ ]	0.3	
ρ	Time preference rate	1/year	0.07	
δ_y	Depreciation rate of econ. capital	1/year	0.1	
δ_d	Depreciation rate of defence capital	1/year	0.1	
κ_m	Frequency of floods	$1/(2\pi)$ /year	1	2
κ_s	Water level of floods	1/2 m	5	10
κ_d	Damage of defence capital relative to physical capital	[ ]	1	0.1
η	Increase in damage due to a higher water level	[ ]	2	
τ_1	Approximation parameter in the damage function	[ ]	0.001	
τ_2	Water peak approximation parameter	[ ]	0	
τ_3	Approximation parameter in the damage function	[ ]	1	
θ_0	Scaling parameter for dike heightening costs [3]	10^9\$/m	0.5	
θ_1	Weight for linear dike heightening costs	[ ]	0.5	
θ_2	Weight for quadratic dike heightening costs	[ ]	0.5	
ξ_d	Additional rise of the water level due to existing defence capital	[ ]	0.5	

θ_0 is calculated due to Slijkhuis et al. (1997) and Bedford et al. (2008)

We may keep the defence capital constant and therefore choose the investment $i_d(t)$ equal to the sum of the depreciation rate of the flood defence capital $\delta k_d(t)$ and the damage $d(W(t), k_d(t))k_d(t)$. The investment in physical capital $i_y(t)$ is then determined by the budget constraint (3). Alternatively, we assume that the total amount available for investments $Y(t) - c(t) = i(t) = i_y(t) + Q(i_d(t))$ is proportionally split between both investment options, i.e. for our simulations we assume $Q(i_d(t)) = 0.3i(t)$ and $i_y(t) = i(t) - Q(i_d(t)) = 0.7i(t)$.

Both cases are shown in the following Figs.4-6 where we plot the water level W as well as the effective water level $W_{eff}(W(t), k_d(t)) = W + \xi_d k_d$ and the dynamics of the state variables $k_y(t)$ and $k_d(t)$. The dynamics are qualitatively similar for both cases: Whenever a flooding hits (the effective water level $W_{eff}(t)$ is above the defence capital $k_d(t)$) damage occurs and reduces the total capital stock $k(t)$ and hence the growth rate of the economy.

We present results of our simulations for two different sets of initial values.

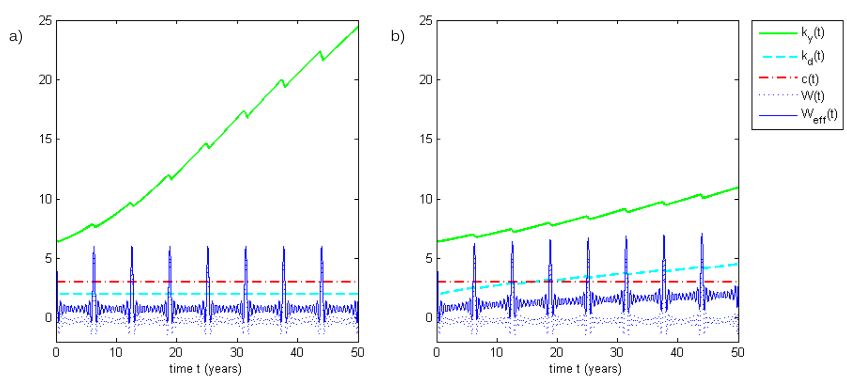

Figure 4: Simulation run of the physical capital $k_y(t)$, the defence capital $k_d(t)$, the consumption $c(t)$, the exogenous water level $W(t)$ and the endogenous effective water level $W_{eff}(t)$. a) Constant $k_d = 2$ with $k_y(t_0) = 6.5$ and b) proportional investments with $k_d(t_0) = 2$ and $k_y(t_0) = 6.5$ lead to economic growth. The unit of $k_y$and $c(t)$ is [$], all the other variables are given in [m].

Higher initial capital stocks ($k_y(t_0) = 6.5$ and $k_d(t_0) = 2$) enable the economy to grow (see Fig.4). Moreover, keeping the amount of defence capital constant (Fig.4 a)) allows even faster growth compared to ever increasing amounts of investment in defence capital (Fig.4 b)) .

A small change in the initial capital stocks can make a significant difference in

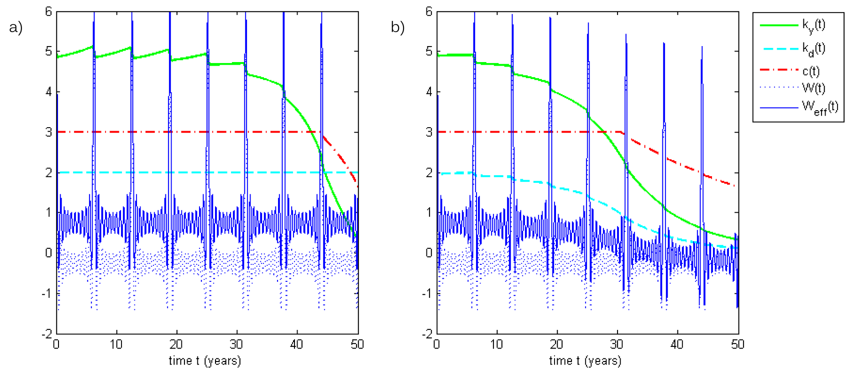

Figure 5: Simulation run of the physical capital $k_y(t)$, the defence capital $k_d(t)$, the consumption $c(t)$, the exogenous water level $W(t)$ and the endogenous effective water level $W_{eff}(t)$. a) Constant $k_d = 2$ with $k_y(t_0) = 5$ and b) proportional investments with $k_d(t_0) = 2$ and $k_y(t_0) = 5$ run into economic desaster.

the long term behaviour of the capital stocks and hence on economic growth. If the economy does not have enough physical capital in terms of infrastructure, machines and buildings to produce economic output, it cannot withstand floods and economic growth will decline in the long run. If the society still tries to keep the level of the defence capital constant (see Fig.5 a)) they even have to invest such a large part of their output in defence capital that their physical capital depreciates and the economy crashes. The situation is not as severe in case two (see Fig.5 b)) where an economy invests in defence capital proportional to the existing capital stock. However, also in this case, the economy will shrink in the long run. In order to avoid such a doomsday scenario when initial capital stocks are too low, an alternative is to reduce the amount of investment. For instance, if $Q(i_d(t))$ is only 25% instead of 30% of the total investments, economic growth is sustainable even for low levels of initial capital stocks (see Fig.6).

Overall, our simulations indicate that constant levels of decision variables that do not adapt to the state of the economy, may in the long run lead to a collapse

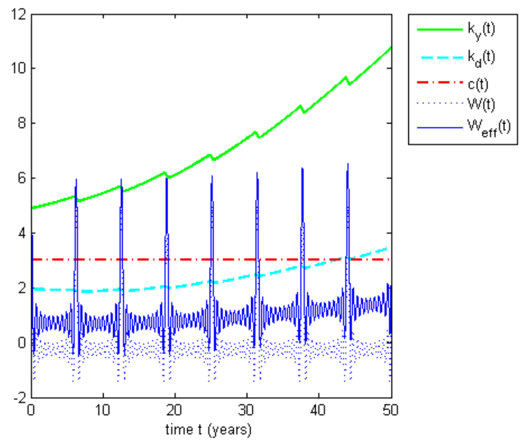

Figure 6: Simulation run where the initial values $k_d(t_0) = 2$ and $k_y(t_0) = 5$ are enough to enable economic growth if the investment in defence capital is only 25% of the total investment.

of the economy. We therefore need to consider dynamic decision rules that react to the state of the model. Dynamic optimization methods are the tools to implement these dynamic decision rules.

3.2. Dynamic Optimization

Given the dynamics of the capital stocks, the exogenous water function, and the functional forms of the damage function and aggregate economic output, the social planner maximizes the discounted flow of utility by choosing the optimal consumption and the optimal amount of investments into defence capital. Since the exogenous function of the water level is periodic, the optimal decisions on consumption and investment will also follow a periodic time path.

3.2.1. Optimal consumption and investment decisions

Before we present detailed analytical and numerical results of the model we give an intuitive explanation of the dynamics of the model. Total aggregate output of the economy is consumed or reinvested into either one of the capital stocks (see equation (3)). Applying optimal control theory (Appendix A), we derive the optimal dynamics of consumption and investment decisions:

$$\begin{aligned}
\dot{c}(t) &= c(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] && (10)\\
\dot{i_d}(t) &= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}\big[A\alpha k_y(t)^{\alpha-1} + (\kappa_d - 1)d(k_d(t), W(t)) \\
&\quad + \kappa_d d'(k_d(t), W(t))k_d + \delta_d - \delta_y\big] \\
&\quad + \frac{1}{2\theta_0\theta_2}[d'(k_d(t), W(t))k_y] && (11)
\end{aligned}$$

Both, the consumption path and the investment path, depend on the exogenous periodic function $W(t)$ and consequently, they will be periodic as well. Note that $W(t)$ also indirectly influences the dynamics because both capital stocks are a function of $W(t)$.

The consumption dynamics are the same as in the standard Ramsey model with a social planner (Ramsey (1928)). A higher marginal product of physical capital (as given by the first derivative of the production function with respect to physical capital) as well as a lower rate of capital depreciation and time preference will positively affect the consumption growth rate. Damage acts like an additional depreciation on the marginal product of physical capital.

The dynamics of the investment in flood defence capital are more complex. The marginal product of physical capital and a lower rate of depreciation of physical capital positively influence the investment rate $\dot{i_d}(t)$, whereas a low rate of depreciation of the defence capital will reduce the optimal investment rate in flood defence capital because less investment is necessary to sustain the defence capital. Moreover, since the factor $(\kappa_d - 1)$ is nonpositive, when damage occurs, investments in defence capital decreases. The latter effect can be explained by the assumption that, in case of $\kappa_d > 1$, the damage to defence capital is more severe than the damage to physical capital. Consequently, investment in defence capital will be reduced. In case the damage rate for both types of capital is the same ($\kappa_d = 1$), damage does not directly influence the investment behaviour. However, the first derivative of damage with respect to the defence capital is zero or close to zero, so neither of the terms affect the investment dynamics. In general, all investment decisions are scaled by the cost parameters θ_0, θ_1 and

θ_2. Lower costs enable higher investments.

3.2.2. Optimal long term capital stocks

Our results indicate that any optimal path of consumption and investment that the social planner decides on will end up in one of two possible long run solutions/limit sets (see Appendix B) depending on the initial conditions. Note, that mathematical limit sets are different from an economic equilibrium which denotes a situation where all markets clear. We name the inner equilibrium which has high capital stocks and therefore high economic output the *rich economy* and the boundary equilibrium which only sustains a comparatively small physical capital stock and no defence capital *poor economy*. This notation will become apparent when we consider the long run economic state of the economy in each case.

To identify both equilibria we solved the optimization problem first analytically using the Pontryagin maximum principle (Pontryagin, 1962) and then numerically using the specific MATLAB® -Toolbox *OCMat* from Grass and Seidl (2013) and the parameter values given in Table 2.

The *rich economy* (Fig.7 a)) invests just enough into flood defense capital to avoid floodings and consequently flood damage. Consequently, the effective water level $W_{eff}(t)$ increases due to the levee effect. Even though the social planner never stops investing into flood risk prevention measures ($i_d(t) > 0$) in the long term, they lower the investments when they are not urgent and rather invest in physical capital $k_y(t)$ to increase the economic output $Y(t)$. In such an economy, the aggregate output is quite high and therefore a constant consumption path is sustainable. These so called smooth consumption paths are characteristic of developed economies (Friedman, 1956) and are also commonly shown to be consistent with economic growth (Acemolu, 2009).

In contrast, *poor economies* (Fig.7 b)) do not invest at all in defence capital. Mathematically they move to a boundary periodic solution with $i_d(t) = 0$.

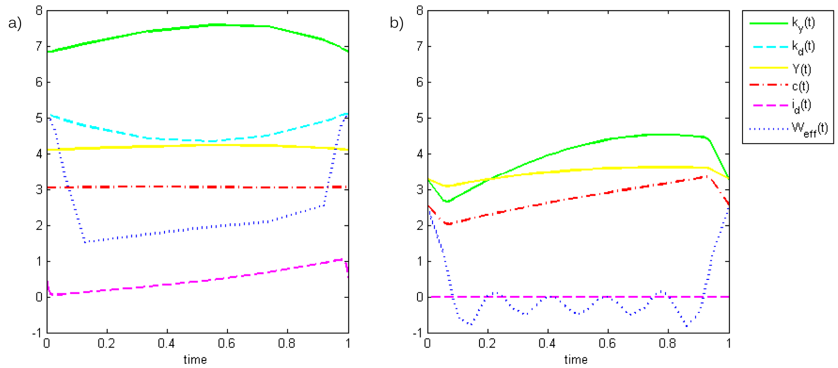

Figure 7: One limit cycle (in normalized time) of the long-term behaviour of a a) rich economy, b) poor economy showing the time series of the physical capital $k_y(t)$, the defence capital $k_d(t)$, the economic output $Y(t)$, the consumption $c(t)$, the investment in defence capital $i_d(t)$ and the exogenous effective water level $W_{eff}(t) = W(t) + \zeta_d k_d(t)$.

Without any investments $i_d(t)$ the defence capital $k_d(t)$ remains zero (and so the effective water W_{eff} level equals the exogenous water level W). Consequently, the society is vulnerable and every time a high water level occurs, flooding hits the economy. The physical capital stock $k_y(t)$ decreases and less economic output $Y(t)$ is produced. Interestingly, the social planner already anticipates the damage shortly before a flood hits and prefers to distribute the output to consumption rather than investment in physical capital. Therefore consumption $c(t)$ strictly increases until a flood hits and less consumption is possible during a flooding event. It takes time to recover and to reach the old consumption level again.

It is useful to highlight the optimal investment strategy for the rich economy: The investments in flood defence capital are always positive and increase before a flood hits. In reality, societies tend to invest in flood defence infrastructure only after big flooding events have occurred. An example is the Danube flood of 1954 which resulted in construction of a flood relief channel in Vienna. Decision processes to invest in flood defence management are mostly based on political decisions and financial considerations and only effective if stakeholders have an immediate memory of past flooding. However, the optimization model shows that investing in flood defence capital before floods would be economically more advisable. Of course we cannot forecast floods, but investing also in times of no flood instead of reacting after flood occurrence is shown to be optimal.

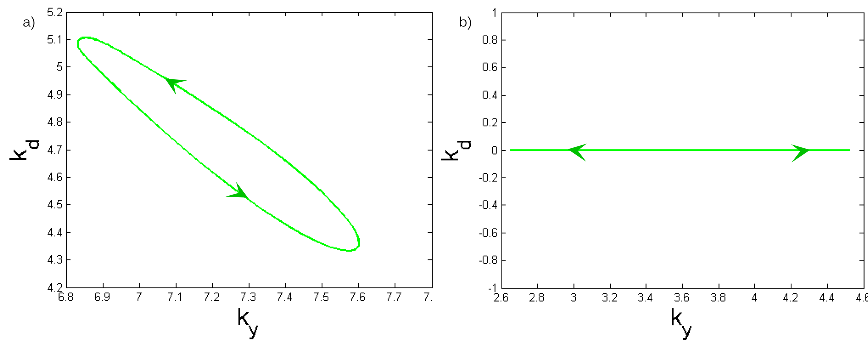

Figure 8: The state dynamics of the a) rich economy, b) poor economy.

The long-term state dynamics of the capital stocks $k_y(t)$ and $k_d(t)$ clearly identify the limit cycle. Note that the cycling is counterclockwise. For the rich economy (Fig.8 a)) we see a negative correlation of the capital stocks: Since the social planner wants to keep consumption smooth, increasing investments in one capital stock lowers the investments in the other capital stock. Moreover, a lower physical capital stock yields less output. This allows less investments and therefore a lower total capital stock. This is always the case after high water levels, when the priority is to build up defence capital. Hence, floods do not only affect the economy directly via damage, but also indirectly through a lower level of output and therefore lower capital stocks.

The limit cycle for the poor economy (Fig.8 b)) is trivial. Since there is no defence capital $k_d(t)$, the physical capital basically increases after a flooding, reaches its maximum slightly before a flooding due to the anticipation effect and decreases quickly when a flood hits the economy.

So far we have studied the long-term behaviour along the limit cycles. It is also important to understand the path towards one of the two limit cycles. Depending on the initial values of the capital stocks k_y and k_d the economy

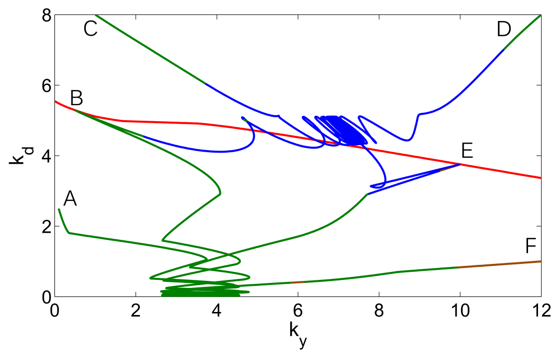

Figure 9: Different initial conditions (points A – F) in several economies induce to different long-term behaviour. The red line (Skiba curve) separates the initial values leading to a rich or to a poor economy. k_y and k_d are the physical capital and the defense capital, respectively.

follows a path to one of the limit cycles, separated by the so called Skiba curve (red line in Fig.9). Starting (slightly) above or below the Skiba curve will lead to a rich economy or poor economy, respectively.

Interestingly, due to the non-autonomous water function, the Skiba curve shifts depending on the starting time relativ to the next flooding event. An economy that e.g. starts slightly below point B but at the same starting time implying that the time it takes to the next flooding has not changed, would converge to a poor economy. However, if in such a situation (i.e. when we start at a point below B) the time to the next flooding would increase as well, the economy would converge to a rich economy.

So it is not only important where the economy starts, but also when the next flood is happening. This allows the paths towards the long term limit cycle to be temporary below the Skiba curve after the starting time.

For the base case where we set the parameters according to Table 2 we choose the set of the starting points A-F and show the different paths in Fig.9. Different colours represent different investment combinations. I.e. along the blue line the economy invests in both capital stocks and also consumes, the green line indicates that the economy is not investing in flood defence, but still in physical capital, and the brown lines at starting points E and F display that

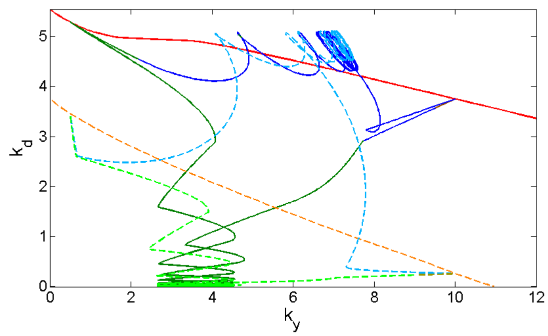

Figure 10: Lower costs of investment in defense capital ($\theta_1 = 0.25$, dashed lines) shift the Skiba curve (orange dashed line instead of red solid line) and enlarge the region where economies become a rich economy compared to the base case ($\theta_1 = 0.5$, solid lines). k_y and k_d are the physical capital and the defense capital, respectively.

the economy consumes all the produced output without investing in any of the capital stocks.

Economies A, B, C with less physical capital first try to build up physical capital. Economies starting at A or slightly below B do not afford to invest in flood defense and it is optimal to prefer consumption over flood defense. Economies starting at C or slightly above B already have enough defense capital and so it is optimal for them to sustain it. In contrast, if we start with a much higher defence capital at point D, which does not bring any extra benefit compared to the long-term level, investments in defence capital are stopped immediately and the defence capital stock depreciates, while investments in economic capital are slightly positive. The main part of the output is consumed directly, unless the defence capital stock has reached the level where it may be too small to prevent damage from floods. So, even if the community could afford more capital, they prefer to only invest as much as necessary to avoid floodings and rather consume the output right away.

Economies starting close to point E, with a lot of physical capital, but slightly too less flood defense capital, are living on the edge. If they always invest at least a small amount in flood defence they manage to turn into a rich economy, whereas choosing to only consume their economic output in the beginning leads to a poor economy. However, it is still optimal to invest at some time into flood defense capital to lower the flood damage, but below a certain level of defense capital it is optimal to not invest in it anymore and consume more. Even if the economy is very rich in terms of physical capital but does not have enough knowledge and flood defence to build on (point F), it will not invest in flood defense and rather consume all the economic output. Because it knows that the next flood will destroy a major part of their capital anyways. It starts investing in physical capital when the additional amount of output pays off the damage. The costs of investment in defense capital are crucial. Fig.10 shows, that decreasing the costs shifts the Skiba curve and significantly enlarges the region where economies develop into a rich economy. I.e. an economy starting with initial values between the red and the orange line would choose the optimal investment given low costs ($\theta_1 = 0.25$) or high costs ($\theta_1 = 0.5$) to end up as a rich or poor economy, respectively.

3.2.3. Higher frequency and higher intensity of floods changes the investment behaviour

So far we have studied the dynamics of the model under one specific set of parameters. We next investigate how the optimal decisions of the social planner will change when she faces a different environment, e.g., a different occurrence of high level water events. We study two cases: First, we assume a higher frequency of floods, and secondly we assume higher water levels which can lead to more pronounced floodings.

Doubling the frequency of high level water events ($\kappa_m = 2$) naturally leads to a smaller time period of the limit cycle. Fig.11 displays less variation in the dynamics of the state and control variables than in the base case. Intuitively, we would expect that a doubling of the flood frequency would translate into a 50%-reduction in the variations of the levels of the state and control variables since the time to accumulate capital without being hit by a flood is only half.

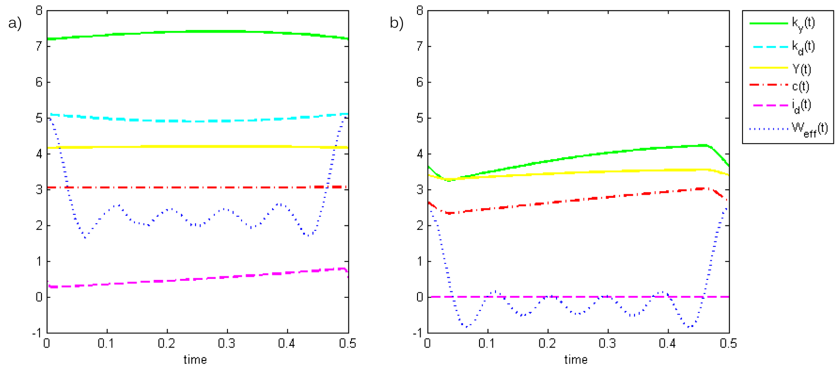

Figure 11: One limit cycle in case of a higher frequency of floods and therefore time period [0,0.5] for a) a rich economy, b) a poor economy. Parameters as in base case of Table 2, but $\kappa_m = 2$. Note, since we plot only one period and the frequency of the periods changed, the time interval is now only [0, 0.5].

However, this is only true for poor economies. For rich economies, the difference between the highest and lowest level of the capital stock along the limit cycle is not even a third in case of double flood frequency. Even more counterintuitive is the finding that a rich economy facing a higher frequency of high water levels manages to have the same consumption rate and even higher capital stocks on average as compared to the case with lower frequencies of high water levels. Both the defence and the physical capital stock are higher on average than in the base case. So only very rich economies manage to stay rich when they are facing higher flood frequencies.

Poor economies suffer from higher flood frequencies. Since more floods lead to shorter flood durations, the damage is not as high, but occurs more often. Not only is the range of the values of the capital stocks smaller than in the base case, also the range of the consumption level is halved. Moreover, on average poor economies facing more floodings consume less and have less economic output.

For the second case we vary the amplitude of the floods ($\kappa_s = 10$) and show the results in Fig.12. In order to protect against higher water levels, rich

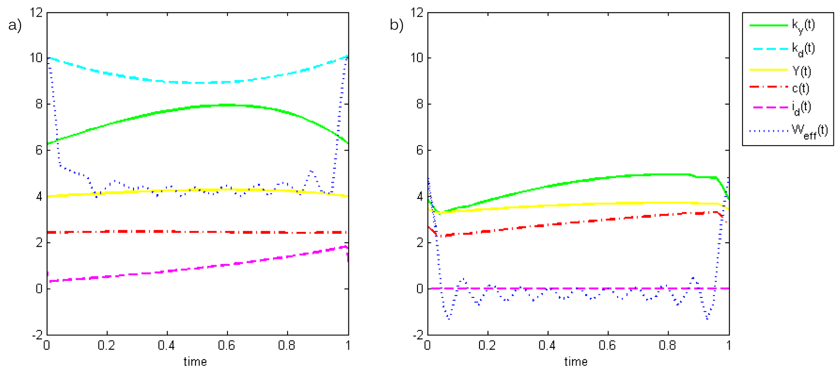

Figure 12: One limit cycle in case of bigger floods for a) a rich economy, b) a poor economy. Parameters as in base case of Table 2, but $\kappa_s = 10$.

economies will start to invest in defence capital earlier and to a larger extent. Consequently, less economic output is left to invest in physical capital or for consumption. Rich economies can consume 20% less than rich economies in the base case scenario. This is the only chance they can keep the physical capital almost at the same level and therefore produce a critical amount of economic output.

Surprisingly, poor economies converge in the case of more pronounced floods to an economic state with higher capital stocks and higher consumption levels compared to the base case scenario. Although floodings hit harder, each flood is shorter which results in a wealthier economy.

When we compare rich and poor economies in case of more pronounced floods, the capital stocks are much higher for rich economies, so they seem to be wealthier. However, consumption and therefore the average utility along one limit cycle of the society is 17% higher for poor economies. This means that poor communities in heavily flooded areas should actually not invest in defence capital but rather invest in physical capital, thereby increasing output and allowing for higher consumption levels, even though they have to give up a smooth consumption path.

The results depend on the parameters and the characteristics of the damage function.

3.2.4. *Less damage in the defence capital stock influences the dynamics of the capital stocks*

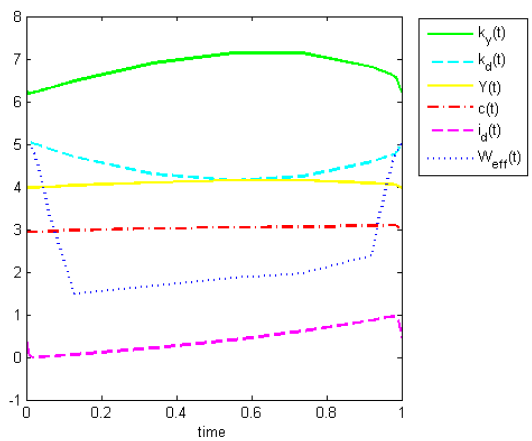

Figure 13: One limit cycle in case of a more robust flood defence capital. Parameters as in base case of Table 2, but $\kappa_d = 0.1$. Note, due to numerical discretization of the solution W_{eff} is displayed different, but has the same oscillating behaviour as in the other figures.

Fig.13 shows a case where the defence capital is not as vulnerable as the physical capital ($\kappa_d = 0.1$). For this case we only need to analyze rich economies, since poor economies do not even have defence capital and therefore defence capital cannot be damaged. Fig.13 shows very similar patterns to the base case. It appears that the floods do not destroy defence capital as heavily as physical capital. Assuming an equilibrium without any damage would simply look like the base case scenario. Since the social planner knows that damage does not affect the defence capital very much, she chooses a lower investment in defence capital than in the base case and therefore allows small flooding events for a very short time, where both capital stocks are damaged. As a consequence, the economic output is slightly lower, but the consumption increases in the time in-between the flooding events. This suggests that, in this model, people do care more about the defence capital, if it is more vulnerable.

3.3. Simulation with stochastic flooding events

The analysis performed so far does not account for the stochasticity of floods. This has been done to obtain analytically results on the long term optimal behavior of the systems. In this section we investigate how these results change if floods occur stochastically, i.e., when the social planner has no complete knowledge of the future flood occurrences and magnitudes. We present simulations of our model assuming a stochastic water level function like in Viglione et al. (2014). The timing of the high water level events is exponentially distributed, as a result of a Poisson process with mean t and arrival rate 0.2 per year, and the height of the water levels is modeled with a generalized Pareto distribution with mean 1 (see Viglione et al. (2014), Section 2.1, for details).

Within such a simulation exercise we compare the two policies we derived in Section 3.2. We assume that an economy consumes 80% of its economic output in both scenarios. A rich economy invests in flood defense capital $i_d > 0$ proportional to the output after consumption and possible damage. If the defense capital is high enough to prevent flood damage they only maintain it and do not invest further. A poor economy splits the output after damage proportional into consumption and investment in physical capital, but does not invest in flood defense capital $i_d = 0$. The obtained scenarios are listed in Table 3 together with the different initial capital stocks. To compare the various simulation runs we record the mean and the variance of the present value (discount rate $\delta = 0.07$) of future utility streams $U_0(T)$ for each simulation scenario choosing the simulation run time $T = 750$ years.

For the stochastic simulation runs displayed in Fig.14 we used the same high water level event series. In the long term the stochastic simulations are comparable to the optimal limit cycles. Economies investing in $k_d(t)$, we again refer to them as rich economies, end up in an almost constant state whereas economies without defense capital (poor economies) fluctuate depending on floods.

The initial conditions do not change the long term behavior in the simulation.[4]

[4]The dynamics with higher initial capital stocks would look similar to those in Fig.14.

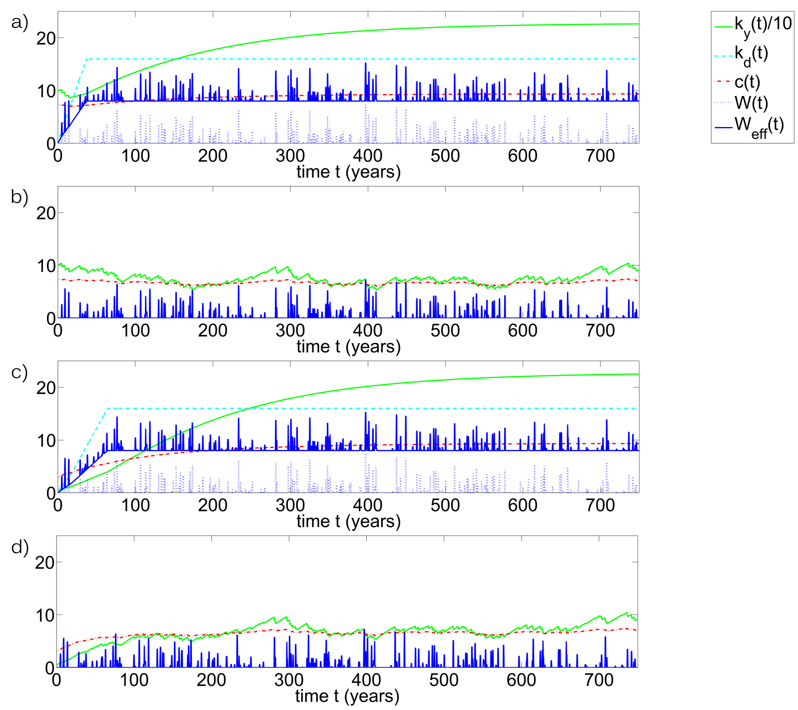

Figure 14: Simulation runs of the physical capital $k_y(t)$, the defence capital $k_d(t)$, the consumption $c(t)$, the exogenous water level $W(t)$ and the endogenous effective water level $W_{eff}(t)$. The unit of $k_y(t)$ and $c(t)$ is [\$], all the other variables are given in [m]. a) and c) show the szenario of the rich economy and b) and d) the poor economy. The initial conditions for a) and b) are $k_y(t_0) = 100$ and $k_d(t_0) = 0$, for c) and d) $k_y(t_0) = 5$ and $k_d(t_0) = 0$

$k_y(t_0)$	$k_d(t_0)$	$i_d(t)$	mean($U_0(T)$)	var($U_0(T)$)	Fig.
100	0	> 0	28.4	0.06	14 a)
100	0	0	27.2	0.14	14 b)
100	6	> 0	28.6	0.01	
100	6	0	28.5	0.05	
5	0	> 0	18.7	0.03	14 c)
5	0	0	19.5	0.04	14 d)
5	6	> 0	18.9	0.01	
5	6	0	19.7	0.04	

Table 3: Stochastic simulation runs for different initial levels of physical capital $k_y(t_0)$ and defense capital $k_d(t_0)$ and two different policies (investing in defense capital ($i_d(t) > 0$) versus not investing in defense capital ($i_d(t) = 0$)) tracking the mean and the variance of the present value of future utility streams $U_0(T)$.

However, the present value of future utility streams $U_0(T)$ increases for larger initial capital stocks. Nevertheless, floods will cause more damage if the physical capital stock is high as indicated by the dip of the capital stock in the beginning of the simulations displayed in Fig.14 a).

The simulations with existing initial defense capital are qualitatively similar to the simulations in Fig.14 a)-d), however these eocnomies reach a higher utility since floodings are avoided in the early years which are discounted less.

In the simulation runs summarized in Table 3 economies do not optimally decide on their investment and consumption, nevertheless, we can compare the discounted stream of utility across the different scenarios. If the initial capital is high, the net present utility value is higher for rich economies, whereas with a low initial capital poor economies are better of in terms of consumption. This coincides with the optimal solution of Section 3.2.

Moreover, the variance indicates the different values in the simulation runs based on different flood time series. Poor economies are more sensitive to the flood time series compared to rich economies.

4. Discussion

In this paper we studied a socio-hydrological model of high water level events potentially causing floodings in an economic decision framework. In the model, a social planner, representing the society, decides how to optimally distribute the economic output between consumption, investment in flood defence capital and investment into physical capital. We apply our model to understand the mechanisms between floods and economic growth if the water level follows a specific exogenous fixed water level function that is time varying. Investments in flood defence capital do not only avoid direct damage in the future, but also safe opportunity costs for reconstruction. This allows investments in physical capital and consequently more economic growth in the future (Hochrainer-Stigler et al., 2013).

We applied dynamic optimization methods to determine the long run optimal solution of our system. Depending on the initial capital stocks of the economy, our system either converges to a rich or a poor economy in the long term. In order to compare the model results to real world data we use macro-economic data for countries, whereas we are aware that usually only parts of a country are under flood risk. So whenever we discuss rich or poor economies, we refer to broader regions or countries that are (partly) affected by floods.

The rich economy manages to build up defence capital to avoid damage and therefore follows a smooth consumption path. The consumption rate of 70% (Fig.7 a)) equals e.g. the rate in the US. [5]. Poor economies, characterized by low levels of initial economic output or initial defence capital, optimally decide not to invest into defence capital and end up with lower capital stocks and lower consumption rates. Every time a flooding hits, physical capital is damaged and consumption decreases strongly. The average consumption rate

[5] http://data.worldbank.org/indicator/NE.CON.PETC.ZS assessed on June 3rd, 2015

of poor economies is higher than 80% of their total output, which is around the rate of third world countries such as Cambodia and Kenya [6]

If defence capital such as levees is built, the water level may increase due to the loss of retention volume (Di Baldassarre et al., 2009; Remo et al., 2012; Heine and Pinter, 2012). Also vulnerability may increase because of the levee effect (Montz and Tobin, 2008; Ludy and Kondolf, 2012). However, economic output and consequently consumption and capital stocks are higher since flood damage can be prevented. If the severity of floods is very high we showed that a rich economy investing in defence capital may end up with consuming less out of the total output compared to a poor economy which does not invest in defence capital. Our results are in line with actual observations. For example, the Netherlands are facing severe floods and invest a lot in their flood management systems (Silva et al., 2004; Eijgenraam et al., 2014). The consumption rate of around 50% in this scenario in our model fits the low consumption rate of the Netherlands.[7] The Netherlands have a higher output and the total per capita consumption is higher than in the mentioned third world countries.

Whether an economy is rich or poor depends very much on its economic capabilities including physical capital of firms and governments, infrastructure and technology, but also on existing flood defence capital. If any one of these components is too small, the economy will never have the strength to become a rich economy. It will stop investing in defence capital because it is not worth the opportunity costs of missed consumption. In reality there is always some investment in flood defense since people want to avoid death or very strong flood impacts to human life. Since this is hard to be displayed in economic values we did not explicitly include it in the model. But assuming a minimum investment in defense capital would not change the results qualitatively[8].

We see this scenario in many poor countries: Without any external help, regions

[6]http://data.worldbank.org/indicator/NE.CON.PETC.ZS assessed on June 3rd, 2015

[7]http://data.worldbank.org/indicator/NE.CON.PETC.ZS assessed on June 3rd, 2015

[8]Introducing a minimum investment in defense capital would only be a small linear transformation in investment and consequently in consumption and aggregate utility.

such as the Mekong floodplains are flooded regularly and the locals are used to the damage (http://www.mrcmekong.org/). Kahn (2005) also found that rich nations suffer less from natural disasters than poor countries. Higher developed economies invest more in prevention of natural disasters and the total losses after a disaster are smaller (Schumacher and Strobl, 2011).

How is it possible to escape the trap into a poor economy? Since environmental conditions cannot be changed easily, only different economic environments can induce a difference. It is essential to invest into physical capital to bring the economy on a path to the equilibrium of the rich economy. If the country cannot afford this by itself, external help is necessary. This help does not only include capital investment but also ensuring strong institutions to accordingly distribute the investments.

As soon as the economy is on the path towards the long term state of a rich economy, our model predicts that it will never revert to a poor economy given the same environmental and economic conditions. Staying rich when the economy is already there does not require any help from outside anymore. This is the case if no surprise will occur (see e.g., Merz et al. (2015)).

In fact, the timing of the expected flooding event plays a crucial role. If a flood is not expected in the near future the optimal behaviour is to invest less in flood defence capital and therefor taking the risk of ending up as a poor economy. This effect is stronger if the costs for flood defense capital are higher.

Fig.15 summarizes the scenarios of this paper. Each scenario is represented in a different colour and we plot the case of a rich and a poor economy for each scenario. The amount of physical capital of the rich economies is quite similar in every scenario. Naturally, the range differs from scenario to scenario: In case of more floods we observe a lower variation of physical capital while the level of both capital stocks is higher compared to the base case.

In the scenario where we increase the severity of floods, the defence capital has to be very high in order for the economy to remain rich. So it is very hard to obtain such a rich economy and the willingness to invest in flood defence capital

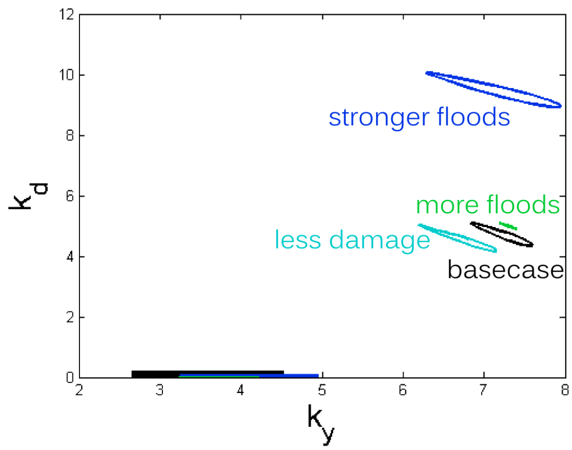

Figure 15: Long-term state dynamics for the cases of Figs. 7-13.

has to be very high, too. We only encounter this case in first world countries that are highly affected by floods such as the Netherlands. This is very much confronted with floods, can afford defence capital, and is willing to invest in it (Vis et al., 2003).

In the scenario of less damage people are minimalists and only invest in their capital stocks as much as necessary to overcome floods. As a consequence, their capital stocks are lower than in any other scenario. Their consumption is just as high as in the base case, but not as smooth since it decreases during flooding events. The consumption cycle in this scenario has similar dynamics as the poor equilibria of the other cases.

In case of poor economies, flood intensity and frequency directly impact the wealth of the economy. More floods more often cause damage of existing physical capital, but the economies have experience with floodings and rebuild the infrastructure quickly. In contrast, if bigger floods happen less frequently, the damage is much higher and the poor economies need longer and also have to invest more into physical capital to regenerate. In total, the consumption is higher than in the scenario with fewer floods. So even if floods hit harder, as long as they do not appear too often, the living standard can be relatively high in between floods.

Overall, the economic output is almost equal for all rich economies independently of the frequency and intensity of floods. Only the amount of defence capital and the variations of physical capital along the long run economic state differs. Furthermore, the economic output in poor economies is much lower than for rich economies, but it is about the same level for any poor economy in various cases.

Besides the higher economic output and the mostly higher consumption for rich economies, they do have the capacities and resources to anticipate damages before a flood hits. On the other hand, poor economies do not have the economic potential and are therefore not flexible to adjust to floods beforehand. The only anticipation is to stop investing into physical capital shortly before a flooding[9], but basically poor economies are affected by floods every time they occur and have to start over again rebuilding capital stocks and increasing consumption.

Optimization is important to use the resources efficiently. The simulation in Section 3.1 shows the dynamics of the model. Even in case of positive economic growth, damage occurs during every high water level event, whereas in the optimization model rich economies can avoid damage in the long run, even though they are investing less, but at the right time. Moreover, in the scenarios with declining economic growth the economies even converge to zero capital stocks. In the optimization case it will never happen that people invest in flood defence capital if they cannot even afford their basic needs for living. They therefore always manage to sustain some physical capital and to have enough resources to consume and invest again in production after a flooding event.

[9]This is true for our model assumptions. In reality the timing of floods is not known in advance and only last minute protections can be build.

Also when we look at simulation runs with stochastic high water level events the present value of utility is larger for rich economies if the initial capital stocks are sufficiently high. This reflects the optimal path towards the limit cycle for rich economies in our dynamic optimization set up. Contrary, the present value of the future utility streams is smaller for economies investing in defense capital than for economies which do not build up a defense capital stock if the initial capital stock is low. This scenario reflects all the paths going towards the limit cycle of the poor economy in our dynamic optimization set up, where the strategy to not invest in defense capital is optimal. Apparently, if the economy starts with low initial capital stocks it will not pay off to invest in flood defense capital and this is the incentive to remain poor and vulnerable. As we have also seen in the optimization, economies do not manage to escape that poor scenario with their own strength, but need external help to do so.

To sum up, if a social planner would base his decision on the present value of future utility given uncertainty of flood events, he would still choose the same policy as in the long term optimization based on a deterministic water level function. For instance, flood frequency analysis is used in hydrology to estimate the expected frequency of exceedence of flood levels for a given time horizon (see, among many, Gumbel (1941, 1958); Chow et al. (1988)). In principle our optimization model with the deterministic exogenous water level function based on this expected parameter values can help identifying optimal investment strategies for the long run. Of course, because of the stochasticity of flooding, sensitivity analysis need to be performed for these optimal scenarios, in order to assess their robustness (Blöschl et al., 2013).

Comparing the results in our paper with the simulation model of Di Baldassarre et al. (2013) and Viglione et al. (2014), on which our model set up is based, we may highlight further important differences: First, they found that, in certain circumstances, investing in flood defence capital may lead to less economic growth than facing frequent small floodings. This is because rare floodings may be catastrophic since societies erroneously consider floodplains more secure after building levees and invest in building and living there. In our optimization model, in which the social planner has the knowledge of flood occurrence and magnitude, rich economies can manage floods and therefore avoid catastrophic floodings.

Second, a lower decay of levees leads to higher growth rates in Viglione et al. (2014). In contrast, in our model the social planner decides to invest just a minimum into flood management and physical capital to consume more than in the scenario with a higher depreciation rate.

Our approach is to conceptualize the interaction of human decision making and flood risk management within a macro-economic framework. Our aim is to understand the mechanisms rather than matching specific cases or predicting the future development of societies. As models cannot and should not capture all details of the reality, we do not claim that this is the only true representation of communities in flood risk areas. However, it enables us to discuss certain dynamics and policies in the field of socio-hydrology.

Starting from the results in this paper, future work will focus on the sensitivity of the model results to the assumptions made, and on the assumption of perfect knowledge of future water levels by the social planner. We expect that, even though uncertainty/stochasticity of natural events will result in more complex dynamics, the results of this work will provide the fundamental baseline over which other mechanism will show up.

5. Acknowledgements

The authors would like to acknowledge financial support from the Austrian Science Funds (FWF) as part of the Vienna Doctoral Programme on Water Resource Systems (DK-plus W1219-N22) and thank their colleagues within the doctoral programme for continuing support and the discussions within the cluster meetings. We also thank four anonymous referees whose comments and suggestions have essentially improved the exposition of the paper.

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Appendix A. Dynamics of the optimal controls

We are analyzing the model analogous to Barro and Sala-i Martin (2004) and Millner and Dietz (2015).

Appendix A.1. The Hamiltonian

To analytically optimize the model given in equations (9) we formulate the Hamiltonian function.

$$\begin{aligned}&\mathcal{H}(c(t), i_d(t), \mu_y(t), \mu_d(t)) \\ &= U(c(t)) + \mu_y(t)[Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y - \delta_y k_y(t)] \\ &+\mu_d[i_d(t) - \kappa_d d(k_d(t), W(t))k_d - \delta_d k_d(t)]\end{aligned} \tag{A.1}$$

The Pontryagin conditions are

$$\frac{\partial \mathcal{H}}{\partial c(t)} = U'(c(t)) + \mu_y(t)[-1] = 0 \tag{A.2a}$$

$$\frac{\partial \mathcal{H}}{\partial i_d(t)} = \mu_y(t)[-Q'(i_d(t))] + \mu_d = 0 \tag{A.2b}$$

$$\frac{\partial \mathcal{H}}{\partial k_y(t)} = \mu_y(t)[A(t)\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y] = \rho\mu_y(t) - \dot{\mu}_y(t) \tag{A.2c}$$

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial k_d(t)} &= \mu_y(t)[-d'(k_d(t), W(t))k_y] + \mu_d[-\kappa_d d'(k_d(t), W(t))k_d - \kappa_d d(k_d(t), W(t)) - \delta_d] \\ &= \rho\mu_d(t) - \dot{\mu}_d(t)\end{aligned} \tag{A.2d}$$

$$\frac{\partial \mathcal{H}}{\partial \mu_y(t)} = Ak_y(t)^\alpha - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y - \delta_y k_y(t) = \dot{k}_y(t) \tag{A.2e}$$

$$\frac{\partial \mathcal{H}}{\partial \mu_d(t)} = i_d(t) - \kappa_d d(k_d(t), W(t))k_d - \delta_d k_d(t) = \dot{k}_d(t). \tag{A.2f}$$

Appendix A.2. The canonical system

We rewrite the first order condition (A.2a), use the ln and take the total time derivative.

$$\mu_y(t) = U'(c(t)) = \frac{1}{c(t)} \tag{A.3}$$

$$\ln(\mu_y(t)) = \ln(\frac{1}{c(t)}) \tag{A.4}$$

$$\frac{\dot{\mu}_y(t)}{\mu_y(t)} = -\frac{\dot{c}(t)}{c(t)} \tag{A.5}$$

Analogous we can use the first order condition (A.2b).

$$\begin{aligned} \mu_d(t) &= \mu_y(t)[Q'(i_d(t))] = \mu_y(t)\theta_0[\theta_1 + 2\theta_2 i_d(t)] && \text{(A.6)} \\ \ln(\mu_d(t)) &= \ln(\mu_y(t)\theta_0[\theta_1 + 2\theta_2 i_d(t)]) \\ &= \ln(\mu_y(t)) + \ln(\theta_0) + \ln(\theta_1 + 2\theta_2 i_d(t)) && \text{(A.7)} \\ \frac{\dot{\mu}_d(t)}{\mu_d(t)} &= \frac{\dot{\mu}_y(t)}{\mu_y(t)} + \frac{2\theta_2 \dot{i}_d(t)}{\theta_1 + 2\theta_2 i_d(t)} && \text{(A.8)} \end{aligned}$$

So we use (A.5),(A.8),(A.2c), (A.2d),(A.2e), and (A.2f) to write the canonical system.

$$\begin{aligned} \dot{c}(t) &= -c(t)\frac{\dot{\mu}_y(t)}{\mu_y(t)} && \text{(A.9a)} \\ \dot{i}_d(t) &= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}\left[\frac{\dot{\mu}_d(t)}{\mu_d(t)} - \frac{\dot{\mu}_y(t)}{\mu_y(t)}\right] && \text{(A.9b)} \\ \dot{\mu}_y(t) &= -\mu_y(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] && \text{(A.9c)} \\ \dot{\mu}_d(t) &= \mu_y(t)[d'(k_d(t), W(t))k_y] \\ &\quad +\mu_d[\kappa_d d'(k_d(t), W(t))k_d + \kappa_d d(k_d(t), W(t)) + \delta_d + \rho] && \text{(A.9d)} \\ \dot{k}_y(t) &= Ak_y(t)^{\alpha} - c(t) - Q(i_d(t)) - d(k_d(t), W(t))k_y - \delta_y k_y(t) && \text{(A.9e)} \\ \dot{k}_d(t) &= i_d(t) - \kappa_d d(k_d(t), W(t))k_d - \delta_d k_d(t) && \text{(A.9f)} \end{aligned}$$

Appendix A.3. Euler equations for optimal controls

The dynamics of the optimal controls are given by the Euler equations. Applying the Pontryagin conditions to this control problem we yield the Euler equations for the optimal controls. We substitute (A.9c) into (A.9a) to describe the optimal consumption and additional (A.9d) and (A.6) into (A.9b) to see the optimal investments in defence capital.

$$
\begin{aligned}
\dot{c}(t) &= -c(t)\frac{-\mu_y(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho]}{\mu_y(t)} \\
&= c(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho] \qquad \text{(A.10)} \\
\dot{i_d}(t) &= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}\Bigg[\frac{\frac{\mu_d(t)}{\theta_0[\theta_1+2\theta_2 i_d(t)]}[d'(k_d(t), W(t))k_y]}{\mu_d(t)} \\
&\quad + \frac{\mu_d[\kappa_d d'(k_d(t), W(t))k_d + \kappa_d d(k_d(t), W(t)) + \delta_d + \rho]}{\mu_d(t)} \\
&\quad - \frac{-\mu_y(t)[A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho]}{\mu_y(t)}\Bigg] \\
&= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}\Bigg[\frac{1}{\theta_0[\theta_1 + 2\theta_2 i_d(t)]}[d'(k_d(t), W(t))k_y] \\
&\quad + [\kappa_d d'(k_d(t), W(t))k_d + \kappa_d d(k_d(t), W(t)) + \delta_d + \rho] \\
&\quad + [A\alpha k_y(t)^{\alpha-1} - d(k_d(t), W(t)) - \delta_y - \rho]\Bigg] \\
&= \frac{\theta_1 + 2\theta_2 i_d(t)}{2\theta_2}\big[A\alpha k_y(t)^{\alpha-1} + (\kappa_d - 1)d(k_d(t), W(t)) + \kappa_d d'(k_d(t), W(t))k_d + \delta_d - \delta_y\big] \\
&\quad + \frac{1}{2\theta_0\theta_2}[d'(k_d(t), W(t))k_y] \qquad \text{(A.11)}
\end{aligned}
$$

Appendix B. Two solutions of the model

To solve the model given in Eqs.9 we proceed as follows. First, to find an initial solution, we redefine the periodic water function $W(\gamma, \bar{W}, t) := \bar{W} + \gamma\Omega(t)$, where $\Omega(t)$ refers to the water function Eq.9f.

For the continuation of the function with a periodic solution we consider the more general boundary value problem (BVP)

$$\dot{x}(t) = f(x(t), W(\gamma, \bar{W}, t)), \quad x(t) \in R^n, \quad t \in [0, 1] \qquad \text{(B.1a)}$$

$$x(0) = x(1) \qquad \text{(B.1b)}$$

with

$$W(\gamma, \bar{W}, t) = \bar{W} + \gamma\Omega(t), \quad \Omega(0) = \Omega(1). \qquad \text{(B.1c)}$$

For $\gamma = 0$ and $\bar{W} = 1$ we found two feasible and optimal solutions $\hat{x}_1$ and $\hat{x}_2$, each corresponding to a different constraint constellation (i.e. $i_d(t) > 0$ and $i_d(t) = 0$). For these two cases the following continuation steps were used: Since

$x(\cdot) \equiv \hat{x}$ is an isolated solution and $f_x(\hat{x}, \bar{W})$ is non-singular, $f_\gamma(\hat{x}, \bar{W}) \neq 0$ and the minimal period of $\Omega(t)$ is one. For an isolated solution there exists $\varepsilon > 0$ such that for every $\gamma \in B_\varepsilon(0)$ a unique solution $x(\cdot, \gamma)$ for B.1 exists. Numerically these solutions can be found e.g. by the pseudo-arclength or Moore-Penrose continuation. As long as $x(\cdot, \gamma)$ itself is an isolated solution and the linearization of Eq.B.1 is non-singular the continuation proceeds.

For the actual computation the Moore-Penrose continuation in the implementation of the specific MATLAB® -Toolbox *OCMat* from Grass and Seidl (2013) was used, whereas it was shown that in the cases of $\hat{x}_1$ and $\hat{x}_2$ the linearization was always non-singular. This was done in two steps:

1. Continuation along γ from 0 to 1.
2. Continuation along $\bar{W}$ from 1 to 0.

So we derived the two solutions for the model given in Eqs.9, whereas the periodic water function is $W(\gamma, \bar{W}, t) = \bar{W} + \gamma\Omega(t) = \Omega(t)$ and therefore equals Eq.9f.