Keeping the Best: Optimal Career Strategies in Academia

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Abstract

Some areas of science face the problem that many people prefer the private sector over academia and so research output is lost. The present paper investigates by means of an optimal control model how the reward of competencies in research and teaching in the private sector affects investments into these skills as well as the decision on whether and when to optimally leave academia. As the decision between academia and industry is obvious if a scientist has strong preference for either, we focus on scenarios where this is not the case. We show that if academic competencies are well rewarded in the private sector, the most competent people will leave academia. We find scenarios in which a scholar will first try to improve his or her skills as much as possible before leaving academia and scenarios in which it is optimal to not put much effort into work and let competencies slowly depreciate before leaving. Even if professors are highly skilled and motivated to stay, if poor working conditions do not support knowledge acquisition, competencies will inevitably fall and academia will only consist of mediocre people in the long run.

Keywords: optimal control, academic career, human capital
JEL classification: C61, I23, D99

1 Introduction

It is the aim of probably most academic research institutions to reach academic excellence in order to produce and disseminate knowledge. Therefore it is crucial to hire and employ the best researchers on a long term basis and to understand the intentions of the scientific staff to invest into improving their competencies. According to a recent study by the Royal Society (see The Royal Society, 2010) only a small part of young PhDs stay permanently in the academic sector. Only 3.5 % will stay in a permanent research position and only 0.45 % will be awarded with a professorship. The rest will end up in careers outside the pure academic sector. I.e. in private firms or private research institutions, that are focused on the profitability of the research (patenting activity, development of new products, etc.) rather than on pure acquisition of scientific knowledge (basic research). Toole and Czarnitzki (2010) study the academic brain drain in terms of the lost research output in the non-profit research sector. In particular, they estimate (under a number of caveats) for the period 1994-2004 that numbers equivalent to 81 % of the cumulative output of journal publications (and 163 % of the cumulative output of patents) of the Massacusetts Institute of Technology were lost in the United States.

Fritsch and Krabel (2012) show that the attractiveness (from the view of academics) of working outside academia differs considerably according to the academic discipline[1], see also

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[1]In the business academic discipline, for instance, doctoral programs gives graduates some options for professional versatility (AACSB, 2013).

Sauermann and Roach (2012). This relates among other things to the commercial potential of the research; consequently there are sometimes significant differences in wages paid in academia and in the private sector. Roach and Sauermann (2010) study career preferences of students and show that inclinations towards academia or industry are related to the preferences for different job attributes such as the salary, the freedom to choose research projects, access to resources, etc.

For the private sector competencies acquirable in academia, in particular research and teaching skills, might be of high value, directly, when a firm works in a related field, and indirectly, when these competencies are seen as a signal for talent and/or the ability to work hard (See Spence, 2002 on signaling). Especially innovative firms view former employees of universities as addition in their research teams. Dietz and Bozeman (2005) investigate the impact of job transformations between the academic and the industry sector on the productivity of a scientist and find positive effects with respect to publications.

Levin and Stephan (1991) use an optimal control model to study the scientific productivity over the academic life cycle of a researcher. Thursby et al. (2007) present a dynamic optimization problem dealing with the question whether and how financial incentives to engage in commercial activities affect a faculty member's basic research activities. Agarwal and Ohyama (2013) compare a scientist's optimal investment into human capital in academia and in industry in a dynamic framework. They find that scientists with high non pecuniary benefits are more likely to choose academia over industry and thus forgo higher earnings. Furthermore, they show that firms and universities have a predilection to hire scientists with high abilities in order to maximize knowledge output.

So far the question of job rotation and how it affects academic productivity has hardly been investigated in literature within a formal model. The present paper extends El Ouardighi et al. (2013), who studies how scientists should optimally invest into research and teaching skills over the course of their career, by including the option to leave academia for the private sector. The following issues are addressed:

- At the scientist's level, to what extent is the trade-off between research and teaching activities affected by the existence of an external option, i.e., a job opportunity in the private sector?

- At the university's level, how is the job rotation affected by the existence of such an external option?

Overall, three career scenarios are essentially considered for a scientist: he or she could leave academia immediately, after some optimally determined time or never. As the skills acquirable in academia might be of different value than in science we approximate in the resulting optimal control problem the gains of entering the private sector by a salvage value function. By varying the parameters of this salvage value function, we are able to analyze the effect of complementary, substitutable and non-academic salvageable competencies on a scientist's decision to leave academia.

Rather obviously, scientists who find a particular high utility in non-commercially exploitable research and teaching activities will stay in academia for their whole career, and scientists who find the private sector much more rewarding will leave academia immediately. Thus, the present paper focuses on intermediate cases where we will show that the decision of a researcher to stay or leave the academic sector crucially depends both on the level of competencies of the scientist and on the structure (substitutability, complementary, etc.) and extent of the expected rewards of the private sector. If the reward of the private sector sufficiently outweighs that of the academic institution the scientist will move sooner or later. Depending on the structure of the

rewards we identify different scenarios for the chances to keep excellent scientists. In most cases only mediocre scientists intend to stay in the academic sector, while really excellent scientists leave for the private sector. This problematic result is only reversed if the reward of the private sector does not (or only on a low extent) depend on teaching or research skills, but is dominated by a constant (i.e. independent of the skills) term.

This paper delivers also some methodologically interesting insights. While there is a growing literature about the phenomenon of history-dependence and indifference points[2] in infinite time horizon models (see e.g. Grass et al., 2008; Zeiler et al., 2010; Kiseleva and Wagener, 2010), little research has focused on finite time horizon models. Caulkins et al. (2010) studies indifference points in free end-time problems. A similar approach is used in the present paper. By evaluating the possibilities of a finite, optimally determined end point and an infinite time horizon solution we find curves where the decision maker is indifferent between two long-run solutions in a linear-quadratic two-state model. We are even able to analytically derive an indifference curve, where the scientist has the choice to either quit immediately or never. We find scenarios where if the initial state values are low, it is optimal to quit immediately, if they are slightly higher to quit after some finite time. For intermediate initial state values it is optimal to never quit, but for high state values to either quit after some finite time or immediately.

The paper is structured as follows: Section 2 describes the model and the underlying assumptions, in Section 3 the analytic results are derived. Section 4 presents the numeric results for different salvage value functions. Section 5 analyses implications for universities. Section 6 concludes.

2 The Model

We extend the setup of El Ouardighi et al. (2013) by adding the possibility of leaving the academic sector after some optimally determined time T. We consider an optimal control model of a scientist[3], who optimally allocates his or her time into improving research and teaching skills. The scientist has the possibility to leave the academic sector and join a firm in the private sector at any time.

Analogously to El Ouardighi et al. (2013) research and teaching skills are represented by the states $x_1(t)$ and $x_2(t)$ respectively. Both competencies can be increased by improvement efforts $u_i(t)$ ($i = 1, 2$), which are the controls of the model, with quadratic costs. Since the time budget is limited we have to assume $u_1(t) + u_2(t) \leq w$. Moreover there is a spillover between the competencies, i.e. teaching skills have a positive effect on research skills and vice versa. These spillovers are modeled linearly with $\chi_i > 0$ ($i = 1, 2$). Finally research and teaching skills depreciate over time by rate $\delta_i > 0$ ($i = 1, 2$). As a result the dynamics of the model[4] reads subject to

$$
\begin{aligned}
\dot{x}_1 &= u_1 + \chi_2 x_2 - \delta_1 x_1, \quad x_1(0) = x_{10}, \\
\dot{x}_2 &= u_2 + \chi_1 x_1 - \delta_2 x_2, \quad x_2(0) = x_{20}, \\
0 &\leq u_i \leq w, \quad i = 1, 2, \\
u_1 &+ u_2 \leq w, \\
x_i &\geq 0.
\end{aligned}
\tag{1}
$$

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[2]Important early works in this field include Sethi (1977, 1979); Skiba (1978); Dechert and Nishimura (1983).

[3]By "scientist" we actually refer to all kinds of scientific staff with permanent employment. Non-permanent employment might alter the problem inasmuch as the individual might then be strongly confronted with uncertainty about whether an academic position can be maintained over the course of his or her career subject to quality evaluations, budget constraints and employment regulations.

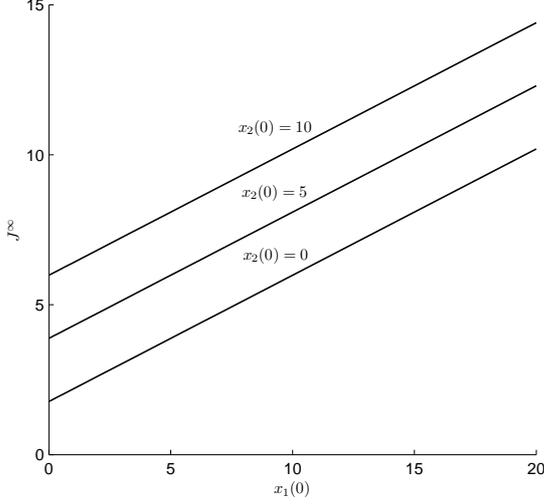[4]In the subsequent we omit time argument t unless necessary.

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Figure 1: Optimal profits of staying in academia forever for different initial state values

As already mentioned in this paper we consider the possibility of leaving the academic sector.[5] Consequently we assume a free end time problem. During the first period (working in the academic sector) the scientist has positive utility of the skills (linear with positive parameters a_1 and a_2) and quadratic costs (with cost parameters $\frac{c_1}{2}$ and $\frac{c_2}{2}$) of time investments. The profit/utility of the second period (working for a firm in the private sector) is approximated by a salvage value function $S(x_1, x_2)$. Thus the scientist faces the following objective function

$$J = \max_{u_1, u_2, T} \int_0^T e^{-rt} \left(a_1 x_1 + a_2 x_2 - \frac{c_1}{2} u_1^2 - \frac{c_2}{2} u_2^2 \right) \, dt + e^{-rT} S(x_1, x_2) \tag{2}$$

where r denotes the time discount rate. The scientist has the option to stay in academia. In this case, we assume that the scientist acts as if he our she would be able to stay in academia forever, and, thus has an infinite planning horizon. For the salvage value function we assume

$$S(x_1, x_2) = b_1 x_1 + b_2 x_2 + b_3 x_1 x_2 + b_4 \tag{3}$$

with non-negative parameters b_i $(i = 1, \ldots, 4)$. This specification implies $\frac{\partial S}{\partial x_i} \geq 0$ $(i = 1, 2)$, $\frac{\partial^2 S}{\partial x_i^2} = 0$ $(i = 1, 2)$ and $\frac{\partial^2 S}{\partial x_i \partial x_j} \geq 0$ $(i, j = 1, 2, \ i \neq j)$. Therefore the salvage value function is convex with respect to both states. Furthermore the specification allows for a number of different scenarios concerning the value of the skills and their interaction at the time of leaving the academic sector, i.e. no value, complements, and substitutes. To approximate gains of one stage in a salvage value problem by a linear function can make sense can be seen in Fig. 1 which actually shows how the profits (i.e. value of the objective function from $t = 0$ to ∞) of the original problem with infinite time horizon (presented in El Ouardighi et al., 2013) depend on the initial state values. From the figure it is obvious that the profit increases linearly in the initial value of x_1 regardless of the initial value of the teaching skill x_2. The same holds for the second stock x_2.

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[5]Within this paper we study the case where the scientist can leave the academic sector and join e.g. a private firm, but switches back are not possible. In general, moving back to academia is possible and leads to interesting questions for further research. However, then the second period in the non-academic sector would also have to be modeled with continuous time.

3 Analysis

In this section we derive optimality conditions as well as properties for the optimal solution of the model analytically. The Hamiltonian for problem (2) subject to (1) is

$$\mathcal{H} = a_1 x_1 + a_2 x_2 - \frac{c_1}{2} u_1^2 - \frac{c_2}{2} u_2^2 + \lambda_1 \left(u_1 + \chi_2 x_2 - \delta_1 x_1\right) + \lambda_2 \left(u_2 + \chi_1 x_1 - \delta_2 x_2\right), \quad (4)$$

where λ_i $(i = 1, 2)$ denote the adjoint variable to the states x_i $(i = 1, 2)$ respectively. By adding the control constraints we obtain the Lagrangian of the problem

$$\begin{aligned}
\mathcal{L} &= \mathcal{H} + \mu_1 u_1 + \mu_2 u_2 + \mu_3 \left(w - u_1 - u_2\right) \\
&= a_1 x_1 + a_2 x_2 - \frac{c_1}{2} u_1^2 - \frac{c_2}{2} u_2^2 + \lambda_1 \left(u_1 + \chi_2 x_2 - \delta_1 x_1\right) + \lambda_2 \left(u_2 + \chi_1 x_1 - \delta_2 x_2\right) \\
&\quad + \mu_1 u_1 + \mu_2 u_2 + \mu_3 \left(w - u_1 - u_2\right),
\end{aligned} \quad (5)$$

where μ_i $(i = 1, 2, 3)$ are the Lagrangian multiplier for the corresponding constraints.

The necessary conditions for the control variables are

$$\mathcal{L}_{u_i} = -c_1 u_i + \lambda_i + \mu_i - \mu_3 = 0, \qquad i = 1, 2. \quad (6)$$

implying the same as in El Ouardighi et al. (2013) paper, i.e.

$$u_i = \frac{\lambda_i + \mu_i - \mu_3}{c_i}, \qquad i = 1, 2. \quad (7)$$

In the case where the control constraint $u_1 + u_2 \leq w$ is active the optimal controls can be determined, by inserting $u_2 = w - u_1$ into the Hamiltonian and considering the necessary conditions, to be

$$u_1 = \frac{1}{c_1 + c_2} \left(c_2 w + \lambda_1 - \lambda_2\right), \quad (8)$$

and by inserting $u_1 = w - u_2$

$$u_2 = \frac{1}{c_1 + c_2} \left(c_1 w - \lambda_1 + \lambda_2\right). \quad (9)$$

The Legendre-Clebsch condition (see e.g. Theorem 3.18 in Grass et al., 2008) is fulfilled. The complementary slackness conditions

$$\mu_1 u_1 = 0, \quad \mu_2 u_2 = 0, \quad \mu_3 \left(w - u_1 + u_2\right) = 0 \quad (10)$$

have to hold.

The adjoint equations are

$$\dot{\lambda}_i = (r + \delta_i)\lambda_i + \chi_i \lambda_j - a_i, \qquad i, j = 1, 2, \quad i \neq j. \quad (11)$$

Note that the above adjoint equations do not depend on the state variable nor on the control due to the problem's linearity in the state and the separability of state and control. This implies that the optimal control, which itself only depends on the costate, only depends on the long-run or terminal state value, respectively.

From (1) and (11) we obtain the following steady-state values of the adjoint variables

$$\hat{\lambda}_i = \frac{(r + \delta_j)a_i + \chi_i a_j}{(r + \delta_i)(r + \delta_j) - \chi_i \chi_j}, \qquad i = 1, 2 \quad (12)$$

In the interior of the admissible region the optimal steady-state controls are

$$\hat{u}_i \quad = \quad \frac{(r + \delta_j)a_i + \chi_i a_j}{c_i\left[(r + \delta_i)(r + \delta_j) - \chi_i\chi_j\right]}, \qquad i = 1, 2. \tag{13}$$

If the constraint $u_1 + u_2 = w$ is active, then the steady state controls become

$$\hat{u}_i = \frac{1}{c_i + c_j}\left(c_j w + \frac{a_i(r + \delta_j - \chi_j) - a_j(r + \delta_i - \chi_i)}{(r + \delta_i)(r + \delta_j) - \chi_i\chi_j}\right) \tag{14}$$

From (1) we obtain the steady-state values for the states

$$\hat{x}_i = \frac{\delta_j \hat{u}_i + \chi_j \hat{u}_j}{\delta_i \delta_j - \chi_i \chi_j}. \tag{15}$$

Assumption 1 *From now on we assume $\delta_i\delta_j - \chi_i\chi_j > 0$ for the rest of the paper.*

From (15) and the control constraints it is obvious that Assumption 1 implies that the steady state value of the state cannot be negative. It is possible that it is zero, but only if both controls are exactly on the boundary, i.e. $\hat{u}_i = 0$ for $i = 1, 2$. The steady state value of the adjoints (12) is always positive since Assumption 1 implies that the denominator is always positive. Since we assume that a_i ($i = 1, 2$) are strictly positive, also the nominator is strictly positive. The same arguments apply to the steady state value of the controls. As a result the steady state value of the state is also strictly positive. From an interpretation point of view Assumption 1 means the product of the spillover rates must be smaller than the product of the depreciation rates. Otherwise it would mean that a scientist could acquire skills without any efforts, i.e. competencies would increase just through spillovers and the long-run solution would diverge.

Proposition 1 *The steady state, defined by (12) and (15), is unique.*

Proof: Since the costate equation (11) does not depend on the state nor on the control, the steady state value of λ_i is always given by (12). The complementary slackness conditions (10) and the necessary conditions (6) imply that $\hat{u}_i$, and as such $\hat{x}_i$, $i = 1, 2$ are unique for any given $\hat{\lambda}_i$. $\square$

To analyze the stability of the above unique equilibrium we formulate the Jacobian evaluated in the equilibrium within the interior of the admissible control region, i.e.

$$J = \begin{pmatrix} -\delta_1 & \chi_2 & \frac{1}{c_1} & 0 \\ \chi_1 & -\delta_2 & 0 & \frac{1}{c_2} \\ 0 & 0 & r + \delta_1 & \chi_1 \\ 0 & 0 & \chi_2 & r + \delta_2 \end{pmatrix}, \tag{16}$$

which has the following eigenvalues

$$\xi_{1,2} \quad = \quad -\frac{1}{2}\left(\delta_1 + \delta_2 \pm \sqrt{(\delta_1 - \delta_2)^2 + 4\chi_1\chi_2}\right)$$

$$\xi_{3,4} \quad = \quad r + \frac{1}{2}\left(\delta_1 + \delta_2 \pm \sqrt{(\delta_1 - \delta_2)^2 + 4\chi_1\chi_2}\right).$$

Assumption 1 implies that $\delta_1 + \delta_2 > \sqrt{(\delta_1 - \delta_2)^2 + 4\chi_1\chi_2}$ holds. Thus the first and the second eigenvalue are negative, while the third and the fourth are positive, i.e. the steady state is a saddle point with a two-dimensional stable manifold.

Next we formulate the following important result.

Result 1 *The optimal controls are constant at their steady state level (i.e. $u_i(t) = \frac{(r+\delta_j)a_i + \chi_i a_j}{c_i[(r+\delta_i)(r+\delta_j) - \chi_i \chi_j]}$ for any t, $i = 1, 2$) along the stable manifold (i.e. on any infinite time horizon solution).*

Proof: The eigenvectors corresponding to eigenvalues ξ_1 and ξ_2 are[6]

$$\nu_{1,2} = \left(1, \frac{1}{2\chi_2}\left(\delta_1 - \delta_2 \pm \sqrt{(\delta_1 - \delta_2)^2 + 4\chi_1 \chi_2}\right), 0, 0\right)'.$$

Note that the coordinates corresponding to the costates are zero. This implies the above result.
$\square$

The intuitive reason behind the result is that the controls only depend on the costates, which remain constant along the stable path. This is due to the linearity in the state and its separability of states and controls which implies that the marginal utility of an additional unit of a skill is constant.

Note that under certain conditions also the eigenvectors of the Jacobian evaluated at a steady state with active constraint $u_1 + u_2 = w$ have coordinates corresponding to the costates which are zero, i.e. on solution paths leading to such a steady state, the controls are also constant.

If on the other hand the finite terminal time T is chosen optimally (with $T < \infty$), we have the following terminal time condition (see e.g. Grass et al., 2008)

$$\mathcal{H}(T) = r\left(b_1 x_1(T) + b_2 x_2(T) + b_3 x_1(T) x_2(T) + b_4\right), \tag{17}$$

together with the transversality conditions

$$\lambda_1(T) = b_1 + b_3 x_2(T),$$
$$\lambda_2(T) = b_2 + b_3 x_1(T).$$

Using the last expressions for the first order conditions it has to hold at the terminal time that

$$u_i(T) = \frac{b_i + b_3 x_j + \mu_i + \mu_3}{c_i}, \quad i = 1, 2, \ i \neq j. \tag{18}$$

Since the scientist can choose T optimally, we have to compare the objective value of three scenarios:

Region I: **Leave immediately:** The scientist collects the salvage value immediately, i.e. $J := S(x_{10}, x_{20})$.

Region II: **Leave after finite time:** It is optimal to leave the academic sector after some finite time $0 < T < \infty$. Then the value of the objective is given by $J^F := \frac{1}{r}\mathcal{H}(0)$.

Region III: **Never leave:** It is optimal to stay in academia forever. The value of the objective is given by $J^\infty := \frac{1}{r}\mathcal{H}(0)$.

It is obvious that the regions in the (x_1, x_2)-plane, where the above three scenarios are optimal, cannot overlap. However, it might happen, that the scientist is indifferent between different long-run outcomes on a line in the (x_1, x_2)-plane separating two regions. If then a scientist starts with (x_{10}, x_{20}) which lies exactly on this line he or she is indifferent between these two cases, i.e. both strategies imply the same profit.

The following two Lemmas present analytic expressions of such curves.

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[6]The third and the forth eigenvalue can also be expressed analytically, however, they are omitted since the expressions are not necessary for the proof. Moreover they are rather long and deliver no additional insight.

Lemma 1 *The indifference curve where the scientist is indifferent between case I (leave immediately) and case III (never leave) is*

$$x_2 = -\frac{1}{a_2 + c_1 \hat{u}_1 \chi_2 - c_2 \hat{u}_2 \delta_2 - r(b_2 + b_3 x_1)} \left[(a_1 + c_2 \hat{u}_2 \chi_1 - c_1 \hat{u}_1 \delta_1 - rb_1) x_1 + c_1 \frac{\hat{u}_1^2}{2} + c_2 \frac{\hat{u}_2^2}{2} + rb_4 \right].$$
(19)

Proof: In the infinite time horizon case the control and costate are constant on any solution path leading to the steady state. Therefore, we can compare the Hamiltonian evaluated at time zero with the salvage value, i.e. the profits of quitting academia immediately. The line separating the two regions consists of all initial state constellations (x_{10}, x_{20}) where both values are equal. Solving $\mathcal{H}(0) = rS(x_{10}, x_{20})$ for x_{20} the equation gives (19). $\square$

Note that the above Lemma only holds for certain parameters, i.e. if (x_{10}, x_{20})-constellations really exist in the admissible region of the state space where the value of the objective of the infinite time horizon solution equals the salvage value. If one solution always dominates the other, the scientist will always choose the better one and never be indifferent between these two cases.

Lemma 2 *For the curve separating case I (leave immediately) and case II (leave after finite time), i.e. the curve describing the optimal end points of the finite time horizon problem, we have to distinguish several cases corresponding to the control constraints. If*

- *no control constraint is active:*

$$x_2(T) = \frac{1}{2b_3 \left(\frac{b_3}{c_1} + \chi_2 \right)} \left(-N_1 \pm \sqrt{N_1^2 - 4b_3 \left(\frac{b_3}{c_1} + \chi_2 \right) M_1} \right)$$
(20)

  with

$$N_1 = a_2 - b_3 x_1(T) (\delta_1 + \delta_2 + r) + b_1 \left(\frac{2}{c_1} b_1 b_3 + \chi_2 \right) - b_2 (\delta_2 + r)$$

$$M_1 = \frac{1}{c_1} b_1^2 + \frac{1}{c_2} (b_2 + b_3 x_1(T))^2 + x_1(T) (a_1 - \delta_1 b_1 + b_2 \chi_1 + b_3 \chi_1 x_1(T) - rb_1) - rb_4$$

- $u_1(T) = 0, u_2(T) = w$:

$$x_2(T) = \frac{1}{2b_3 \chi_2} \left(-N_2 \pm \sqrt{N_2^2 - 4b_3 \chi_2 M_2} \right)$$
(21)

  with

$$N_2 = a_2 - b_3 x_1(T) (\delta_1 + \delta_2 + r) + b_1 \chi_2 - b_2 (\delta_2 + r)$$

$$M_2 = (w + \chi_1 x_1(T)) (b_2 + b_3 x_1(T)) + x_1(T) (a_1 - \delta_1 b_1 - rb_1) - rb_4 - c_2 \frac{w^2}{2}$$

- $u_1(T) = w, u_2(T) = 0$:

$$x_2(T) = \frac{1}{2b_3 \chi_2} \left(-N_3 \pm \sqrt{N_3^2 - 4b_3 \chi_2 M_3} \right)$$
(22)

  with

$$N_3 = a_2 - b_3 x_1(T) (\delta_1 + \delta_2 + r) + b_3 w + b_1 \chi_2 - b_2 (\delta_2 + r)$$

$$M_3 = \chi_1 x_1(T) (b_2 + b_3 x_1(T)) + b_1 w + x_1(T) (a_1 - \delta_1 b_1 - rb_1) - rb_4 - c_1 \frac{w^2}{2}$$

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- $u_1(T) = 0, u_2(T) = 0$: *can be excluded for $\lambda_i(T) > 0$, $i = 1, 2$, which happens if either $b_{1,2} > 0$ and/or $b_3 > 0$ as the analysis is restricted on positive state values.*

- $u_1(T) + u_2(T) = w$ *with $u_{1,2}(T) < w$: Using $u_1 = \frac{1}{c_1+c_2}(c_2 w + \lambda_1 - \lambda_2)$ implies*

$$u_1 = \frac{1}{c_1 + c_2}(c_2 w + b_1 + b_3 x_2 - b_2 - b_3 x_1)$$
$$u_2 = \frac{1}{c_1 + c_2}(c_1 w - b_1 - b_3 x_2 + b_2 + b_3 x_1).$$

(23)

The analytic expression for the curve is omitted in this case due its size.

Proof: If the terminal time is a positive finite number, it has to hold that

$$\lambda_i(T) = b_i + b_3 x_j(T), \ i = 1, 2, i \neq j$$
$$u_i(T) = \frac{b_i + b_3 x_j(T) + \mu_i(T) + \mu_3(T)}{c_i}, \ i = 1, 2, i \neq j$$

(24)

and plug these into (17). By reordering we obtain x_2 as function of x_1. We have to distinguish the case where the controls are in the interior of the admissible region and where a control constraint is active at the end point. $\square$

4 What Drives Scientists to Leave Academia? A Numerical Investigation

In this section we numerically analyze the model defined in the preceding section 2. This allows to figure out the implications of different situations of the private market (as captured by the salvage value function). The three subsections discuss different implications of the skills x_1 and x_2 on the expected profit in the private market. In particular, we aim to study which scientists have the highest incentives to leave academia, how this decision affects the investments into improving skills and what this means for research institutions trying to achieve academic excellence by employing the best staff possible.

For the numerical solution a boundary value approach and a continuation algorithm, which are described in Grass et al. (2008); Grass (2012), are used. For the calculations let us assume the following benchmark parameters

$$r = 0.1, a_1 = 0.08, a_2 = 0.08, c_1 = 1, c_2 = 1, \chi_1 = 0.01, \chi_2 = 0.01,$$
$$\delta_1 = 0.1, \delta_2 = 0.1, w = 1.$$

The parameters are chosen in a manner that the optimal controls are in the interior of the admissible region for the infinite time horizon solution. Note that in this case both skills are symmetric (i.e. they are equally valuable, costly, and have the same depreciation and the same spillover rates). Assumption 1 is fulfilled and the steady state values are

$$\hat{x}_i = 4.6784, \ \hat{\lambda}_i = 0.42105, \ \hat{u}_i = 0.4211, \quad \text{for } i = 1, 2.$$

(25)

Note that it is practically impossible for universities to observe job applicants' personal utilities of academic skills and of leaving for the private sector, particularly with respect to non-pecuniary rewards, before hiring them. Thus, our implications for universities are provided under the assumption that the underlying parameters are representative for a scientist working in this

area under the conditions that a university is able to provide. Thus, scientists only differ in their observable[7] initial research and teaching skills.

In the following we will study different parameter constellations reflecting different preferences of scientists (which are of course not known by the research institutions before hiring them), first with respect to the gains by leaving for the private sector, later with respect to the utility of academic competencies within academia. This case study is crucial for research institutions since they need to know how to set the research environment (e.g. by influencing the spillovers) in order to hold the best scientists.

4.1   Complementary Salvageable Competencies

Complementary competencies imply that x_1 and x_2 are both important. For example, someone aiming to work as a reputable consultant should have *both* expert knowledge and didactic skills to be able to communicate his or her expertise. There is no value if both skills are zero and a high x_i can only partly compensate for a low x_j $(i \neq j)$. For the salvage value consider the parameters

$$b_1 = 0, b_2 = 0, b_3 = 0.1, b_4 = 0. \tag{26}$$

In this scenario acquiring research and teaching competencies is rather costly compared to the revenues when working in academia, thus a person of this type or working under such conditions would never work at the maximum of his or her capacities if planning to stay in academia, because it simply is not rewarding enough. As already derived analytically, the costate and consequently the optimal controls are constant on any admissible solution path leading to the steady state. The reason for this is that in this case the marginal utility of an additional unit of any of the competencies is also constant due to the linear quadratic nature of the problem. It is optimal to approach the steady state if the initial competencies are low.

The phase portrait is shown in Fig. 2. It consists of three areas separated by a dashed, a dotted and a dashed-dotted line. The meaning of the three curves is as follows. If the scientist starts[8] at the

dashed line: he or she is indifferent between staying in academia forever (Region III) and leaving it in finite time (Region II).

dashed-dotted line: he or she is indifferent between staying in academia forever (Region III) and leaving it immediately (Region I). This line is analytically given by (19).

dotted line: he or she will leave academia immediately. This curve is also the border between Region I and II, and shows where the scientist leaves academia after having spent a finite time in academia, i.e. the points where the conditions $\mathcal{H}(T) = rS(x_1(T), x_2(T))$ and $\lambda_1(T) = S_{x_1}, \lambda_2(T) = S_{x_2}$ are fulfilled.

The solid lines in Fig. 2 are exemplary solution paths for different initial state values.

If both the initial teaching and research competencies are high, it is optimal to leave academia immediately (Region I), because due to the abilities it pays better to change for the private sector.

If only either the initial teaching or the research competency is high (i.e. below or left of the dotted line (Region II)), the scientist would also quit academia, but only after acquiring skills in the area in which he or she lacks those. The reason for this is that the mixed derivative of

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[7]E.g. by publication lists, the extent of previous teaching activities, rewards, evaluations etc.

[8]By the term "scientist starts at the dashed (dotted, dashed-dotted) line" we mean that (x_{10}, x_{20}) lies on the dashed (dotted, dashed-dotted) line.
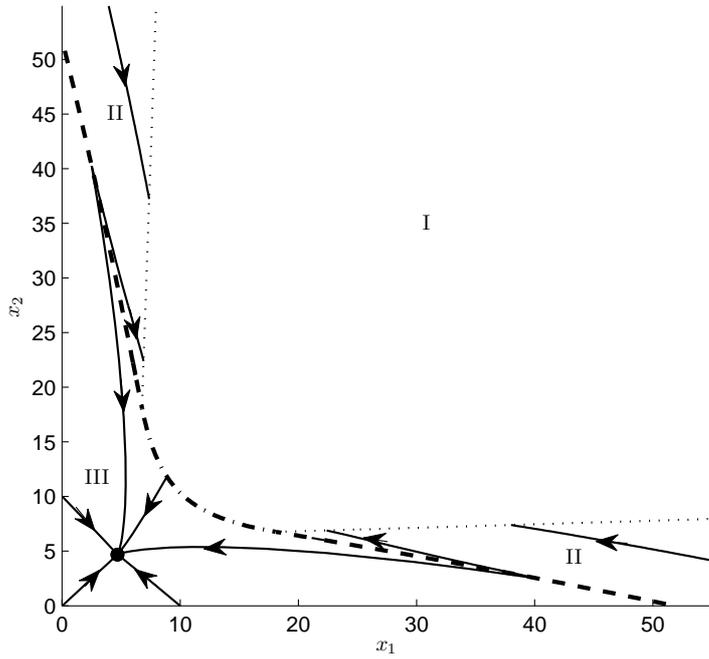
Figure 2: Phase portrait with salvageable complementary competencies ($b_3 = 0.1$, $b_{1,2,4} = 0$)
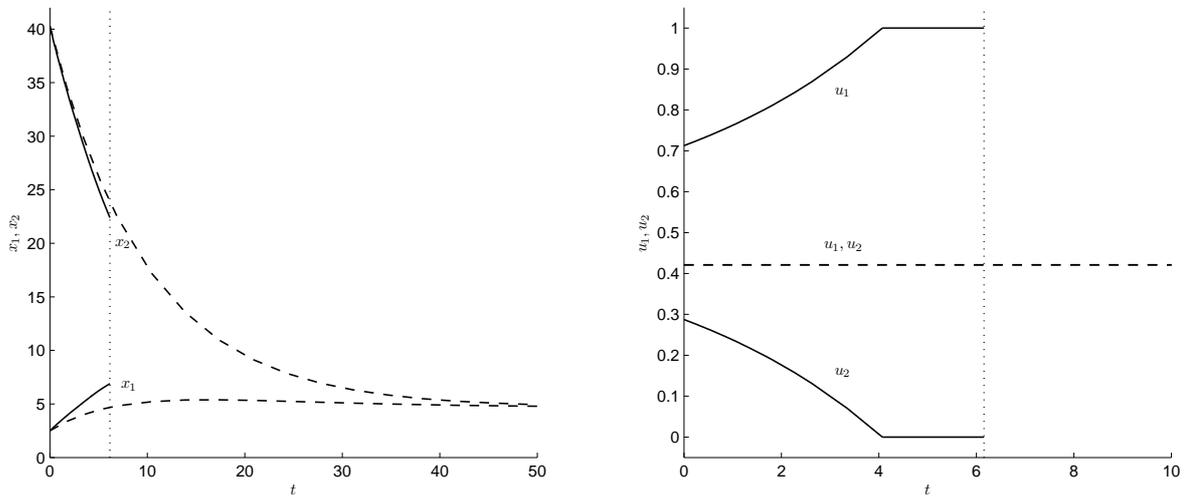

Figure 3: Timepath for solution paths starting on the Skiba curve with initially low research competency and large teaching competency ($x_1(0) = 2.5, x_2(0) = 40.255$). The left panel depicts the development of the state variables over time, while the right panel shows the optimal controls. The solid line depicts the finite time horizon solution, the dashed line the solution with infinite time horizon.

the salvage value with respect to the states is not zero, i.e. the benefits of leaving his or her position are not particularly high if the scientist only has skills in one of the two considered areas. If the dotted line in Fig. 2 has been reached the scientist leaves academia. Note that on the dotted left vertical line the constraint $u_1 = 1$ and $u_2 = 0$ is active. This makes sense at this point the teaching competencies are rather high, so our scientist would do all he or she can to compensate their lack of research skills. Analytically this curve is given by (22). On the other hand on the dotted right horizontal line the scientist only would invest in his or her teaching competencies and neglect research, see (21). If the initial skills of the scientist are low in both areas (left of the dashed and the dashed-dotted line (Region III)), it is optimal to stay in academia forever. In this case the control boundaries are never reached, i.e. the scientist will never work as much as it would be possible, since that is simply not rewarded enough.

In Fig. 3 we can see an example for a solution path starting on the dashed line where the scientist is indifferent between staying in academia forever and leaving after some optimally determined time: For this particular solution path the initial research competencies are small, but teaching skills are high. In case of an infinite time horizon the controls are constant as described. For the initial state values this would mean that teaching skills would fall and research competencies increase when investing the same amount into each of these skills. In case of the finite time horizon, the optimal strategy looks different: Initially the scientist would invest into both skills, but more into research to compensate his or her lack of skills. This would lead to a decrease of teaching but an increase of research activities. As time proceeds the decline of teaching competencies becomes less steep which is on the one hand due to the particular depreciation of skills which is proportional to the existing skills, but slightly also due to an increase of spillovers resulting from research activities. After some time this would mean that the scientist can focus entirely on improving his or her research skills to make out the most of the competencies when entering the private sector.

The given parameters imply that the most competent scientists always quit, while less competent scientists who even do not work as much as they would be able do stay in academia forever. This means that if (e.g. for strategic reasons) a university wants to employ people for a long time, this goal might not be achievable if it only accepts the best scientists available as employees. Furthermore, it is impossible under these circumstances to hire personnel which is very competent both in research and teaching, because for those scientists it does not pay off to work in academia. Thus, if a university wants to employ excellent people, it is only possible to get scientists who are only competent in one of these skills, not in both and it has to accept that this in a sense overqualified staff will leave sooner or later. This is probably not a desirable, but also not an (entirely) unrealistic outcome for universities. It might explain the magnitude of the flow of scientists that leave academia particularly in areas where expert knowledge is commercially exploitable.

If we would decrease the benefits of quitting, i.e. decrease parameter b_3 the indifference curve would shift upwards, i.e. it gets only interesting to quit if competencies are really high.

Increasing parameter b_3, i.e.

$$b_3 = 0.25, \tag{27}$$

substantially affects the optimal solution. This scenario is depicted in Fig. 4. In this case a scientist would always quit academia sooner or later. If both skills are high, he or she would leave immediately (or never enter academia at all). In other words, academia is a launching pad for the private sector. If both skills are low, a scientist would invest into both skills. However, when the competencies are low both controls are in the interior of the admissible region, meaning that it is too costly to dedicate the whole time for improving academic skills. This changes, however, when skills are sufficiently high, i.e. one gets closer to quitting, because then an increase in competencies has a higher impact. If research competencies are low but
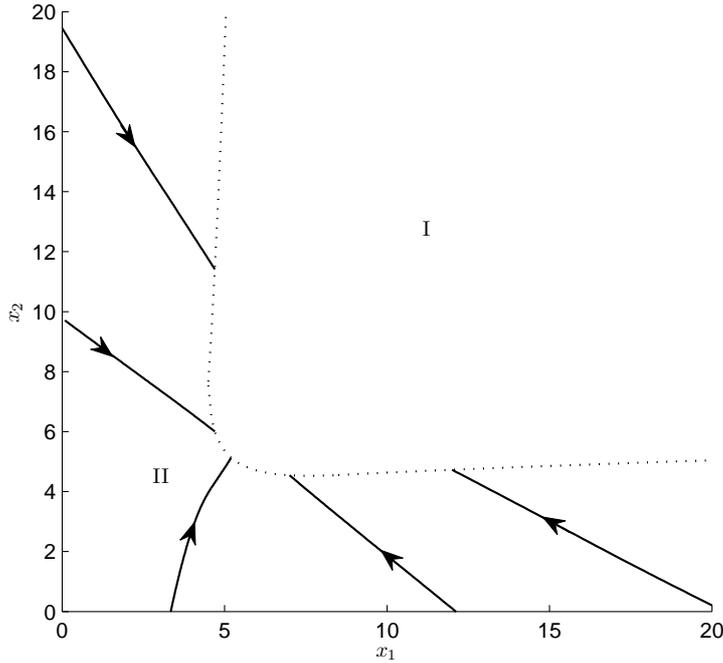
Figure 4: Phase portrait for greater salvage value of complementary competencies ($b_3 = 0.25, b_{1,2,4} = 0$)

teaching skills are high, one would work harder on improving research and vice versa. Note that for high research competencies the constraint $u_1 + u_2 = 1$ with $u_2 = 1$ is active at the optimally determined end point and for high teaching competencies the constraint $u_1 + u_2 = 1$ with $u_1 = 1$. If the extent of research and teaching competencies are similar, then $u_1 + u_2 = 1$ with $u_{1,2} < 1$. Thus, in a sense the university faces a trade-off when deciding who to hire as it only has the choice between people who are neither competent and initially do not work as hard as they could, but stay for a longer time period, and people who are competent and hard working, but quit soon. Note that in this scenario the steady state which we calculated before and the infinite time horizon solution paths approaching it are still admissible, however, they are never optimal.

4.2 Substitutable Salvageable Competencies

Now we look at the case where both competencies are values independent of the other one. For instance, a scientist might have the choice to work at another academic institution. We consider

$$b_1 = 0.5, b_2 = 0.5, b_3 = 0, b_4 = 0, \tag{28}$$

This scenario is depicted the left panel of Fig. 5. For high initial competencies it is optimal to leave academia immediately, otherwise one would remain in academia forever since it is too costly to increase the skill to a level where the salvage value is more profitable. In view of the institutions this is a very stable but not very promising scenario. Every scientist, that can be hired, will stay forever. But there is no way to hire excellent people, not even for a small period.

If b_1 and b_2 exceed a certain size, the finite time horizon solution is always optimal, only for small initial x_1 and x_2 one would stay in academia for some time. For small b_1 and b_2 the infinite time horizon solution is always optimal.
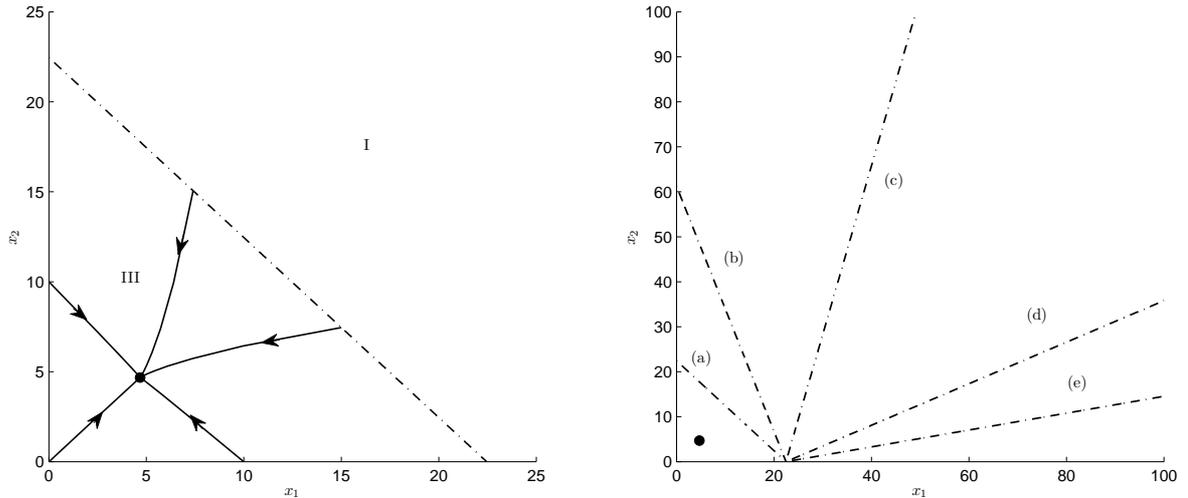
Figure 5: Phase portrait for substitutable salvageable competencies ($b_1 = 0.5, b_2 = 0.5, b_3 = 0, b_4 = 0$) (left panel) and for increasingly asymmetric salvage values with $b_1 = 0.5, b_3 = 0, b_4 = 0$, and (a) $b_2 = 0.5$, (b) $b_2 = 0.45$, (c) $b_2 = 0.4$, (d) $b_2 = 0.25$, and $b_2 = 0$ (right panel)

The right panel of Fig. 5 shows what happens if research and teaching skills are valued differently in the private sector. For example, teaching skills might be of little relevance for someone doing commercially usable research. We keep parameter b_1 constant and decrease b_2. It can be seen that the curve where one is indifferent between leaving immediately and never becomes steeper with a lower b_2, and even turns, and the region where staying in academia is optimal becomes larger. This is of course due to the lower valuation of teaching skills in the private sector which makes it more attractive particularly for scientists whose strength lies in teaching to stay in academia. If teaching skills lead to no or very little benefits outside academia, only people with a strong research background will leave academia. Note, however, that even though academia becomes more attractive to work in with lower gains in the private sector due to a lower parameter b_2, the investments into the skills are not affected when planning to keep the academic position forever. As already explained the reason for this is constant marginal benefit from skills in the infinite time horizon problem.

Furthermore we can consider that the skills are not only substitutable but also complementary (as discussed in the previous section). We assume[9]

$$b_1 = 0.05, b_2 = 0.05, b_3 = 0.1, b_4 = 0, \tag{29}$$

The result as plotted in Fig. 6 is now again back to the qualitative shape of Fig. 2. Only the values of the curves change a bit. Since the salvage value is now larger for any state value, the indifference curve shifts downwards and the area for which one would leave academia becomes larger. For the given parameters we have again three regions: If the initial state values are small, it is optimal to stay in academia, otherwise it is optimal either to leave after some finite time or immediately.
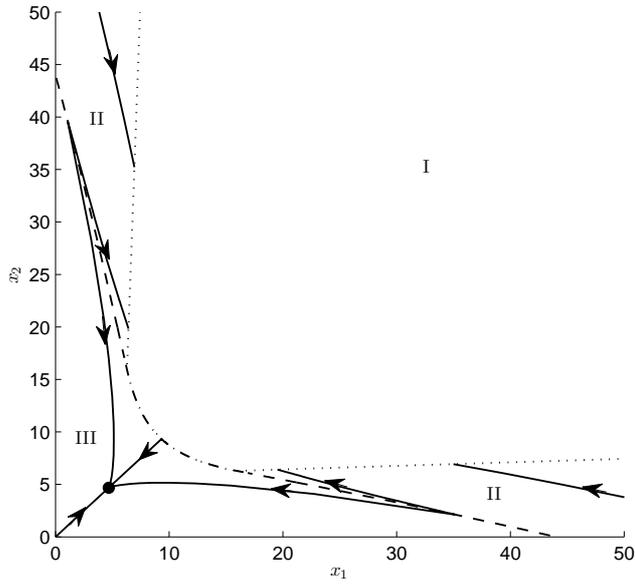
Figure 6: Phase portrait for a salvage value with $b_1 = 0.05, b_2 = 0.05, b_3 = 0.1, b_4 = 0$
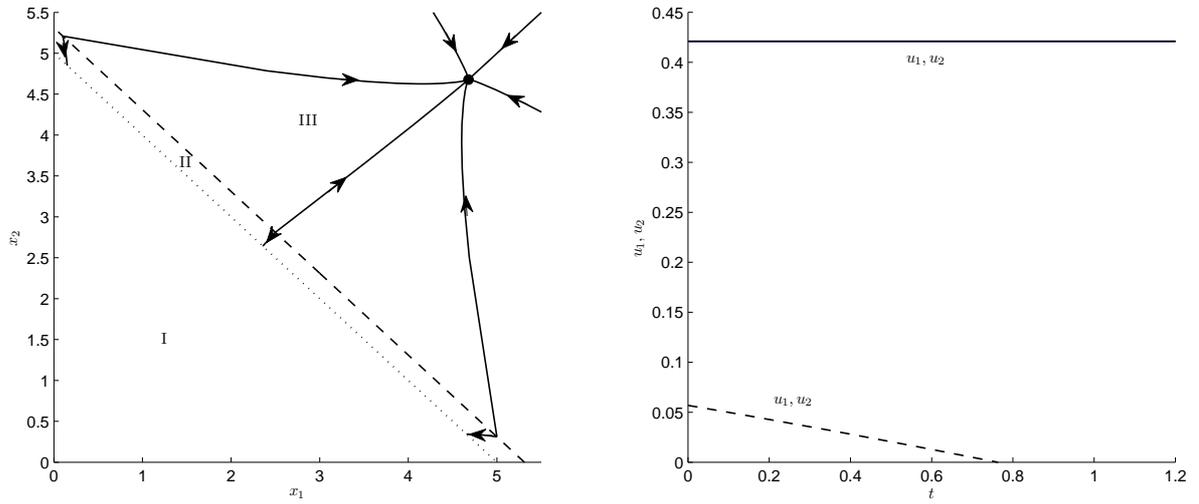

Figure 7: Phase portrait for salvageable non-academic competencies ($b_1 = 0, b_2 = 0, b_3 = 0, b_4 = 4$)(left panel) and time path for the initial state values $x_1(0) = 2.6$, $x_1(0) = 2.71$

4.3 Non-Academic Salvageable Competencies

In the previous subsections we focused on the effect of the skills on the salvage value function and the corresponding implications for the optimal strategy. In the following we will also add an additional constant term, i.e. $b_4 > 0$. In the beginning the research or teaching skills are of no value for the private sector at all. This might apply for scientists working in areas which are commercially not exploitable. In such a case all parameters in the salvage value but b_4 are zero. Of course, if b_4 is very large, it is optimal to quit immediately, if b_4 is small one would never leave academia. The phase portrait for the parameter choice

$$b_1 = 0, b_2 = 0, b_3 = 0, b_4 = 4 \tag{30}$$

is depicted in the left panel of Fig. 7. The result of this scenario is opposite to the results of the previous sections. If the initial skills are high, it is optimal to approach the steady state. If the initial skills are low it is optimal to leave immediately, because payoffs are higher in a different sector. However, there is an intermediate region in which it is optimal to stay in academia for a certain time period. In this period one would not work as hard as if one planned to stay in academia forever. Actually, there it would be optimal to be quite lazy and do very little, such that skills do not depreciate too quickly and one has some profits of one's existing skills while remaining in academia. Of course, this would mean that competencies get lower and after some time it is optimal to leave academia completely. The time path for the optimal controls can be seen in the right panel of Fig. 7. The solid line depicts the optimal controls for the infinite time horizon case and the dashed line for the finite time horizon solution starting at $x_1(0) = 2.6$, $x_1(0) = 2.71$. Note that in case of a salvage value, which does not depend on the state values, $\lambda_i(T) = 0$, and thus $u_i(T) = 0$, for $i = 1, 2$. Since the optimal controls only depend on the costate and the competencies are equally valued, a scientist would invest the same efforts into improving both skills independent of the initial state values. Like in the infinite time horizon case, the reason here is that the marginal value is the same for both skills along the solution path.

This kind of scenario actually reflects some phenomenon which can be observed from time to time, i.e. when people, knowing they cannot gain anything by actively engaging in their work as they plan to leave their job soon, do only what is absolutely necessary and nothing more, possibly to the disadvantage of their employers and coworkers. An important implication of such a scenario is that a university should be very careful only to hire competent people as it is optimal for lesser skilled scientists to exploit the system.

Having discussed the *pure* effect of non-academic salvage value, we now combine it with substitutable salvageable competencies. We assume

$$b_1 = 0, b_2 = 0, b_3 = 0.05, b_4 = 4. \tag{31}$$

The resulting phase diagram, which is depicted in Fig. 8, is a combination of the case with substitutable salvageable competencies (discussed in subsection 4.1) and the case of pure non-academic salvageable competencies. The optimal behavior is analogous to that discussed at Fig. 2 with an additional region near the origin, where it is optimal to leave academics immediately. Excellent people will never work for the academic institution. If the skills of the scientist are below the dashed and the dashed-dotted line, he or she will stay in academic forever. If his skills are in between the dashed and the dotted line, he or she will work for the institution for a finite time in order to increase his or her skill before leaving academia. In the current case

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[9]We assume here parameters b_1 and b_2 to be smaller than the parameters used for the calculations of Fig. 5. For $b_1 = b_2 = 0.5$, the additional complementary term would just increase the attractiveness of the private sector such that it would always be optimal to leave.
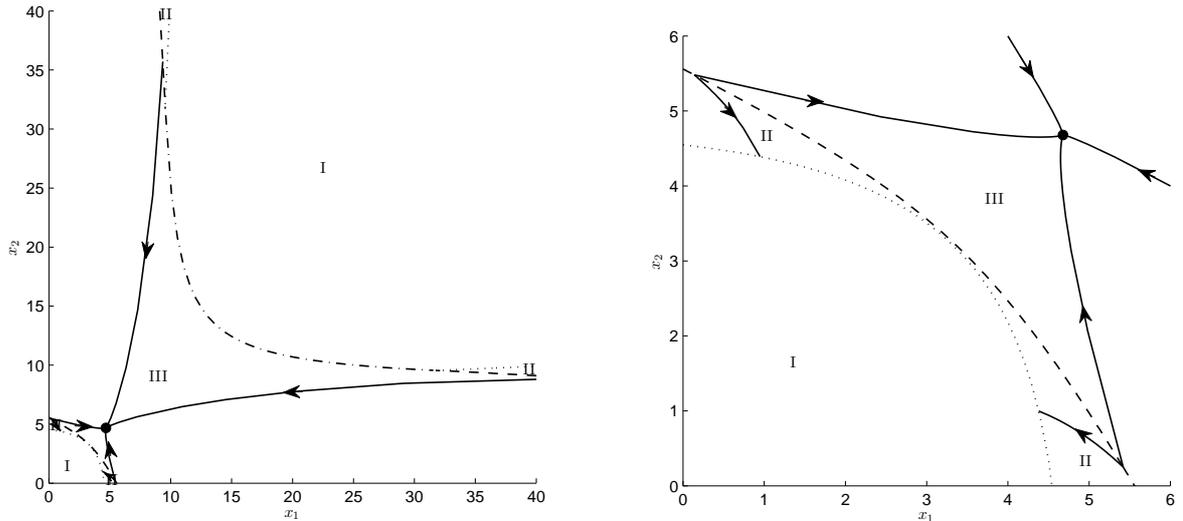
Figure 8: Phase portrait for a low salvage value of substitutable academic competencies and large salvage value of non-academic competencies with $b_{1,2} = 0, b_3 = 0.05, b_4 = 4$; the right panel is a zooming

additionally the most incompetent people will never start to work in the academic sector, and they will immediately collect the salvage value. Also around this area in the phase diagram, there is a small region, where the scientist works for a finite time in academia before he or she leaves it. From the view of academic institutions aiming for academic excellence, the situation has a little bit improved (compared to pure substitutable or complementary salvageable competencies). Now scientists with very poor skills will never even start to work for the institution. Furthermore, in this case less skilled scientists have at least some incentive to improve their skills as they derive at least some utility of it later on. However, the excellent ones are still not available.

Let us now consider the opposite case, i.e. pure non-academic with complementary salvageable competencies. Here we assume the following parameter constellation[10]

$$b_1 = 0.5, b_2 = 0.5, b_3 = 0, b_4 = 4. \tag{32}$$

The result, plotted in Fig. 9, shows that for small initial competencies it is optimal to leave immediately, for higher initial competencies the optimal strategy depends on the initial state values. As for the given parameters research and teaching skills are equally valuable, if one of the skills exceeds the other, one would work to improve the weaker skill, and slightly neglect the stronger competency. This scenario is quite pessimistic: Neither the incentives to stay in academia nor the incentives to improve or keep the competencies are strong, so people would become in a sense lazy (and finally all leave the academic sector).

Finally, we combine all possible types of salvageable skills in the salvage value function, i.e. complementary, substitutable and non-academic. In Fig. 10 we plot the phase diagram for

$$b_1 = 0.05, b_2 = 0.05, b_3 = 0.05, b_4 = 4. \tag{33}$$

Again for the given parameters it is optimal to stay in academia forever if the initial skills are high, to leave after some finite, optimally determined time if the initial skills are intermediate

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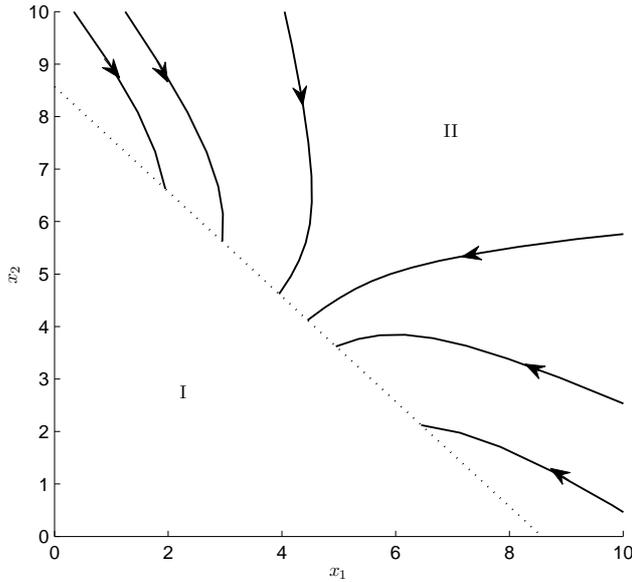[10]Parameter b_1 and b_2 are the same as in Fig. 5.

Figure 9: Phase portrait for a low salvage value of complementary academic competencies and large salvage value of non-academic competencies ($b_1 = 0.5, b_2 = 0.5, b_3 = 0, b_4 = 4$)

and to quit immediately if the skills are low. We see that the effect of the term where we multiply the states in the salvage value dominates in the sense that the curve where it is optimal to quit is nonlinear and that an academician is strongly advised to improve the weaker skill and neglect the stronger one. Due to their complementarity, if skills are very large it is optimal to leave academia. Thus, incompetent people leave, intermediate people stay and very competent people leave academia. For the incompetent, they profit from leaving by collecting the relatively large reward for non-academic competencies. Highly skilled people profit from their skills most in the private sector. We have indifference curves both for incompetent and competent people. For certain initial state values people have the choice to either neglect their skills and stay in academia for some time or to never quit. For higher competencies they are - for certain initial state values - indifferent between either quitting immediately or never, for other initial state values to stay in academia for some time and to improve on the weaker skill or to never quit.

5 Improving Academic Performance

In the previous section we discussed the optimal behavior of scientists over their academic career depending on different salvage value functions (representing the profit from working in the private market). In the given problem formulation the academic institution could not influence the optimal behavior of scientists as well as the profit from working in the private market. Nevertheless, to ensure academic excellence it is important to be aware of the (optimal) behavior of scientists over their working career in particular when academic institutions want to hire a new scientist.

In this subsection we pose the question differently. We ask whether academic institutions can actively improve (and to what extent) something to employ better scientists. By the term *improving actively*, we refer to the reward for the skills or the the quality of the research environment, i.e. parameters a_i $(i = 1, 2)$ or χ_i $(i = 1, 2)$ respectively.
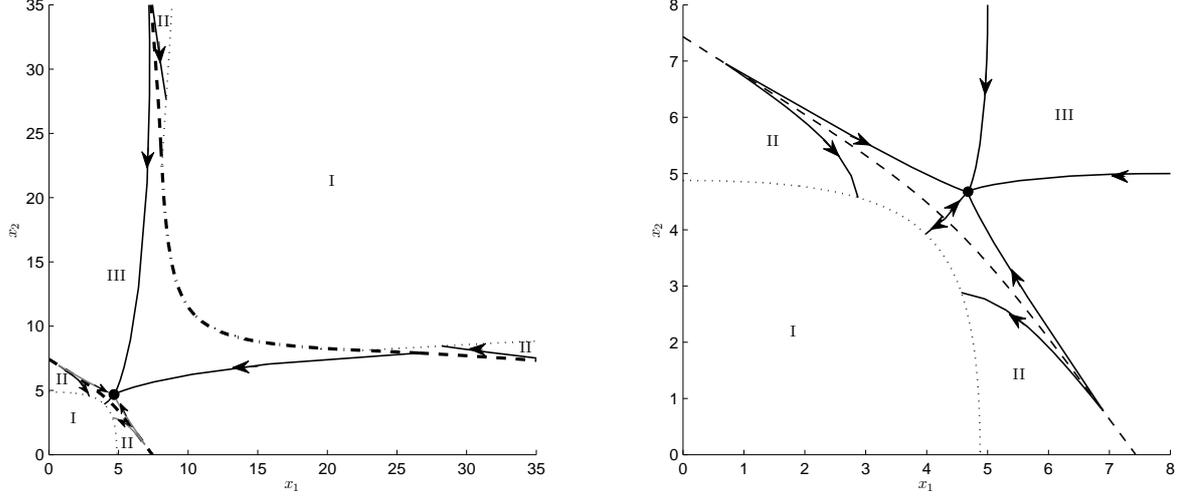
Figure 10: Phase portrait for a salvage value with $b_1 = 0.05, b_2 = 0.05, b_3 = 0.05, b_4 = 4$

Let us now assume the institution is able to increase a_i $(i = 1, 2)$, i.e. it is more rewarding to work in academia (or this particular university, respectively), i.e.

$$a_1 = 1, a_2 = 1. \tag{34}$$

To ensure comparability we assume same as in Fig. 2 $b_1 = b_2 = 0$ and $b_3 = 0.1$.

The corresponding phase portrait can be seen in Fig. 11. Compared to Fig. 2 we see that the indifference curve shifts upwards. Thus, only extremely skilled scientists would choose to quit academia either immediately or at some optimally determined time. Compared to the first scenario the steady state value is slightly higher, i.e.

$$\hat{x}_i = 5.5556, \quad \hat{\lambda}_i = 5.2632, \quad \hat{u}_i = 0.5. \tag{35}$$

Here the constraint $u_1 + u_2 \leq 1$ is binding with equality, meaning the scientist really puts all his or her efforts into the academic career. In this case the analytic expression for the steady state in the symmetric case (where $\delta = \delta_1 = \delta_2$, $\chi = \chi_1 = \chi_2$, $a = a_1 = a_2$ and $c = c_1 = c_2$) is

$$\hat{x} = \frac{w}{2 (\delta - \chi)}. \tag{36}$$

Note that this expression (in contrast to the steady state value with interior control, which in the symmetric case can be written as $\hat{x} = \frac{(r+\delta+\chi)a}{c(\delta-\chi)[(r+\delta)^2-\chi^2])}$) does not depend on parameter a. Thus, even if an academic career might be extremely rewarding for an individual, this reward does not affect the skills anymore when a person works at the maximum of his or her capacities.

Thus, even if either of the skills is initially high, the decision to remain in academia would extremely negatively affect this particular skill. This suggests that remaining in academia will turn highly skilled into mediocre profiles. Even though the said constraint is active on the whole solution path leading to the steady state, working at the maximum of one's capacities is not sufficient to be able to keep the skills high.

This means that increasing salaries in academia / rewards for academic achievements is not enough to have the best scientists. The implication for universities is that if they wish to keep highly skilled scientists, they not only have to make the work more rewarding, they also
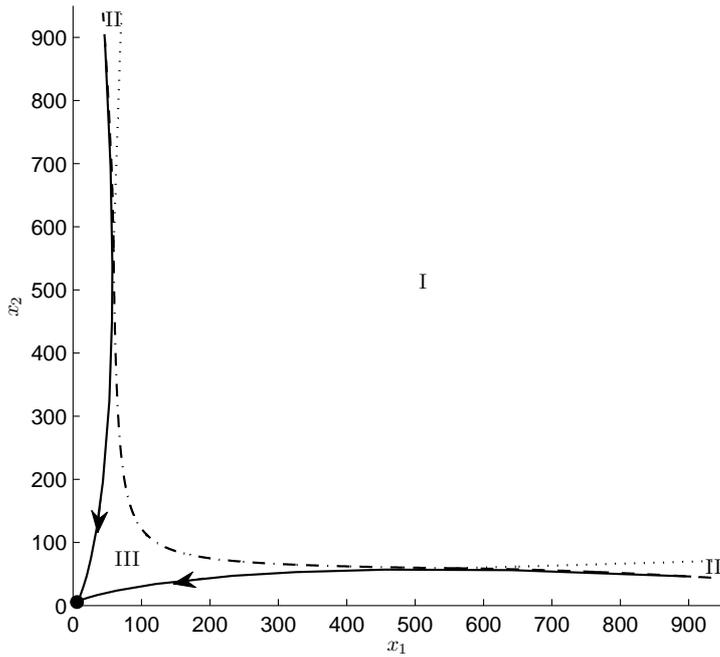
Figure 11: Phase portrait for greater rewards of academic competencies ($a_1 = a_2 = 1$)

should provide working conditions which make knowledge acquisition and maintenance more efficient. In particular they should implement measures (e.g. less distractions by administrative duties, better equipment, etc.) which decrease the depreciation of skills (parameter δ), increase spillovers (parameter χ), and increase the time budget available for improving competencies (parameter w).

If, on the other hand, the academic institution is able to increase the spillover rates (while keeping rewards at $a_1 = a_2 = 0.08$) to

$$\chi_1 = 0.05, \quad \chi_2 = 0.05, \tag{37}$$

and keep the parameters of the salvage value function at

$$b_1 = 0.05, b_2 = 0.05, b_3 = 0.05, b_4 = 4, \tag{38}$$

(i.e. the same salvage value function parameters as in the beginning of subsection 3 and in Fig. 10) we can observe the following: In Fig. 12 the steady state competencies are due to the higher spillovers larger than in Fig. 10. While before one was indifferent between quitting in finite, positive time or never at certain initial state values with rather low competencies, now one has only the choice between quitting immediately or never. The reason is that due to the higher spillovers it is much easier to acquire skills for incompetent people, thus, it makes an academic career more attractive even for less skilled people than before. Furthermore, the upper indifference curve shifts upwards as keeping skills high requires less efforts, making science more attractive for more competent people.

All in all, the institution can improve the situation actively by making research work more attractive. However, it cannot turn around the dilemma that the best scientist tend to leave academics when it is possible to make higher profits in the private market.
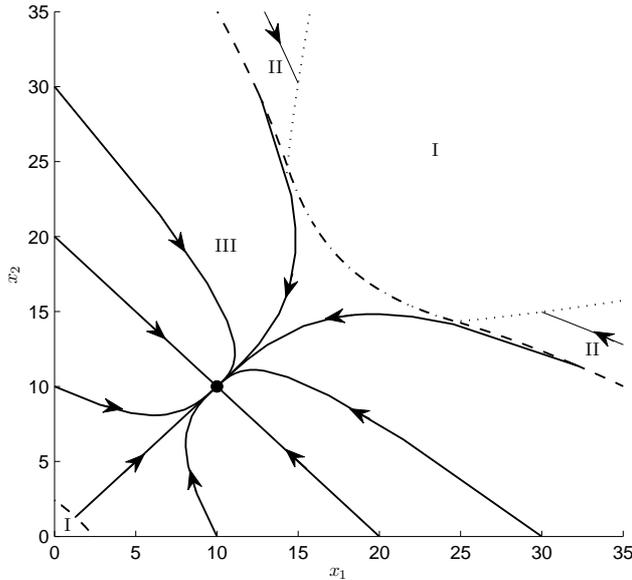
Figure 12: Phase portrait for a salvage value with $b_1 = 0.05, b_2 = 0.05, b_3 = 0.05, b_4 = 4$, $\chi_{1,2} = 0.05$

6 Conclusions

It is not surprising that when gains of leaving academia significantly outweigh the advantages of staying, a scientist will quit the job sooner or later. When the private sector does not offer sufficient rewards, a scientist will stay. For intermediate rewards of leaving, the decision on whether to leave or stay in academia as well as the efforts put into work, crucially depends on the initial competencies. Particularly, we can show that if rewards like the wage in the private sector do not depend on academic skills, it is the less competent people who will leave academia. As it does not pay off to invest much into competencies, in such a scenario people with mediocre skills are strongly tempted to neglect work and simply exploit advantages of an academic job before leaving. People with low skills will quit immediately. If the gains of leaving academia significantly increase with the competencies, the most competent people have the highest incentives to quit.

Depending on whether the salvageable competencies are substitutable or complementary, a scientist will either invest into his or her weaker skill before leaving or not at all. We found a scenario where scientists with low and high skills have the highest incentives to leave academia, and mediocre people will stay. Depending on the initial skills, a scientist who will leave academia puts more efforts into his or her weaker competency. A scientist planning to stay will invest the same efforts into research and teaching if these skills are equally valued at his or her institution.

In order to prevent the most competent people from leaving, universities might try to stronger reward skills. We show, however, that while improving the incentives to stay in academia might affect a scientist's decision on whether to leave or not, if the poor working conditions at the institution do not support knowledge acquisition or preservation, competencies will inevitably fall and people will be mediocre at best in the long run.

Furthermore, we analyze the impact of spillovers between research and teaching on the decision to leave. We are able to show that if the academic job allows such spillovers, people are more likely to stay in academia. The reason for this is that high spillovers make it much easier

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for incompetent people to acquire and for highly skilled people to keep rewarding academic skills.

The results we gain are not only interesting from an application point of view, but also methodologically. By comparing finite and infinite time horizon solution, we are able derive indifference curves in a linear-quadratic two-state model, in some cases even analytically. We apply numerical methods to find curves, where for certain initial state values one has the choice between quitting academia immediately or never, and between leaving after some optimally determined time and never.

There are several possibilities for extensions: Within a multi-stage framework, one could formulate and study the optimization problem of the scientist after leaving academia. Furthermore, many academicians are confronted with rigorous evaluations of their work. Thus, it would be interesting how the timing and frequency of review processes affect the efforts put into one's work particularly under the threat of having to face severe consequences by negative evaluations. Another important extension would be to consider a dynamic game between the scientist and the dean, who might have different preferences with respect to teaching and research efforts. Also interesting would be to study how annoying, but necessary administrative obligations affect academic careers. Furthermore, one could explicitly model effects of aging and thereby gain insights about its impact on academic achievements and the decision to pursue a career in the private sector.

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