Least-Squares-Based Deep Learning for Sentinel-2 Derived Bathymetry: A Case Study on Anegada's Southern Coast

<u>Yushan Liu</u> (yliu74@tudelft.nl)
PhD candidate, TU Delft
Control & Operations

Mirjam Snellen
Professor, TU Delft
Control & Operations

Alireza Amiri-Simkooei
Associate professor, TU Delft
Control & Operations

Roderik Lindenbergh
Associate Professor, TU Delft
Geosciences and Remote Sensing

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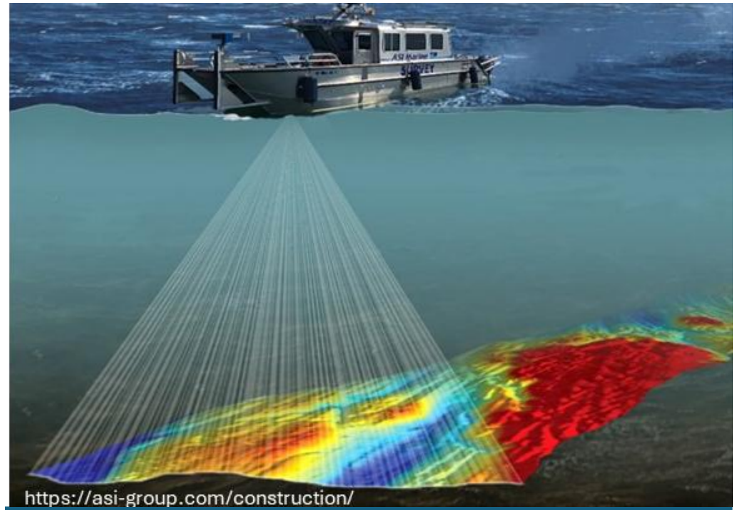
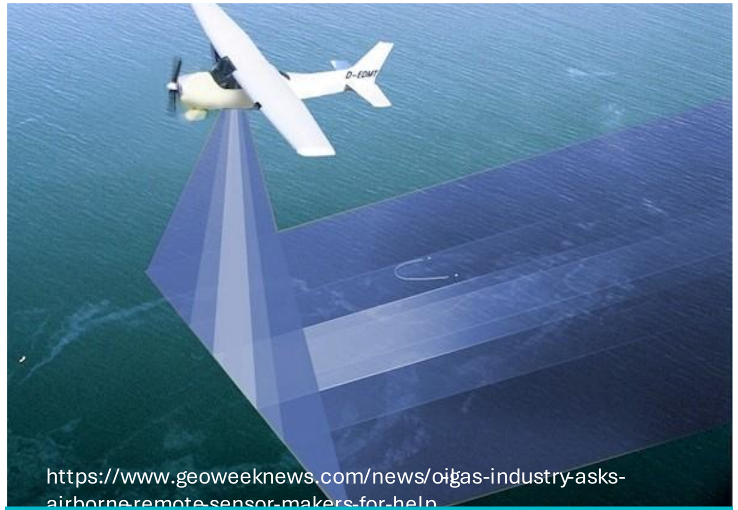
Introduction

Why Sentinel-2 based bathymetry?



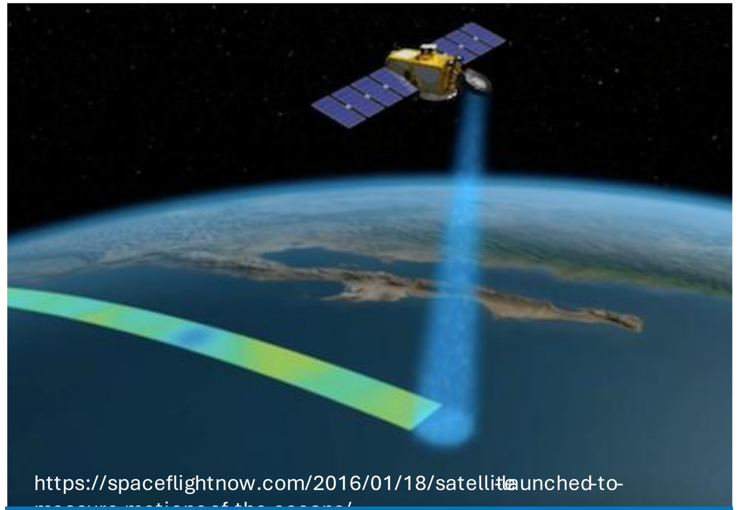
Shipborne

Multi-beam echo sounders (MBES)



Airborne

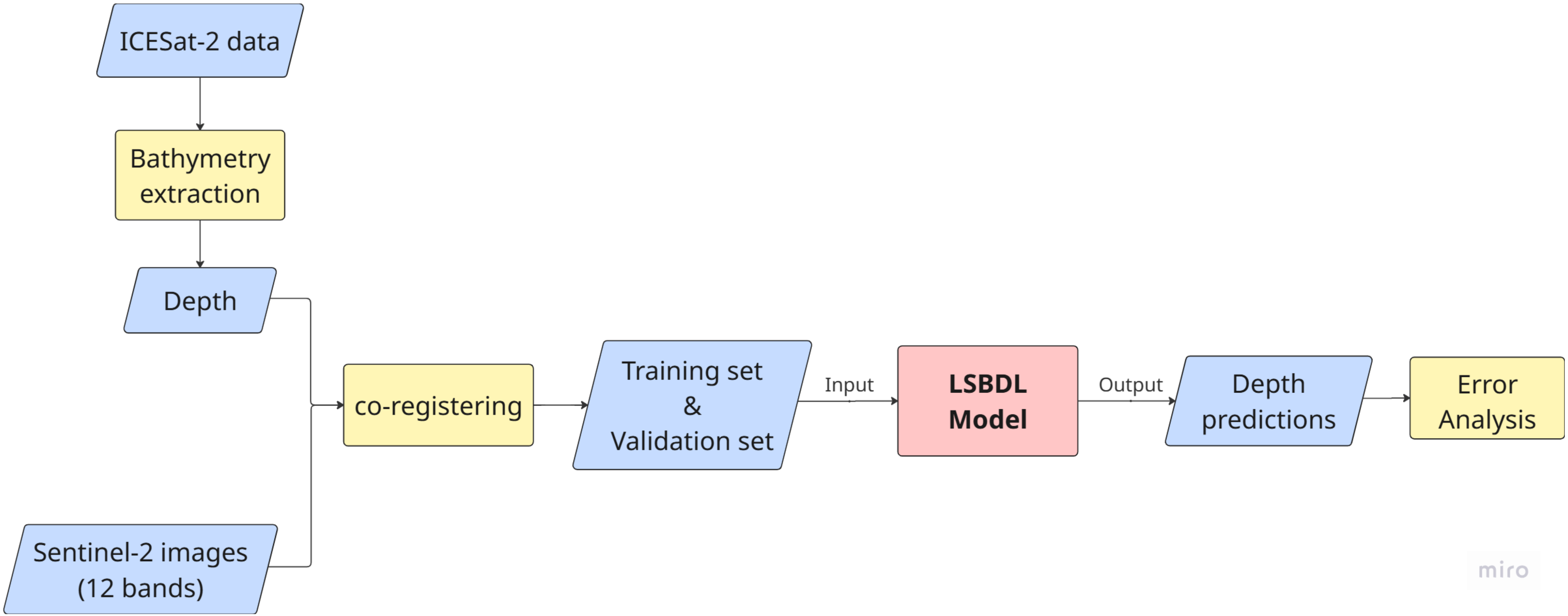
Light detection and ranging (LiDAR)



Spaceborne

Satellite-derived bathymetry (SDB)

Workflow of this paper



Contribution

- Completely Satellite based
- LSBDL Model applied in SDB

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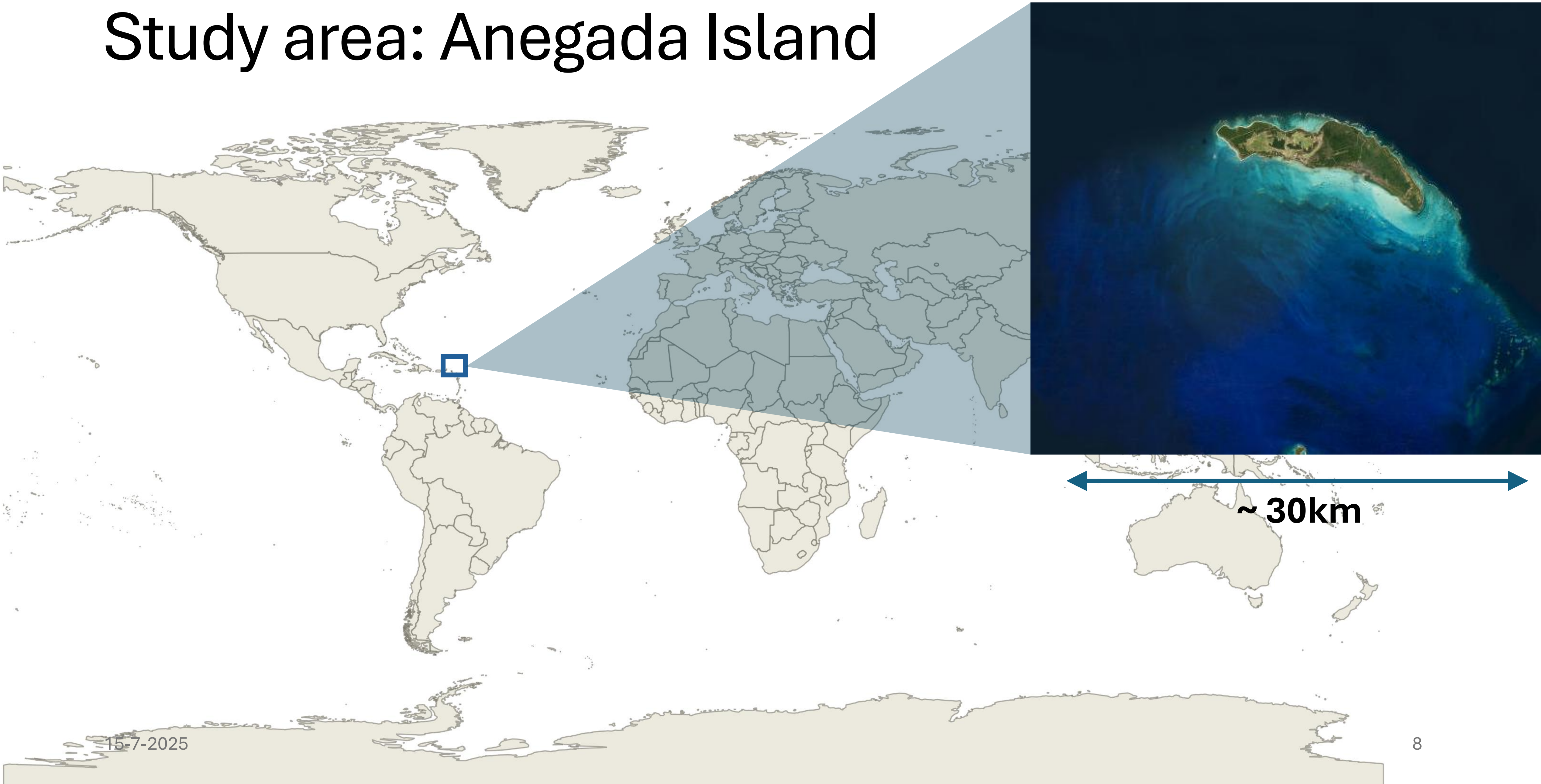
Study Area & Data

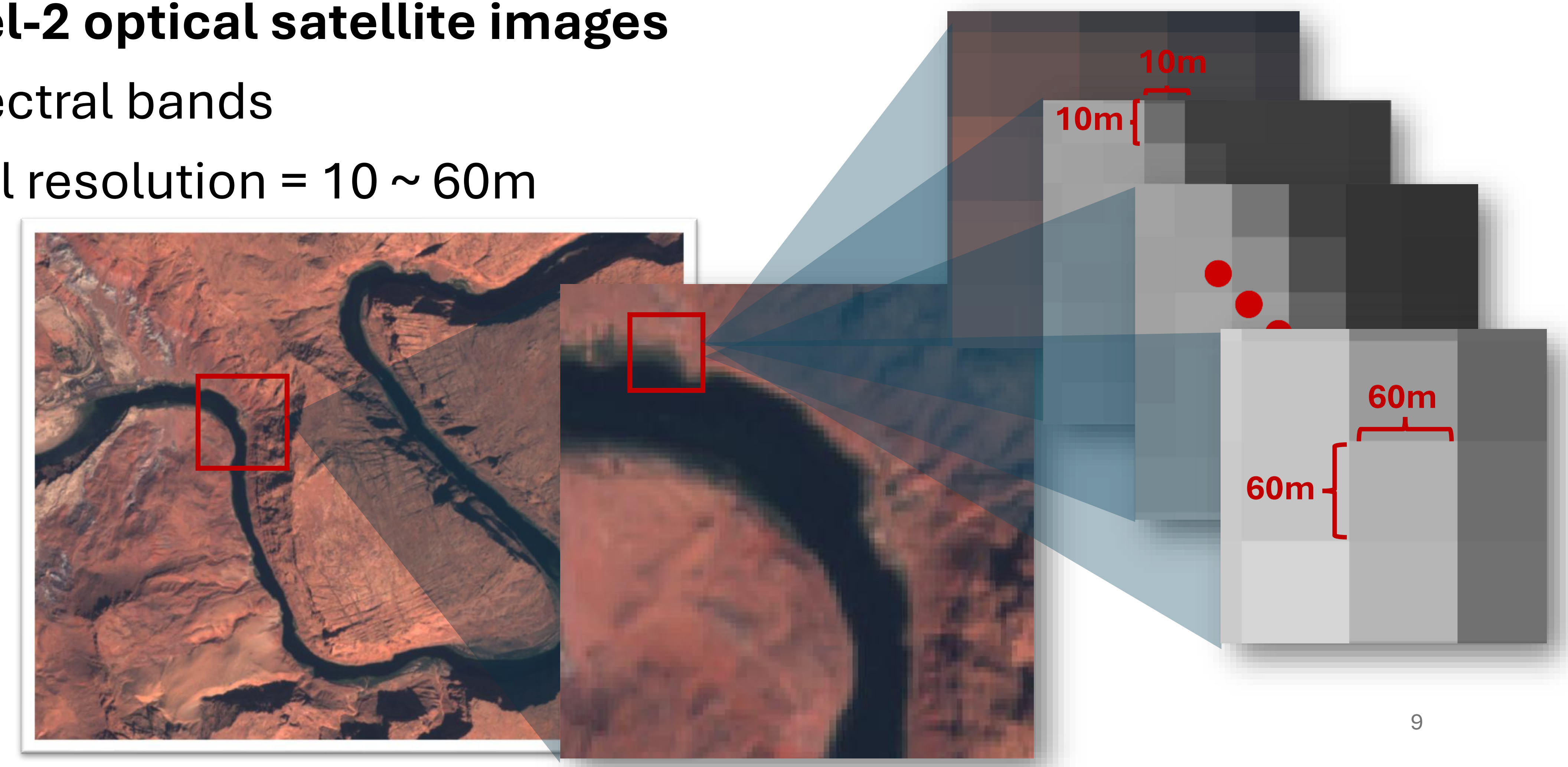
Study area: Anegada Island



Data Used

Sentinel-2 optical satellite images

- 13 spectral bands
- Spatial resolution = 10 ~ 60m



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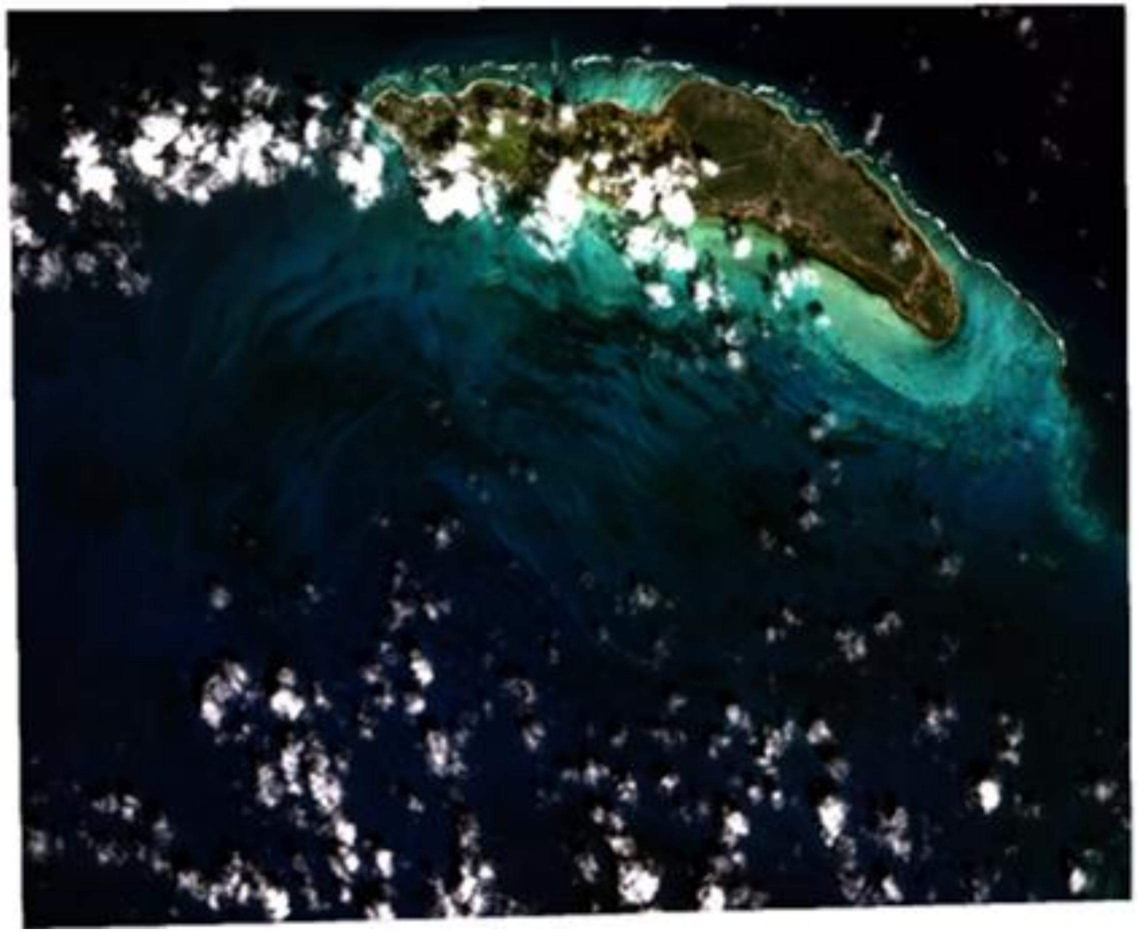

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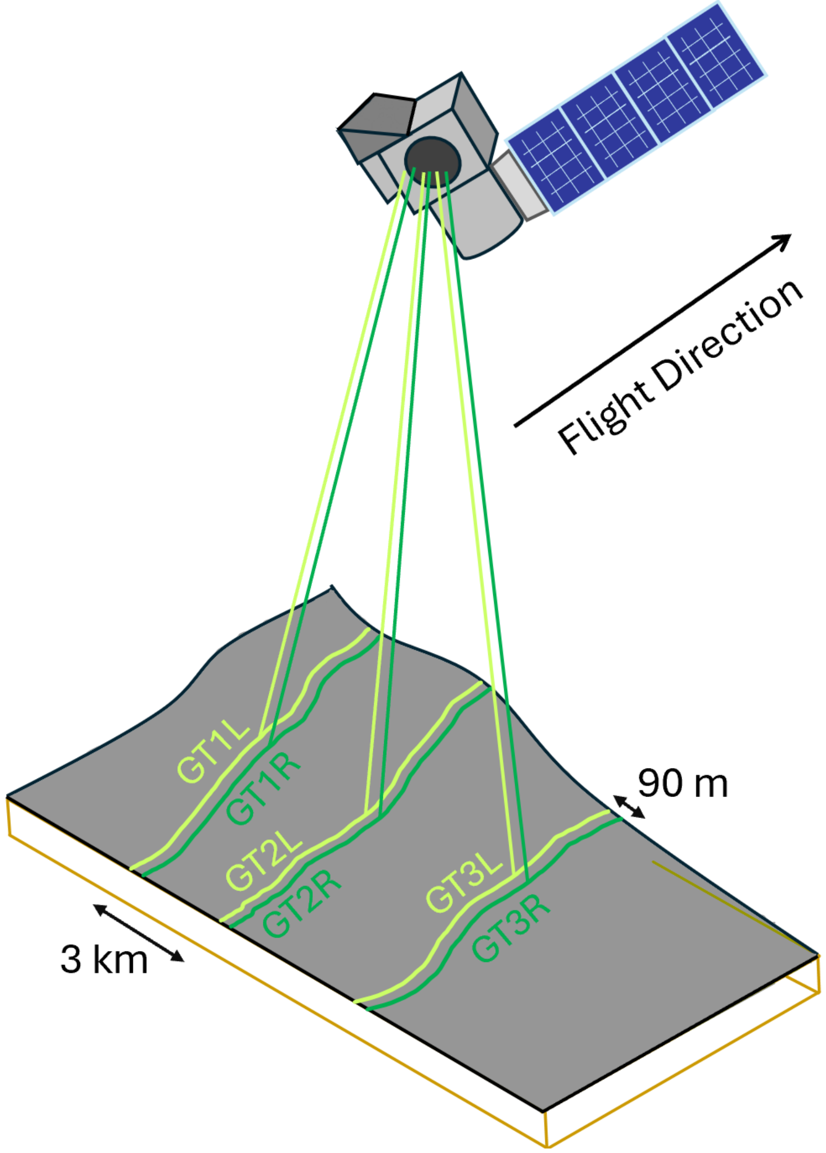
Data Used

ICESat-2 LiDAR data

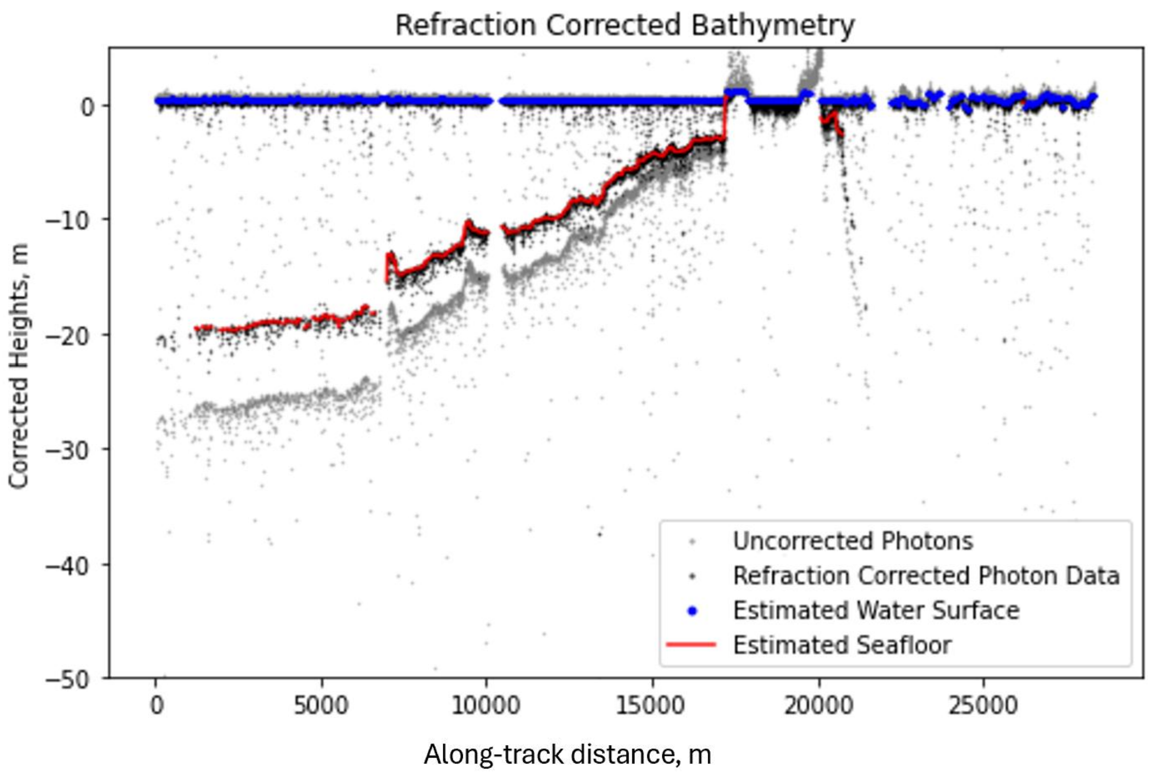
- Measure elevation via laser pulses

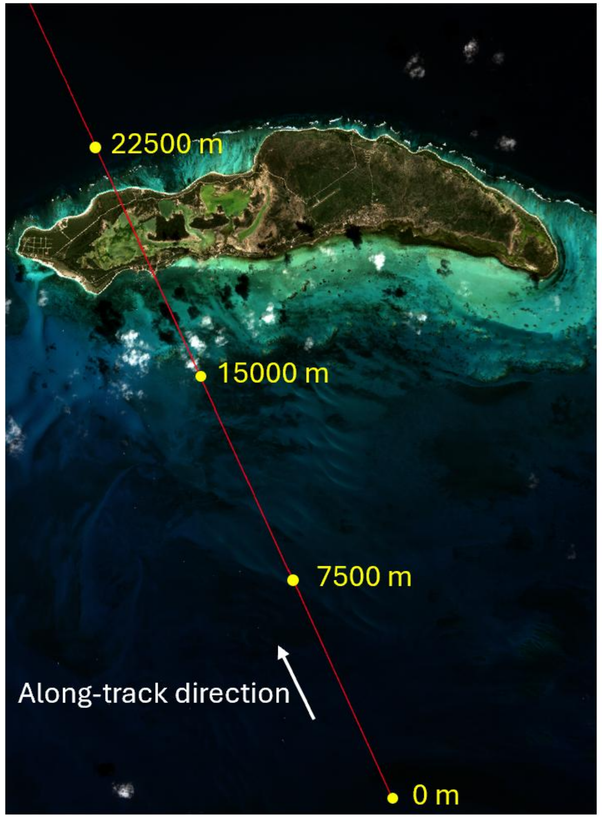


Data Used

ICESat-2 LiDAR data

- Depth Profiles can be computed

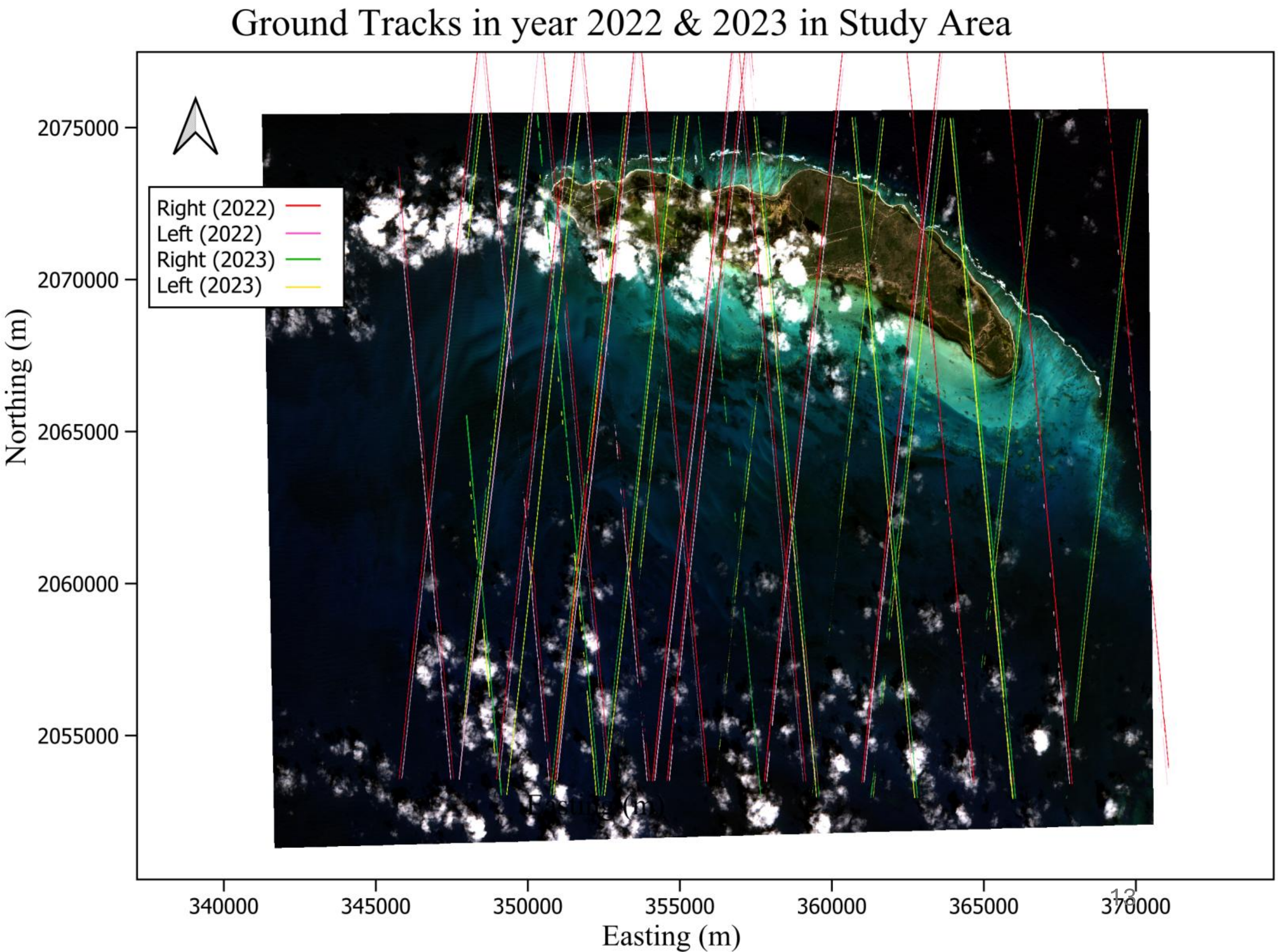




Data Used

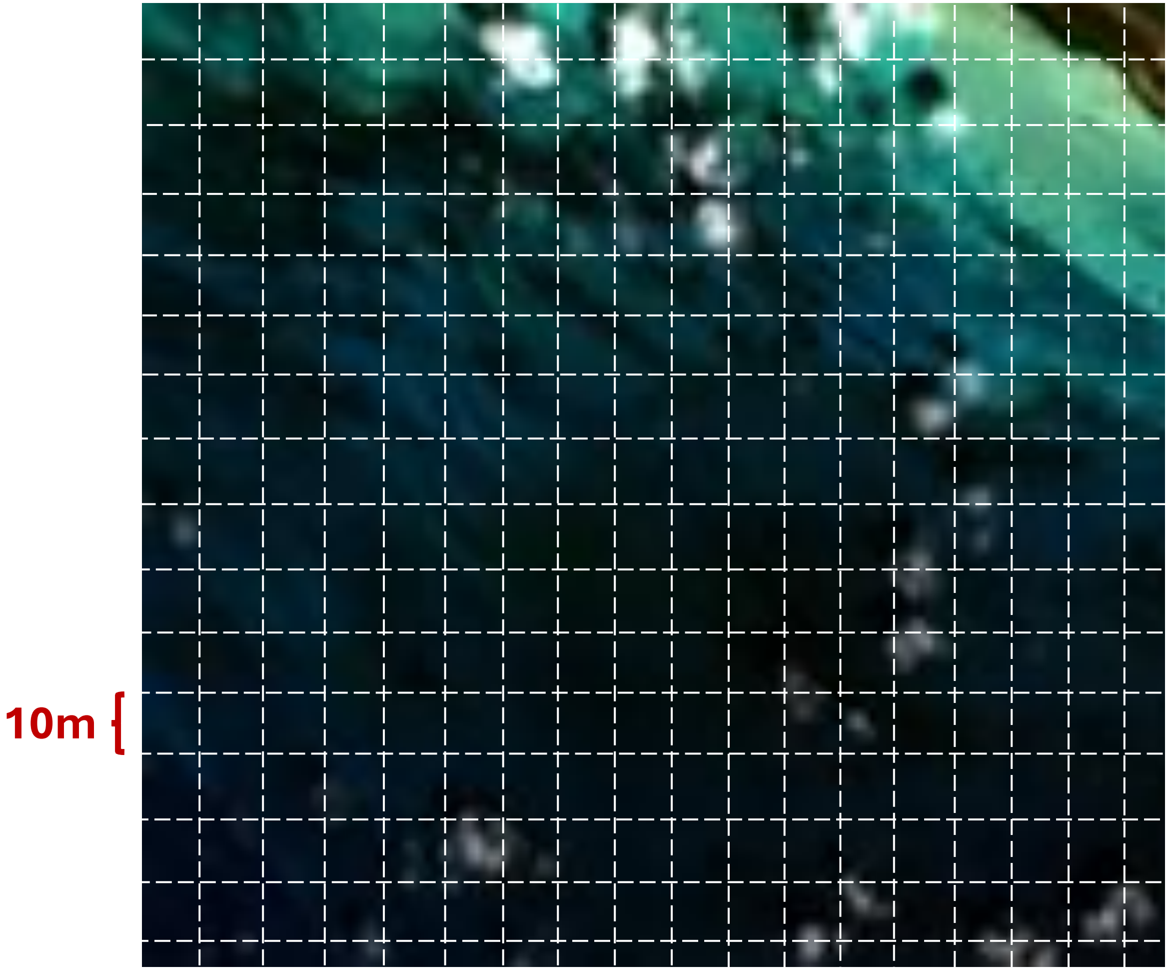
ICESat-2 LiDAR data

- 88 ground tracks
- 40,000 data points



For each data point, we have:

- 12 reflectance values (from different spectral bands of Sentinel-2)
- 1 depth (derived from ICESat-2 data)



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Methodology

Least-squares-based deep learning

Known:

Input:

- Feature matrix D (12 bands)
- Observation vector Y (depth)

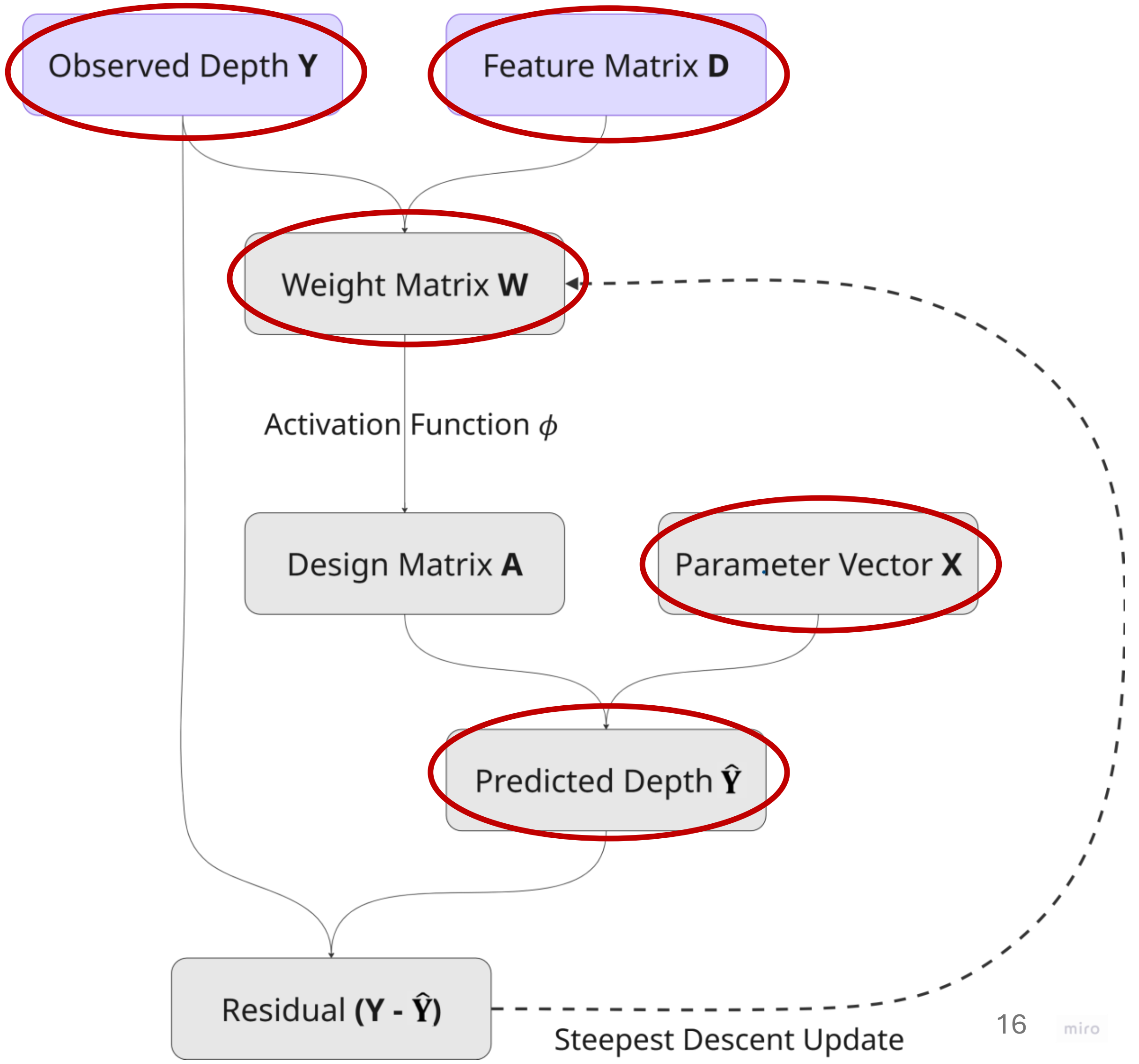
Unknown:

Model Parameters:

- Weight Matrix W
- Parameter Vector X

Output

- Predicted Depth $\hat{\mathbf{Y}}$



Least-squares-based deep learning

Main Strengths:

- Uncertainty Estimation
- Enhanced Explainability

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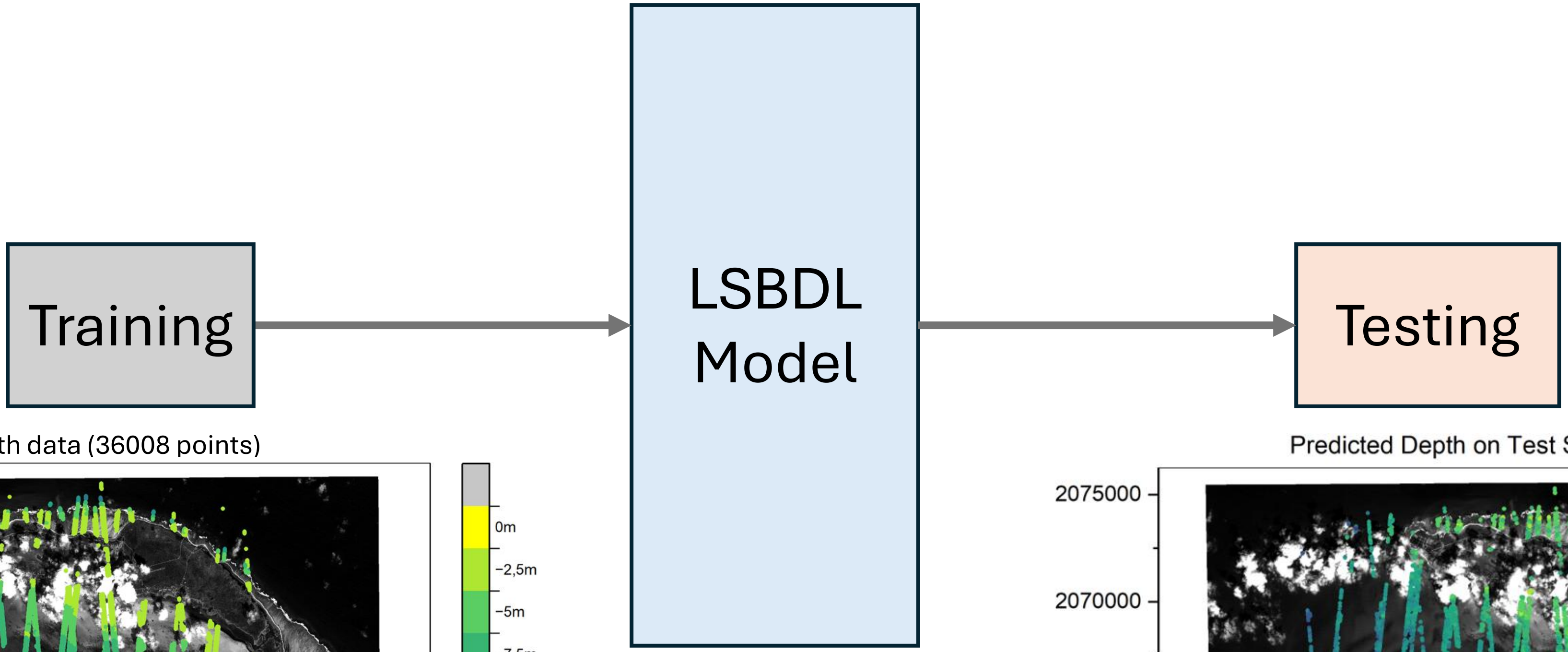
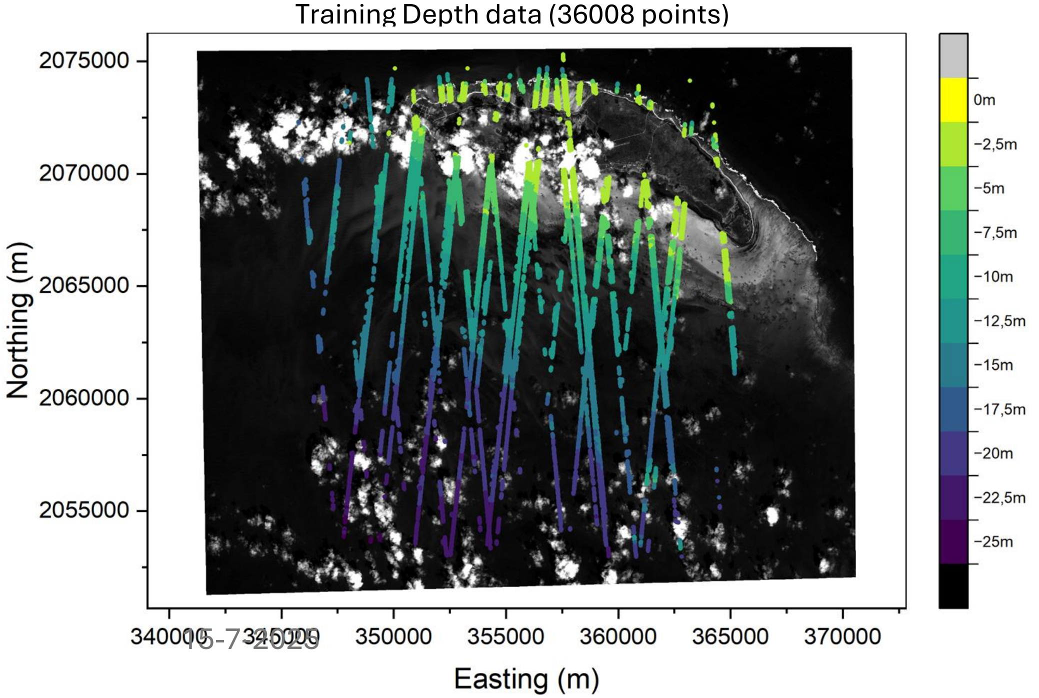
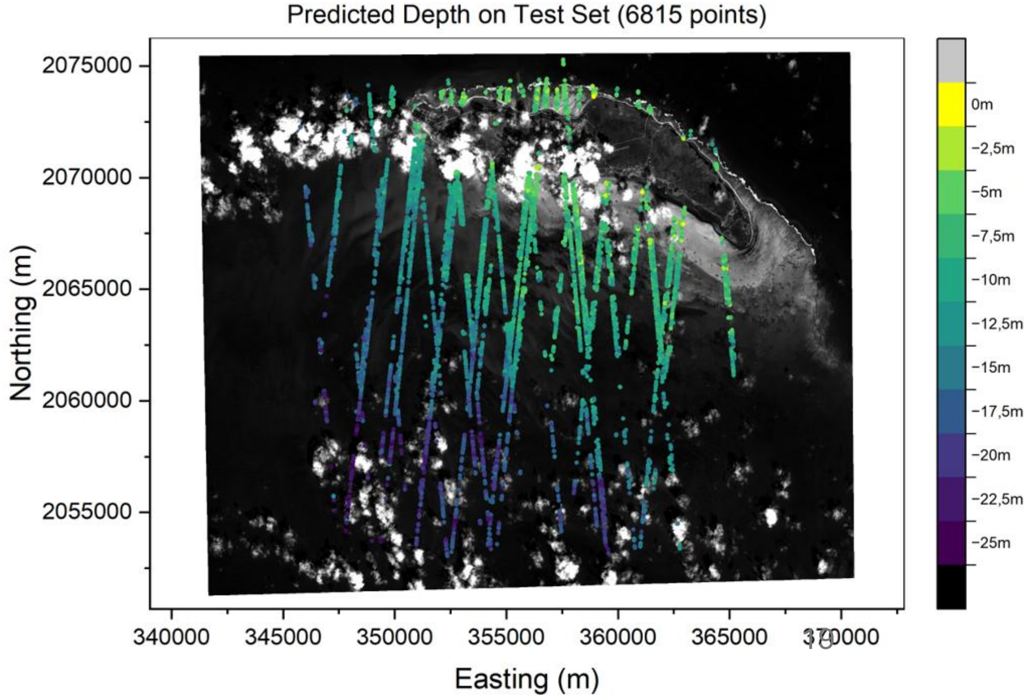
Results & Discussion

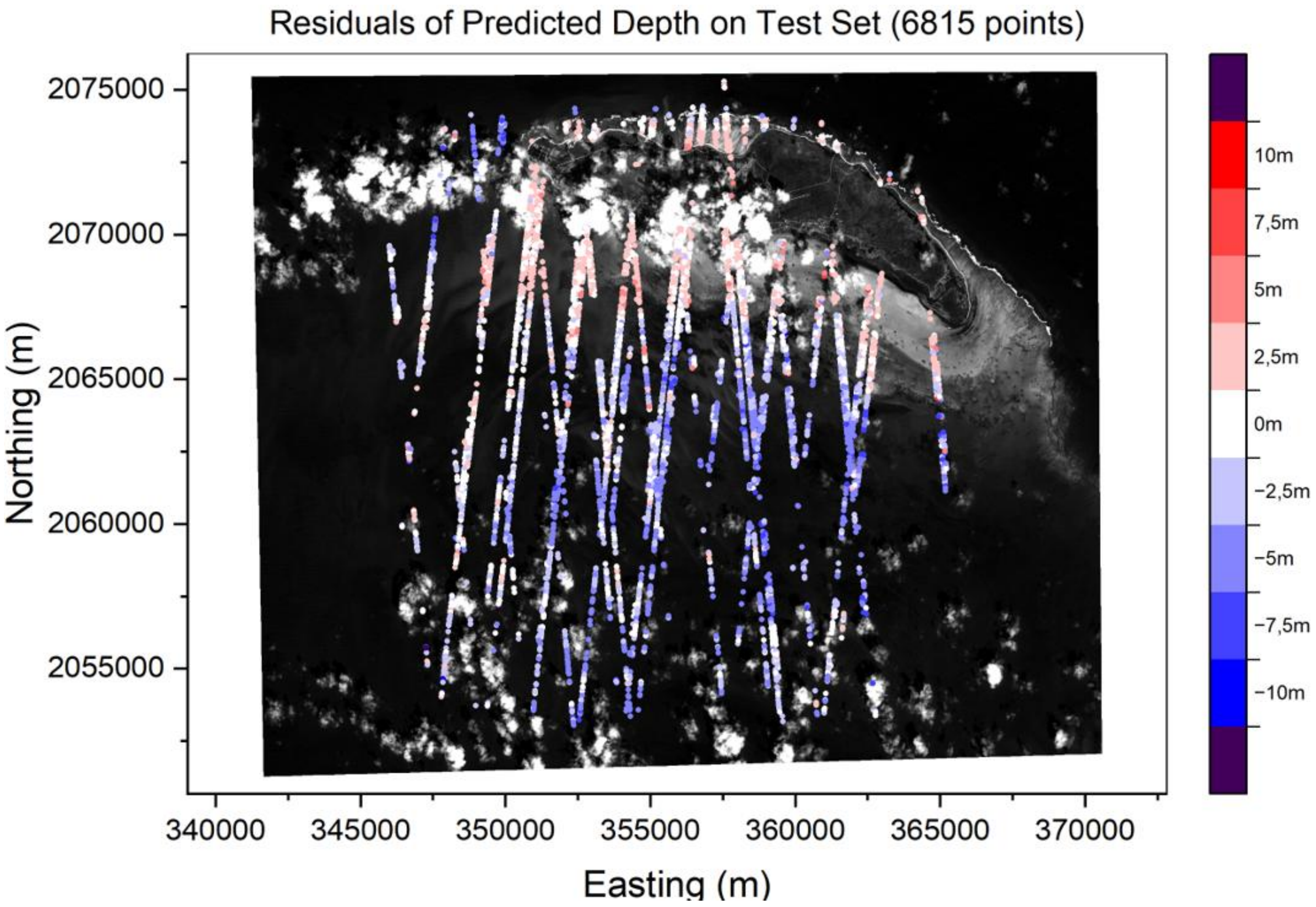
LSBDL model prediction







Accuracy of LSBDL model



RMSE = 2,73 m

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Conclusion & Future Work

- **LSBDL** produces bathymetric maps with accuracy and consistency comparable to traditional methods.

- The **uncertainty estimation** of LSBDL model makes the model more transparent and explainable.(change combine with first one)

- **ICESat-2** bathymetry is accurate, accessible, and serves as a strong reference for true depth.

- Future work:
  - Consider incorporate other parameters
  - Multi-sensor
  - Other types of water areas

Thank you for your attention!

Yushan Liu (yliu74@tudelft.nl)

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Sentinel-2

Sensor	MSI (Multispectral Instrument)
Product Used	L2A (Level-2A) — Bottom-of-atmosphere (surface reflectance) product
Spectral Bands	13 bands total; commonly used for SDB: B2 (Blue), B3 (Green), B4 (Red), B8 (NIR), B11/B12 (SWIR)
Spatial Resolution	10 m (B2, B3, B4, B8); 20 m (B5, B6, B7, B8A, B11, B12); 60 m (B1, B9, B10)
Swath Width	290 km
Revisit Time	5 days (at the equator with both satellites active)

ICESat-2 (Ice, Cloud, and land Elevation Satellite-2)

Instrument	ATLAS (Advanced Topographic Laser Altimeter System)
Product Used	ATL03 V006 — Global Geolocated Photon Data
Wavelength	532 nm (green)
Along-track Resolution	~0.7 m (spot spacing); 6 laser beams (3 pairs: strong/weak)
Vertical Accuracy	~2–3 cm for land/ice; typically ~10–30 cm for bathymetry in clear, calm water
Depth Detection Range	~0–45 m in clear water (best <20 m); returns diminish with turbidity/depth

ICESat-2 data

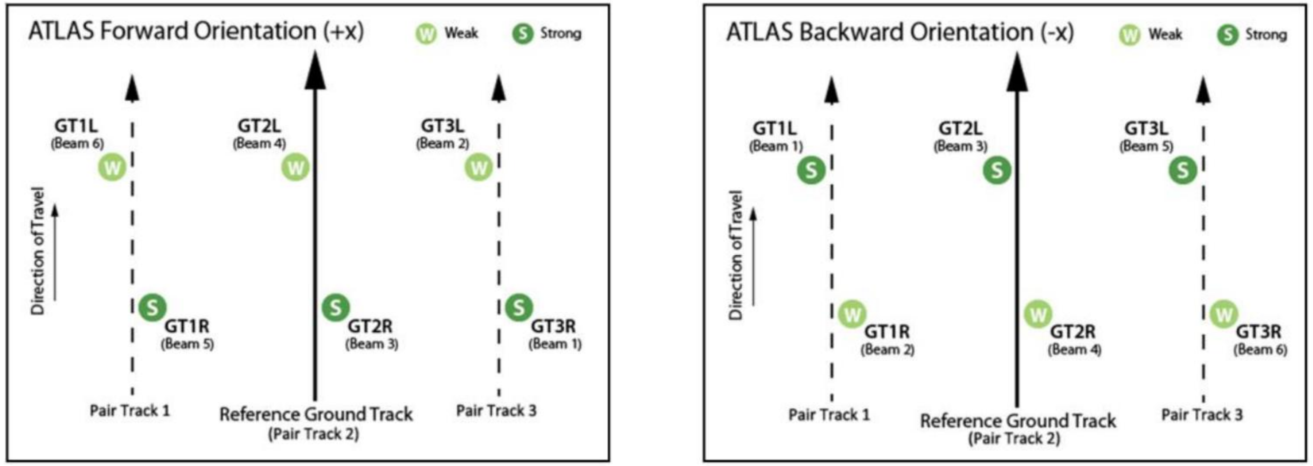


Figure 1. Spot and ground track (GT) naming convention with ATLAS oriented in the forward (instrument coordinate +x) direction and backward (instrument coordinate -x) direction.

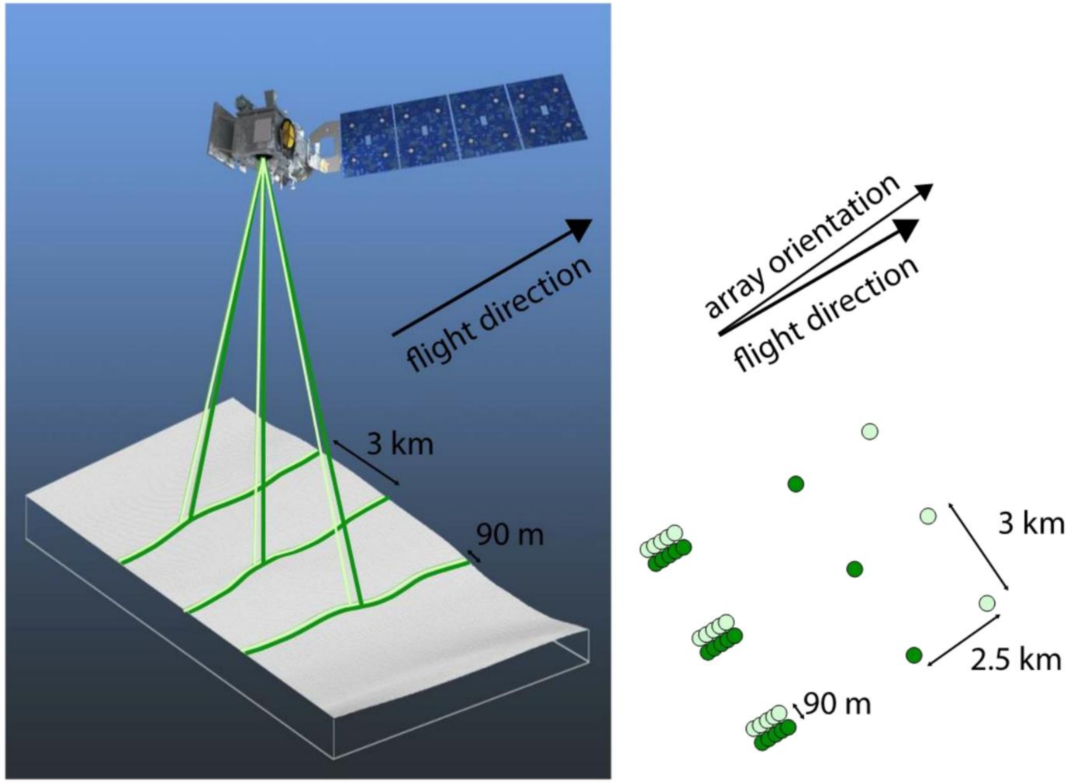


Figure 6. ATLAS idealized beam and footprint pattern. Adapted from the ATL03 ATBD.

Both images taken from USER GUIDE: ATLAS/ICESat-2 L2A Global Geolocated Photon Data, Version 6

Least-squares theory

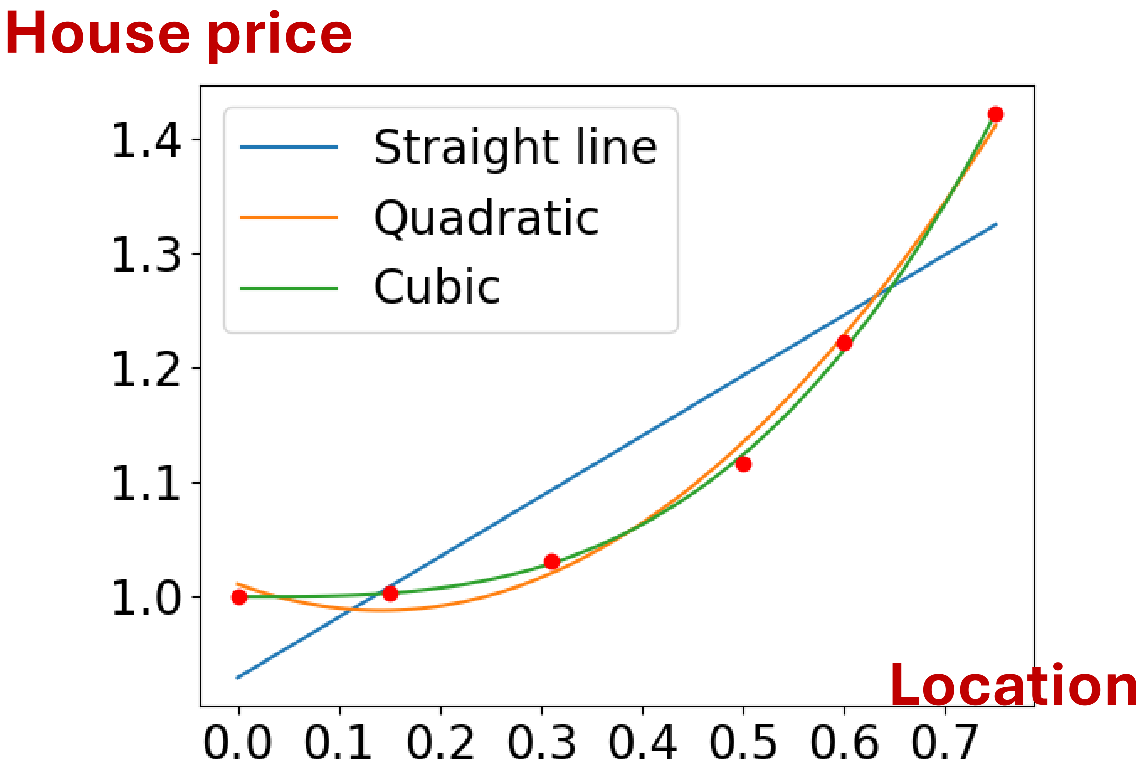
$$\mathbf{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}, \quad \boldsymbol{D}(\boldsymbol{y}) = \boldsymbol{Q_y}$$

$$\hat{x} = (A^T Q_y^{-1} A)^T A^T Q_y^{-1} y$$

$$\hat{y} = P_A y$$

$$\hat{e} = P_A^{\perp} y$$

(where $P_A = (A^T Q_y^{-1} A)^T A^T Q_y^{-1}$, and $P_A^{\perp}$ = I - P_A)



Graph source:
https://en.wikipedia.org/wiki/Linear_least_squares

Least-squares theory

$$\mathbf{Y} = \boldsymbol{\mathcal{A}}(\boldsymbol{DW})\boldsymbol{X} + \boldsymbol{E}, \quad \boldsymbol{Q}_{\boldsymbol{VEC(Y)}} = \boldsymbol{\Sigma} \otimes \boldsymbol{Q}$$

Activation function: $\mathcal{A}(\cdot)$

Feature matrix: $D = \begin{bmatrix} b1_1 & b2_1 & \cdots & b12_1 & 1 \\ b1_2 & b2_2 & \cdots & b12_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ b1_m & b2_m & \cdots & b12_m & 1 \end{bmatrix}$

Design matrix: $A = \mathcal{A}(DW)$

Weight matrix: includes weights (w) and biases (b)

Least-squares-based deep learning

$$\mathbf{Y} = \mathcal{A}(\boldsymbol{DW})\boldsymbol{X} + \boldsymbol{E}, \quad \boldsymbol{Q}_{\boldsymbol{VEC(Y)}} = \boldsymbol{\Sigma} \otimes \boldsymbol{Q}$$

Algorithm 1: Implementation of LSBDL using SD method

Input:
- obtain data/feature matrix D
- observation matrix Y
- matrices Σ and Q

Initialization:
- initialize weight matrix $W^{(0)}$ and $\Delta W^{(0)} = 0$
- set regularization parameter κ
- set learning rate parameter α
- set momentum parameter μ
- set softening parameter s
- set maximum iteration t_{max}
- provide convergence threshold ϵ

BEGIN

Do for $t = 0$ till t_{max}
- compute the activation matrix A and its derivative A'
- obtain the least squares estimates $\hat{X} = (A^T Q^{-1} A + \kappa I_n)^{-1} A^T Q^{-1} Y$
- compute the least squares residuals $\hat{E} = Y - A\hat{X}$
- compute the gradient matrix $\nabla\phi = -D^T(Q^{-1}\hat{E}\Sigma^{-1}\hat{X}^T \odot A')$
- soften the gradient by transformation $\nabla\phi = \mathrm{sgn}(\nabla\phi) \odot |\nabla\phi|^s$
- compute weights' corrections $\Delta W^{(t+1)} = -\alpha\nabla\phi + \mu\Delta W^{(t)}$
- update the weights $W^{(t+1)} = W^{(t)} + \Delta W^{(t+1)}$

if $\|W^{(t)} - W^{(t-1)}\| > \epsilon$
  - break loop

else
  - increase counter $t = t + 1$

end if

End do

END

Algorith Source: Amiri-Simkooei, A., Tiberius, C. and Lindenbergh, R., 2024. Deep learning in standard least-squares theory of linear models: Perspective, development and vision. Engineering Applications of Artificial Intelligence, 138, p.109376.

Uncertainty evaluation of LSBDL:

Covariance matrix of $\hat{\boldsymbol{y}}$:

$$y_{test} = \mathcal{A}\left(D_{test}\widehat{W}\right) = A_{test}\hat{x}$$

$$Q_{y_{test}} = A_{test}\, Q_{\hat{x}}\, {A_{test}}^{T}$$

$$\textbf{std}(\hat{\boldsymbol{y}}) = \sqrt{\boldsymbol{diag}(\boldsymbol{A}_{\boldsymbol{test}}\, \boldsymbol{Q}_{\hat{\boldsymbol{x}}}\, {\boldsymbol{A}_{\boldsymbol{test}}}^{\boldsymbol{T}})}$$

Uncertainty Estimation