Regularity properties of mappings and applications

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Most of the mathematical models involve various kinds of mappings, both in their formulations and their analytical and numerical/computer investigation. An example from economics can be the mapping that describes the set of economic equilibria (of markets) as a function of exogenous data. A fundamental issue in economics is, whether small changes in the exogenous factors may lead to dramatic changes in the equilibria (even to their disappearance). A negative answer to these questions resembles a certain kind of regularity of the mapping involved.

As another example, if an optimization problem under constraints is to be solved, one has to employ a solution procedure that involves approximation methods and numerical algorithms. The following (different) questions arise: does the procedure guarantee that any solution it produces is close to a solution of the optimization problem at hand? Can every solution of the optimization problem be approximated by one generated by the procedure? Since the approximation methods can usually be regarded as specific "disturbed" versions of the original problem, the above questions can be translated as questions about different kinds of regularity of the mapping that maps disturbances in the optimization problem to solutions. Often, in order to apply solution procedures, the optimization problem is replaced with a system of optimality conditions, which contains not only equations but also inequalities or inclusions, in general. This brings the necessity to investigate the regularity of set-valued mappings (sometimes involving differential equations or inclusions). That is, stability of mappings defining generalized equations has to be investigated.

The investigation of various kinds of regularity of mappings made a substantial progress in the past few decades, but new problems arising in science, engineering and economics, as well as new mathematical techniques, create new challenges. The aim of this project is to develop several aspects of the regularity theory (e.g. directional, global, semi-, sub-regularity) for generalized equations and to estimate the "radius of regularity", which measures how robust is a given regularity property. An important part of the project is devoted to verification and application of the regularity theory to design and error analysis of numerical algorithms for solving optimization problems. The focus of the applications will be on dynamic optimisation problems for systems described by ordinary or partial differential equations.