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With this paper, dedicated to Gustav Feichtinger on the occasion of his 70-th anniversary, the authors—all being his former students and/or collaborators—want to summarize and honor his remarkable scientific contributions and academic activities, and wish him many years of a further active and fruitful life.

Gustav studied mathematics and physics at the University of Vienna (1958–1963), completed his PhD in algebra in 1963, worked for a year at the IBM research laboratory in Vienna, and continued his academic career as assistant and associate professor at the University of Bonn (1965–1972), where he also received his habilitation in statistics. In 1972 he became full professor of operations research at the TU Vienna and founded the *Institute of Operations Research* (originally named *Institut für Unternehmensforschung*) which, after several metamorphoses, still prospers as *Research Unit for Operations Research and Control Systems*. Soon after the foundation of this institute, the Viennese school of optimal control theory gained international reputation and paved the way into the academic world

for many scholars, who now hold senior positions in various universities in Austria and Germany as well as in other countries.

Gustav carried also a number of additional duties: department director at the *Vienna Institute of Demography*, part-time researcher at the *International Institute for Applied Systems Analysis* (Laxenburg), member of the advisory board of the *Max Planck Institute for Demographic Research* (Rostock), corresponding member of the *Austrian Academy of Sciences*, chairman of the *Austrian OR Society*, associate editor of eight international journals, and many others. He also organized a number of very successful international conferences: a series of nine *Viennese Workshops of Optimal Control, Dynamic Games and Nonlinear Dynamic Systems*, a huge OR conference in 1990 (with more than 1200 participants), as well as dozens of other scientific meetings in Vienna.

One of the most admirable features of Gustav's academic profile is his broad research interest, which resulted in substantial contributions in such diverse fields as optimal control theory, operations research, demography, economics, management science, population dynamics, social sciences, etc. It is hard to give a systematic account of his work, which is why the following exposition is partly chronological, partly thematic, and in no way comprehensive.

1. Demography and Population Economics. In the early 1970s Gustav's work in demography focused mainly on statistical and formal demography. One of the key publications of this time is [33] which includes research on the representation of demographic quantities as stochastic processes and, more generally, the description and analysis of demographic processes using stochastic models (in particular Markov models [35]). Applications range from individual-based micro processes such as fertility and nuptiality, competing risk models, and stochastic decrement tables [34] to macro models with special reference to the stable population model. During these years Gustav also published a book on population statistics [36].

Most interestingly, already as early as in the mid 1970s, Gustav started studying stationary and shrinking populations [40] with particular focus on the demographic and economic implications of these processes. Many of the topics he dealt with during this time [37, 38, 41] are currently of high priority for ageing societies. Formal contributions at this time relate to applications of demographic concepts to manpower planning [39], research on quasi-stable populations [98], statistical measurement of the family life-cycle [42], the relation between period and cohort measures of demographic processes [99], and optimal growth of stable populations in neoclassical growth models [23]. This work culminated in the book [43] which still constitutes one of the most advanced German textbooks on formal demography.

By the end of the 1980s the focus of Gustav's work in population dynamics moved to demo-economics. Related to his other fields of study his attention was drawn to research on non-linear relations between demographic and economic/environmental dynamics. Based on the Easterlin model, which relates fertility behavior to opportunities of the labor mar-

ket and the level of consumption aspiration formed when young, he studied models of self-generated fertility waves [94] and the possibility of Easterlin cycles [63]. Whether demographic processes can indeed generate complex behavior has been reviewed in [89]. The possibility of chaotic dynamics has been shown in a Solow model with endogenous population [114]. Further topics include the interrelationship between population and resource dynamics in primitive societies [113, 111, 118, 115, 116, 117]. Essentially these types of models are closely related to prey (the resource)–predator (the population) models where economic forces determine the interaction mechanisms. During the last couple of years, Gustav has introduced the concept of age-structured optimal control models into population [90] and health economics [110] (see Section 7 below).

2. Manpower Planning. Gustav's publications in the area of manpower planning have been greatly influenced by his work on population dynamics. By applying the Perron-Frobenius theorem he obtained a generalization of stable age distributions from discrete-time demographic models to manpower models with given recruitment trajectories [39]. In [85], which appeared in the first volume of the prestigious journal *Mathematics of Operations Research*, a recruitment trajectory capable of generating a given sequence of manpower stocks is determined and various results concerning the limiting behavior of recruitment trajectories and manpower stock vectors are proved.

3. Advertising. After Gustav had started working on optimal control theory, one of the first areas of application was marketing, where a stock of goodwill or sales is controlled by instruments such as price, advertising, or quality [45, 46, 24]. The typical solution method in these first approaches was phase plane analysis in order to obtain important insights into the structure of the optimal solution (e.g. monotonicity or convergence) in very general classes of problems. In [67] marketing was combined with production in order to obtain the simultaneous optimal solution. Gustav also recognized the importance of strategic interaction and competition. Consequently, he and his coauthors developed a series of differential games in which a firm plays against potential or existing competitors in the market [47, 50, 51, 79, 25, 26, 82], see also Section 12.

While the papers on marketing were initially mainly published in OR journals, the work of Gustav was also recognized by the marketing community which culminated in the publication of [83] in the top journal *Marketing Science*. Here, a store's price image and advertising determine the number of customers attracted into the store, while sales per visit depend only on the prices actually observed in the store. One of the most recognized surveys out of the many that were written or co-authored by Gustav is the one on dynamic optimal control models in advertising in *Management Science* [75].

At the end of the 1980s Gustav joined the general enthusiasm about interesting and strange behavior of optimal solutions. Paper [74] provided the first continuous time model that was able to explain the (empirically observed) superiority of cyclical advertising policies to constant or monotonic ones; see also [86] and Section 9 below. The next step was the investigation of chaotic behavior which led to a series of papers [54, 77, 57, 105] show-

ing that even more complicated outcomes can be the result of dynamic advertising models (with and without optimization) – see Section 11 for the continuation of this work.

4. Machine Maintenance. Another early application area of optimal control theory in Gustav's work was machine maintenance. In order to keep the capital stock productive, the machines can be subject to maintenance in order to prevent breakdowns, or the machines can be repaired when broken down. Furthermore, the intensity of usage also influences the state of the machines. In this context some early optimal control models [66, 48, 53] and differential games [49] were developed.

5. Social Interactions. After the turn of the millennium, Gustav launched investigations on social interactions, partially linked to the issue of tipping points also mentioned below in Section 10 and to papers by Glaeser, Scheinkman, and others. Typically for Gustav, these extensions centered around the introduction of dynamics into initially static models. In [123] a widely quoted paper by Krugman is reconsidered showing that the claim—increasing returns to scale are the mechanism to induce history-dependent (or even expectations-dependent) outcomes—is wrong. The papers [12, 15] deal with the problem of segregation and the optimal management of controlled migration. The recent paper [124] considers how social interactions can induce (or reduce) obesity, a quite topical problem in industrialized societies, in particular in North America.

6. The Economics of Crime: Drugs, Corruption, and Terror. In 1995, as at many other times, Gustav and his group wanted to raise research money from the *Austrian Science Fund* (FWF). Already at that time, the research funding system was very competitive, so Gustav spent some time to think about a topic for the research proposal that would have a high success probability. Inspired by the seminal work of the Nobel laureate Gary Becker, the idea was to establish dynamic optimization in the economics of crime literature, which at that time was dominated by static optimization models. During the careful preparation of the proposal, Gustav's group established contact with Jon Caulkins from CMU (Pittsburgh), at that time also co-director of RAND's *Drug Policy Research Center*. This was the beginning of both a very fruitful collaboration and a distinct focus on optimal control models of drug use. The research proposal was granted, and many follow-up projects originated from it.

The work by Gustav and his group in this area resulted in dozens of publications, most of which appeared in prestigious journals such as *Operations Research* and *Management Science* [5, 119, 14]. In addition to the broad acceptance by the operations research community, it is remarkable that some of the work not only appeared in drug policy oriented outlets [1, 8] but also had an influence on real life problems. For instance, Gustav's group has contributed to the understanding of the analogies between "drug epidemics" and "classical" epidemics (such as HIV), the driving factors being social interactions (see also Section 5) and the "infection" of non-users by users. A prominent example is [7], which is among the most cited papers co-authored by Gustav. Furthermore, several striking and sometimes

provoking results were derived. For example, in [6] it is shown that for present-oriented societies it may be optimal to undergo cycles of drug epidemics.

Apart from the many publications on drugs such as cocaine and heroin (including numerous Master and PhD theses), Gustav's group has also worked on corruption [91, 102], terror [4, 16, 13], and the economics of crime in general [65, 104]. All this work allowed the group to write the monograph [106], which is a state-of-the-art textbook on optimal control theory with applications in drugs, corruption, and terror.

The work on the economics of crime has also led to a lot of methodological advances in areas such as the dynamic optimization of age-structured systems (see Section 7) or the fruitful work on multiple equilibria and tresholds (see Section 10). The recent focus of Gustav's group on multi-stage optimal control [8, 9] and stochastic Skiba sets [10] was motivated by the Australian heroin drought (sudden and significant reductions in the availability of heroin around Christmas 2000).

7. Dynamic Optimization of Age-Structured Systems. Thanks to his profound knowledge of population dynamics, in the last years of the 20th century Gustav initiated several new topics of research involving dynamic optimization models with an age/duration-structure. Formally, these are first-order PDE optimal control models with non-local dynamics, extending the classical McKendrick population model. Such types of models, involving contagious phenomena, were successfully developed and investigated in a number of papers by Gustav and his collaborators, starting with the problem of optimal prevention and treatment of illicit drug epidemics [2], and expanding to a broader epidemiologic context, e.g. [96, 3].

About the same time, a new wave of vintage-capital accumulation models emerged, based on the striking analogy between the dynamics of physical capital and that of populations, and employing PDE optimal control models. Gustav understood quite well the importance of this topic, which resulted in a series of investigations published in [95, 90, 71, 72, 73] and many other papers by his collaborators. He also organized the first *Viennese Vintage Workshop* in 2003, aimed at promoting age-structured optimal control models in economics, epidemiology and social sciences, and at bridging the involved mathematical techniques. This was the beginning of a fruitful series of multi-disciplinary workshops on heterogeneous dynamic modeling and its mathematical foundation, taking place in Vienna every second year.

The contributions of Gustav and his co-authors in the publications mentioned in the above two paragraphs employ age-structured PDE models to analyze several crucial issues in economics and in population economics: the role of learning for the diffusion of new technologies, the influence of the anticipation of future technological innovations on the investment policy, the impact of an environmental tax on the economic performance, the investment behavior of firms with balanced budget encountering a technological breakthrough, optimal migration policy regarding long-term economic effects, etc. Gustav also contributed to the development of the mathematical techniques needed for the investiga-

tion of the above issues. Many follow-up studies by the international network of Gustav's collaborators contributed to mathematical epidemiology for heterogeneous populations, management of biological resources, formation of human capital, etc.

One aim of Gustav was to bring a normative intertemporal optimization flavor into demographic considerations. This found another manifestation in his works on the optimal dynamics of the age-distribution of fixed-size populations [97, 18]. In the spirit of this general aim, Gustav initiated a new line of investigations on optimal long term policies in health economics, in particular, individual versus socially optimal health investments [110] and the concept of the reproductive value in age-structured optimal control models [125].

8. Qualitative Analysis of Optimal Solutions and Equilibria. Gustav was always interested in developing, refining, and applying methods for the qualitative analysis of the solutions of optimal control problems and the equilibria of differential games. His book (coauthored by R.F. Hartl) [68] devotes considerable space to this topic. Various applications of phase diagram analysis to non-linear optimal control models or differential games are described in other parts of this survey. An essential theoretical contribution to the analysis of the structure of solutions of linear optimal control models with a single state variables appeared in [69]. In that contribution Gustav together with R.F. Hartl provided a rigorous proof of the most rapid approach path (MRAP) property. The MRAP property had long been known in the literature but the set of assumptions under which the property holds was incomplete. In [69] this missing condition was provided along with a counterexample in which the MRAP property fails in the absence of this assumption. Gustav's work on state separable differential games [29] (see also Section 12) can also be considered as a contribution to the qualitative analysis of differential games. Further contributions to the general topic of qualitative structures of optimal and equilibrium solutions will be further discussed in the following sections.

9. Limit Cycles in Dynamic Optimization. Gustav initiated a substantial amount of research (theoretical as well as applied) explaining that "driving in circles" (to quote Niki Lauda) can be an optimal strategy in intertemporal decision problems. Starting point is [92], where it is shown via the Hopf bifurcation theorem that even constant demand can lead to production cycles if production is characterized by increasing returns and adjustment (or set up) costs. This paper provides a closed form application of the Hopf bifurcation theorem, an approach that is rarely found in the literature. This intuitive mechanism (also applied to marketing [32] and employment [76]) was then complemented by investigating strictly concave optimization problems, which ensure that the first-order conditions are also sufficient but where the economic rationalisation is more subtle. In [28] it is shown that strictly concave preferences can induce cyclical consumption as optimal stationary outcome given sufficient adjacent and distant complementarity. This property requires nonlinear interaction between control and state; hence, the analysis is usually very cumbersome. A particularly simple route to limit cycles, developed in collaboration with F. Wirl, proved to be very useful for a general adjustment cost framework [88] and in a

number of applications, e.g. to regulation [100], politics [102], and drug consumption [120].

10. Thresholds in Dynamic Optimization. Towards the turn of the century, Gustav came forward with the idea to investigate thresholds, or tipping points. The tools were again from optimal control theory, more precisely from the works of Sethi, Skiba, and Dechert and Nishimura. These papers have triggered considerable research by prominent economists like Brock, Dechert, and Schmalensee (just to name a few) in the 1980s. Given the dates of these publications and the substantial body of literature that existed at that time, it seemed that there would be little that can be added. However, Gustav proved again to be very innovative and established with his co-workers new insights concerning pathways to thresholds under diminishing returns in [103, 122]. The papers [70, 107] demonstrate in a simple model of relative adjustment costs the possibility of continuous policy functions in case of a non-concave dynamic maximization problem and an overlap if the unstable steady state is a node. All these distinctions are typically excluded in the related literature, which relies exclusively on phase diagrams linking an unstable steady state to a jump in the control without explicitly computing the eigenvalues and the corresponding stable manifolds. Paper [22] is a first survey of different mechanisms leading to thresholds in economic intertemporal optimization problems.

More recently, thresholds in higher dimensions (thus not points but curves and surfaces) have been studied by collaborators of Gustav, some of them with non-trivial and even puzzling properties [109, 14, 11, 108, 13].

11. Complex Dynamics in Economics and Management. In the 1980s, several macroeconomists demonstrated that complex dynamics resembling stochastic time series can result from equilibrium behavior in deterministic frameworks with standard structure, like overlapping generations models or optimal growth models. The basic question underlying this work was, for which types of intertemporal decision problems complex behavior is compatible with standard assumptions about the economic framework, individual decision making, and preferences. Gustav was one of the first to pose similar questions also for different areas of management science and initiated a vivid research agenda in this direction with his research group in Vienna. A wide variety of analytical and numerical methods and results that had been developed in the nonlinear dynamics literature were employed and extended to characterize properties of dynamics arising in models well-founded in different areas of management science and related fields.

One of the first optimal control applications in economics was to find the trade-off between economic performance and environmental quality or the production-pollution dilemma as described in [84]. A whole chapter in [68] is devoted to applications in environmental economics. The paper [80] is among the first contributions to demonstrate that complex trajectories might arise from the use of a well-documented behavioral decision rule by a firm repeatedly choosing the intensity of its R&D activity. A number of subsequent papers deal with complex dynamics emerging in different advertising models, where on the one hand behavioral models [77, 64] and, on the other hand, frameworks with

intertemporally optimizing firms [58, 60] were considered. Further work of Gustav and his co-workers explored the implications of nonlinear dynamic effects in various other fields of operations research and management science including machine maintenance problems [59] and ressource exploitation [62]. In [78] Gustav and his co-authors extended the literature on complex dynamics in economics and management, which previously mainly dealt with one-dimensional systems, substantially by providing a characterization of complex dynamics in a two-dimensional model of addiction, and [56] is among the first papers applying chaos control techniques in a socio-economic framework.

In addition to his numerous papers dealing with different instances of chaotic dynamics in economics and management, Gustav had a large impact on this field of research via several survey articles (e.g. [81, 55]) and many stimulating presentations at international conferences.

12. Differential Games. In the early 1980s, when Gustav developed his interest in optimal control theory and dynamic optimization, he also started to work on differential games. In this field Gustav made several very important contributions both methodologically as well as in terms of economic and social science applications. Immediately after he started to work on differential games he identified a class of games for which the open-loop equilibrium also qualifies as a degenerate feedback equilibrium and coined the terminology of state separable games [44]. This working paper later turned into a joint publication [29] in which state separable games are fully characterized and several applications are discussed. The main characteristic of this class of games rests on the linearity of the players' value functions in the state variables. The identification of state separable games led to a sequence of papers that covered differential game applications in marketing including duopolistic pricing [25], strategic advertising [51, 24], strategic price and advertising interactions [26], and production [49]. Exploiting the probabilistic approach pioneered by Kamien and Schwartz in which the probability function of a stochastic event is modeled as the state variable that is directly controlled by the actions of rivals, Gustav was the first to find out that this approach leads to state separable differential games This resulted in the contributions [52, 26, 31].

In terms of applications of differential games, Gustav studied optimal share cropping as a differential game [93], non-cooperative exploitation of fisheries [30], the benefits of environmentalists for society [27], and family economics [121]. Among the non-standard applications are the classical battle of the sexes [101], dynastic cycles [87], and a game between the ruling class and the tabloid press [20]. The last two papers also make an important methodological contribution. They demonstrate that limit cycles and indeterminacy can be the equilibrium outcome of differential games. The paper [61] shows that indeterminate and cyclical outcomes can emerge even in a game with open loop Nash strategies.

In more recent years, Gustav's attention was caught by differential game applications in law enforcement, punishment, and crime. In this area his modeling provides very valuable

insights into aspects of real life that are characterized both by strategic interactions and dynamics, see [17, 19, 104, 21, 4, 112].

13. Gustav Feichtinger and his students. Gustav has started his studies in a teacher-training program in the subject area of mathematics. It is therefore not utterly surprising that he likes to teach and work with students. What is, however, remarkable is how successful he was in inspiring students for a research career in general and for a specific research question that he himself was working on in particular.

When Gustav started academic career as an assistant professor at the University of Bonn, university life was in uproar. The revolutionary year 1968 was almost there. Being in class at those times did not mean that one was able to teach a subject without interruption, but the students would often gather in class to discuss topics of mutual political and social interests. And quite often, students did not even attend classes, because they were out on the streets for rallies and demonstrations. It was certainly not the most comfortable environment for delivering one's first lectures. At that time, a strong character and the ability to assert oneself were necessary to survive in the class room.

After Gustav's move to the University of Technology in Vienna it was his task to introduce the new scientific field of operations research to the curriculum. During the summer of 1972 Gustav sat every working day several hours in the library to shape the new courses, and he spent hardly less time on preparing his lectures in each of the following twenty years.

One of the authors, J. Haunschmied, remembers: "Towards the end of these two decades I personally was fortunate to attend one of his classes. I can give an account of an excellent instructor, always focused on generating students' interest in the topic. He develops his presentation in a strong logical way, sophisticated, and with fascinating examples. One thing I want to mention here is that when unperceptive students rattled him and he ran the risk to loose his concept, he focused on some static object and continued with his presentation fully concentrated on the contents of his talk. It had happened that this static object was even not in the class room such that he was talking several minutes while gazing out of the window."

After two decades at the University of Technology in Vienna Gustav partly delegated his teaching duties to his assistant professors, because he was eager to visit many other universities, to establish scientific partnerships, and to give lectures abroad. In the following years, he taught classes at many universities around the globe, not only in Europe and North America but also in Australia, Chile, Colombia, Russia, Iran, Vietnam, Japan and China. In all of these places students and academics alike were captivated by Gustav Feichtinger. For instance, once an African PhD student expressed his admiration of Gustav by driving him on his motorbike through the countryside of Uganda.

Gustav has supervised a myriad of theses. It seems like he has been working day and night on actual scientific problems. When he arrived at the office in the morning full of new ideas, he started immediately feeding his graduate students with intellectual nutrition.

His instructions have become an igniting spark for so many successful academic careers, and he has always tried to boost the careers of his scholars.

14. Gustav Feichtinger and the OR societies. If Gustav had not pushed the formation and growth of ÖGOR (Austrian Society of Operations Research), this society would not have experienceded such a positive development during its first two decades. Correspondingly, this journal, CJOR, would not have been launched together with other central European OR societies and would not have become a success story and finally a SCI journal. If CJOR had not been launched, we would not have "gathered here" to celebrate Gustav's 70th anniversary. Those who know Gustav can confirm that he has always been ready to make huge efforts to promote matters of common interests. In this section we want to honor Gustav's role for OR societies in strict rotation.

Due to his time in Germany till 1972, Gustav Feichtinger had intensive contacts with the German OR societies, especially with GMÖR (German Society of Mathematics, Economics and Operations Research), but also with DGOR (German Society of Operations Research), and was involved in the fusion of these two societies to GOR in 1998. This and other achievements led to his election as an honorary member of the German OR Society (GOR) in 2008.

As head of the Institute of Operations Research at the Vienna University of Technology, Gustav was closely connected with the development of ÖAGOR (Austrian Consortium of Operations Research) and the foundation of ÖGOR. It is not a surprise that the founding venue was the Vienna University of Technology on Oct. 23, 1978. Gustav was member of the first executive board. From 1988 to 1991, he served as ÖGOR president. In 1990 Gustav agreed on the hard job to organize a joint GMÖR – DGOR – SVOR – ÖGOR meeting at the Technical University of Vienna. Thanks to him and his team this became a rather successful conference with more than 1200 participants and many distinguished keynote speakers.

Gustav was representing ÖGOR at the European Association of Operational Research Societies (EURO) for many years. Due to his extraordinary efforts ÖGOR has excellent relations to the European OR Societies, especially in the neighboring countries. He has contributed to many of the ÖGOR's activities and is still member of its advisory board. It was also a great pleasure for ÖGOR to decorate Gustav Feichtinger with an honorary membership at the ÖGOR's 30th anniversary meeting in 2008.

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