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Unconditional convergence and optimal complexity of adaptive iteratively linearized FEM

slides

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Outline

Motivation and context

Adaptive iteratively linearized FEM

Main results

Numerical experiments

Motivation and context

Workflow

a posteriori estimator $\eta_\ell(u_\ell)$ $\approx$ discretization error $|||u^\star - u_\ell^\star|||$

a posteriori estimator $\zeta_\ell(u_\ell)$ $\approx$ solver error $|||u_\ell^\star - u_\ell|||$

MODELING → DISCRETIZATION → SOLVER

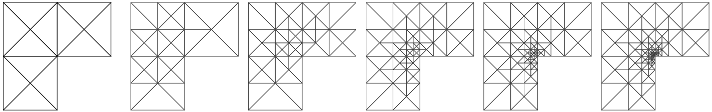
Partial Differential Equation (PDE) $u^\star \in \mathcal{X}$

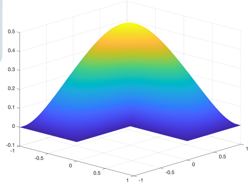
Finite Element Method (FEM) $u_\ell^\star \in \mathcal{X}_\ell^p$

computational mesh $\mathcal{T}_\ell$
polynomial degree p

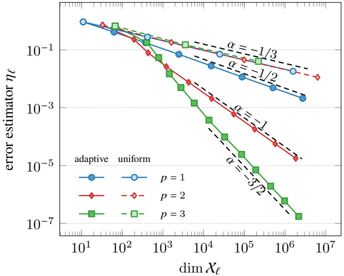
$u_\ell \in \mathcal{X}_\ell^p$

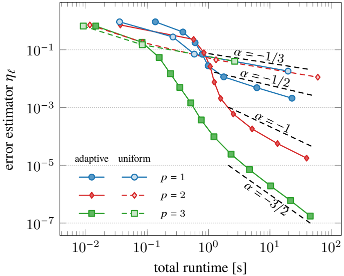
Example: Laplace problem $-\Delta u = 1$ in L-shaped domain with known exact solution
AFEM: SOLVE & ESTIMATE – MARK – REFINE driven by $\eta_\ell(u_\ell)$





Goals: Optimal convergence wrt. dofs and time





Best possible rate is $\alpha = p/d$, we want to achieve it in practice wrt:

dofs: i.e., $\dim \mathcal{X}_\ell^p \eqsim \#\mathcal{T}_\ell$ for fixed p

$$\sup_{\ell \in \mathbb{N}_0} (\#\mathcal{T}_\ell)^\alpha \eta_\ell < \infty$$

overall computational work: assuming

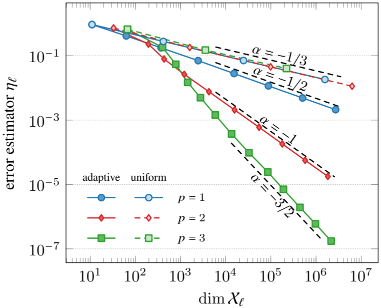
$$\texttt{work}(\mathcal{T}_{\ell'}) \eqsim \#\mathcal{T}_{\ell'}$$

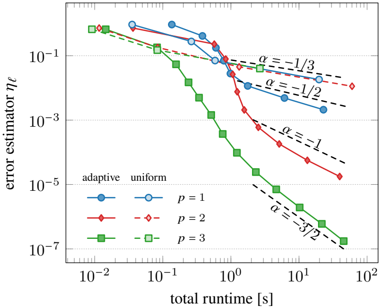
$$\sup_{\ell \in \mathbb{N}_0} \Big(\sum_{\ell'=0}^{\ell} \#\mathcal{T}_{\ell'} \Big)^\alpha \eta_\ell < \infty$$

Unconditional convergence:

The adaptive algorithm should guarantee convergence for arbitrary choice of parameters.

Towards **optimal complexity**





Optimal complexity of AFEMs requires each of its modules to be realized in linear complexity:

- SOLVE is critical
- ESTIMATE ✓
- MARK (Stevenson 2007, Pfeiler-Praetorius 2020 for minimal cardinality marking) ✓
- REFINE (Binev-Dahmen-DeVore 2004, Stevenson 2008) ✓

Adaptive iteratively linearized FEM

Non-linear problem with energy structure

$\Omega \subset \mathbb{R}^d$, $d \in \{1,2,3\}$ a polyhedral Lipschitz domain $\quad f \in L^2(\Omega) \quad \|\cdot\| := \|\nabla \cdot\|_{L^2(\Omega)}$

Scalar nonlinearity: $M\,(t-s) \leq \mu(t^2)t - \mu(s^2)s \leq L\,(t-s) \quad \forall\, 0 \leq s \leq t$

$$-\mathrm{div}\left(\mu(|\nabla u^\star|^2)\nabla u^\star\right) = f \quad \text{in } \Omega, \quad u^\star = 0 \quad \text{on } \partial\Omega$$

Weak formulation: find $u^\star \in H_0^1(\Omega)$ s.t.

$$\langle \mathcal{A}\, u^\star\,,\, v\rangle := \langle \mu(|\nabla u^\star|^2)\nabla u^\star\,,\, \nabla v\rangle_{L^2(\Omega)} = F(v) \quad \forall\, v \in H_0^1(\Omega)$$

- strongly monotone $\quad M\,\|u-v\|^2 \leq \langle \mathcal{A}u - \mathcal{A}v\,,\, u-v\rangle$
- Lipschitz continuous $\quad \langle \mathcal{A}u - \mathcal{A}v\,,\, w\rangle \leq L\,\|u-v\|\,\|w\|$

$\Longrightarrow$ existence and uniqueness of weak solution $u^\star \in H_0^1(\Omega)$

Energy setting

- energy functional $\quad \mathcal{E}(v) := \frac{1}{2}\int_\Omega \int_0^{|\nabla v(x)|^2} \mu(t)\,\mathrm{d}t\,\mathrm{d}x - F(v)$
- energy distance $\quad \mathbb{D}^2(u,v) := \mathcal{E}(v) - \mathcal{E}(u) \qquad$ note: $\mathbb{D}^2(u^\star, v) \approx \|u^\star - v\|^2$

Zeidler: (1990)

Discrete problem and linearization

- $\mathcal{T}_\ell$ a simplicial triangulation of Ω and $p = 1$
- finite element space given by $\mathcal{X}_\ell^p := \{v_\ell \in H_0^1(\Omega) : v_\ell|_T \in \mathbb{P}_p(T)\ \forall T \in \mathcal{T}_\ell\}$
- seek $u_\ell^\star \in \mathcal{X}_\ell^p$ solution to

$$\langle \mathcal{A}\, u_\ell^\star\,,\, v_\ell\rangle := \langle \mu(|\nabla u_\ell^\star|^2)\nabla u_\ell^\star\,,\, \nabla v_\ell\rangle_{L^2(\Omega)} = F(v_\ell) \quad \forall\, v_\ell \in \mathcal{X}_\ell^p.$$

Idea: combine mesh-refinement (ℓ) with a linearization iteration (k) and algebraic solver (j) to compute

$$u_\ell^{k,j} \approx u_\ell^{k,\star} \approx u_\ell^\star \approx u^\star \quad (\ell,k,j) \in \mathcal{Q} \subset \mathbb{N}_0^3$$

Kačanov linearization: the iteration mapping $\Phi_\ell \colon \mathcal{X}_\ell^p \to \mathcal{X}_\ell^p$ given as

$$\langle \mu(|\nabla u_\ell|^2)\, \nabla\Phi(u_\ell)\,,\, \nabla v_\ell\rangle_{L^2(\Omega)} = F(v_\ell) \quad \text{for all} \quad u_\ell, v_\ell \in \mathcal{X}_\ell$$

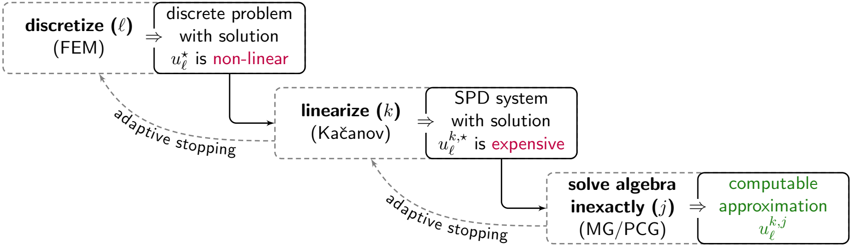
Parameter-free method

- growth condition: $0 < M \le \mu(|\nabla u_\ell|^2) \le L/3 \quad \forall u_\ell \in \mathcal{X}_\ell^p$
- uniform energy contraction : $q_{\rm lin}^\star \in (0,1)$ such that $\mathbb{D}^2(\Phi_\ell(u_\ell), u_\ell^\star) \le q_{\rm lin}^\star\, \mathbb{D}^2(u_\ell, u_\ell^\star) \quad \forall u_\ell \in \mathcal{X}_\ell^p$
- note that: $\mathbb{D}^2(u_\ell^\star, v_\ell) \eqsim \|u_\ell^\star - v_\ell\|^2 \quad \forall v_\ell \in \mathcal{X}_\ell^p$

Heid, Praetorius, Wihler: *Comput. Methods Appl. Math.*, 21 (2021)

Adaptive approach with nested solvers

Adaptive iteratively linearized finite element method (AILFEM) denote $u_\ell^{k,\star} := \Phi_\ell(u_\ell^{k-1,J})$



Requirements from the linearization and algebraic solver

- uniform energy contraction: $q_{\rm lin}^\star \in (0,1)$ such that $\mathbb{D}^2(u_\ell^{k+1,\star}, u_\ell^\star) \le q_{\rm lin}^\star\, \mathbb{D}^2(u_\ell^{k,J}, u_\ell^\star) \quad \forall k \in \mathbb{N}_0$
- uniform norm contraction: $q_{\rm alg} \in (0,1)$ such that $\|u_\ell^{k,\star} - u_\ell^{k,j}\| \le q_{\rm alg}\, \|u_\ell^{k,\star} - u_\ell^{k,j-1}\| \quad \forall j \in \mathbb{N}_0$
- linear cost: computing $u_\ell^{k,j+1}$ from $u_\ell^{k,j}$

Chen, Nochetto, Xu: *Numer. Math.*, 120 (2012)

Wu, Zheng: *Appl. Numer. Math.*, 113 (2017)

Innerberger, Miraçi, Praetorius, Streitberger: *ESAIM Math. Model. Numer. Anal.*, 58 (2024)

Algorithm

Input *initial mesh* $\mathcal{T}_0$, *initial guess* $u_0^0 := 0 \in \mathcal{X}_0^p$,
adaptivity parameter $0 < \theta \leq 1$, *solver-stopping parameter* $\lambda_{\mathrm{lin}} > 0$

For each $\ell = 0, 1, 2, \ldots$ *repeat* [mesh-refinement loop]

- **SOLVE & ESTIMATE** *For* $k = 1, 2, \ldots, K$, *repeat* [linearization loop]
  - *Define* $u_\ell^{k,0} := u_\ell^{k-1,J}$ *and set up linearized problem with solution* $u_\ell^{k,\star} = \Phi_\ell(u_\ell^{k-1,J})$
  - *For* $j = 1, 2, \ldots, J$, *repeat* [algebra loop]
    compute $u_\ell^{k,j} \approx u_\ell^{k,\star}$ *from the previous step* $u_\ell^{k,j-1}$ *by the algebraic solver*
  - *until* **[algebra-criterion]**

  until $\mathbb{D}^2(u_\ell^{k,J}, u_\ell^{k-1,J}) \leq \lambda_{\mathrm{lin}} \eta_\ell(u_\ell^{k,J})^2 \longrightarrow$ *idea: balance linearization and discretization error*
- **MARK** *choose* $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ *such that* $\theta \sum_{T \in \mathcal{T}_\ell} \eta_\ell(T, u_\ell^K)^2 \leq \sum_{T \in \mathcal{M}_\ell} \eta_\ell(T, u_\ell^K)^2$
- **REFINE** $\mathcal{T}_{\ell+1} := \mathtt{refine}(\mathcal{T}_\ell, \mathcal{M}_\ell)$

  $u_{\ell+1}^{0,0} := u_\ell^{K,J} \longrightarrow$ *nested iteration with error control on all* $u_\ell^{k,j}$ *except* $u_0^{0,0}$

Output *Discrete solutions* $u_\ell^{K,J}$ *and corresponding estimators* $\eta_\ell(u_\ell^{K,J})$

Termination of algebraic solver I

- the stopping criterion should guarantee that nested linearization-algebraic solver contracts in energy

$$\mathbb{D}(u_\ell^\star, u_\ell^{k,J}) \le q_{\rm ctr}\, \mathbb{D}(u_\ell^\star, u_\ell^{k-1,J}) \qquad 0 < q_{\rm ctr} < 1$$

Equilibration criterion [HPSV21]

- stop algebraic solver if $\|u_\ell^{k,J} - u_\ell^{k,J-1}\| \le \mu\,[\,\eta_\ell(u_\ell^{k,J}) + \|u_\ell^{k,J} - u_\ell^{k-1,J}\|\,]$
- stop linearization if $\|u_\ell^{K,J} - u_\ell^{K-1,J}\| \le \lambda\,\eta_\ell(u_\ell^{K,J})$

$\Longrightarrow$ full R-linear convergence for arbitrary λ but sufficiently small μ

Note: there exists $C_{\rm nrg}^\star > 0$ st. $C_{\rm nrg}^\star \|u_\ell^{k,\star} - u_\ell^{k-1,J}\|^2 \le \mathbb{D}^2(u_\ell^{k,\star}, u_\ell^{k-1,J})$

Energy-based criterion [MPS24+]

- enforce algorithmically $\|u_\ell^{k,J} - u_\ell^{k,J-1}\| \lesssim \mathbb{D}(u_\ell^{k,J}, u_\ell^{k,J-1})$ **(parameter-free)**
- stop linearization if $\|u_\ell^{K,J} - u_\ell^{K-1,J}\| \le \lambda\,\eta_\ell(u_\ell^{K,J})$

$\Longrightarrow$ full R-linear convergence for arbitrary $\lambda > 0$

Haberl, Praetorius, Schimanko, Vohralík: *Numer. Math.*, 147 (2021)

Miraçi, Praetorius, Streitberger: arXiv: 2401.17778 (2024)

Termination of algebraic solver II

Recall: there exists $C^\star_{\rm nrg} > 0$ st. $C^\star_{\rm nrg} \| u_\ell^{k,\star} - u_\ell^{k-1,J} \|^2 \le \mathbb{D}^2(u_\ell^{k,\star}, u_\ell^{k-1,J})$

Energy-based criterion [MPS24+]

- Initialize threshold bounds $\alpha_{\min} = 100$, $J_{\max} = 1$, and reduction factor $\rho = 0.5$
- In each inner algebra loop, in addition to $u_\ell^{k,j}$ compute:

$$\alpha_\ell^{k,j} := \mathbb{D}^2(u_\ell^{k,j}, u_\ell^{k-1,J}) / \| u_\ell^{k,j} - u_\ell^{k-1,J} \|^2$$

  Until either $\alpha_\ell^{k,j} \ge \alpha_{\min}$ or $u_\ell^{k,j} = u_\ell^{k-1,J}$ or $[\alpha_\ell^{k,j} > 0$ and $j > J_{\max}]$.

- If $J[\ell,k] > J_{\max}$, then update $J_{\max} \leftarrow J[\ell,k]$ and $\alpha_{\min} \leftarrow \rho\, \alpha_{\min}$.

Proposition (uniform bound on algebraic steps)

- There exists an index $j_0 \in \mathbb{N}$ such that $J[\ell,k] \le j_0$ for all $(\ell,k,0) \in \mathcal{Q}$
- There exists $0 < C_{\rm nrg} < C^\star_{\rm nrg}$ such that

$$C_{\rm nrg} \| u_\ell^{k,J} - u_\ell^{k-1,J} \|^2 \le \mathbb{D}(u_\ell^{k,J}, u_\ell^{k-1,J}) \quad \text{for all } (\ell,k,0) \in \mathcal{Q} \text{ with } k \ge 1.$$

Main results

A posteriori error control

Note that for each iterate u_ℓ^k computed by the adaptive algorithm, there holds

$$\underbrace{|||u^\star - u_\ell^{k,j}|||}_{\text{overall error}} \le \underbrace{|||u^\star - u_\ell^\star|||}_{\text{discretization error}} + \underbrace{|||u_\ell^\star - u_\ell^{k,\star}|||}_{\text{linearization error}} + \underbrace{|||u_{k,\ell}^\star - u_\ell^{k,j}|||}_{\text{algebraic error}}$$

$$\overset{\text{reliability}}{\lesssim} \underbrace{\underbrace{\eta_\ell(u_\ell^\star)}_{\text{discretization error estimator}} + |||u_\ell^\star - u_\ell^{k,\star}||| + |||u_\ell^\star - u_\ell^{k,j}|||}_{\text{quasi-error } \mathrm{H}_\ell^{k,j}}$$

$$\overset{\text{stability}}{\le} \eta_\ell(u_\ell^{k,j}) + (C_{\text{stab}} + 1)\,|||u_\ell^\star - u_\ell^{k,\star}||| + (C_{\text{stab}} + 1)\,|||u_\ell^\star - u_\ell^{k,j}|||$$

$$\overset{\text{contractive solvers}}{\lesssim} \eta_\ell(u_\ell^k) + \underbrace{|||u_\ell^{k-1,J} - u_\ell^{k,j}|||}_{\text{linearization error estimator}} + \underbrace{|||u_\ell^{k,j-1} - u_\ell^{k,j}|||}_{\text{algebraic error estimator}}$$

Main results I: full R-linear convergence

Theorem (full R-linear convergence of the quasi-error)

Consider arbitrary $0 < \theta \leq 1$, $\lambda_{\mathrm{lin}} > 0$. For all $(\ell', k', j'), (\ell, k, j) \in \mathcal{Q}$ with $|\ell', k', j'| > |\ell, k, j|$, there holds

$$\mathrm{H}_{\ell'}^{k',j'} \leq C_{\mathrm{lin}} \, q_{\mathrm{lin}}^{|\ell',k',j'|-|\ell,k,j|} \, \mathrm{H}_{\ell}^{k,j}, \quad C_{\mathrm{lin}} > 0, \; 0 < q_{\mathrm{lin}} < 1.$$

Contraction regardless of algorithmic step: mesh-refinement or algebraic solver step.

1. algebraic solver norm contraction $\quad |||u_\ell^{k,\star} - u_\ell^{k,j}||| \leq q_{\mathrm{alg}} \, |||u_\ell^{k,\star} - u_\ell^{k,j-1}|||$
2. linearization energy contraction $\quad \mathbb{D}^2(u_\ell^{k,\star}, u_\ell^\star) \leq q_{\mathrm{lin}}^\star \, \mathbb{D}^2(u_\ell^{k-1,J}, u_\ell^\star)$
3. reduction, Dörfler, nested iteration $\quad \eta_{\ell+1}(u_{\ell+1}^{K,J}) \leq q_\theta \, \eta_\ell(u_\ell^{K,J}) + C \, |||u_{\ell+1}^{K,J} - u_\ell^{K,J}|||$
4. energy orthogonality $\quad \mathbb{D}^2(u_{\ell+1}^\star, u^\star) + \mathbb{D}^2(u_\ell^\star, u_{\ell+1}^\star) = \mathbb{D}^2(u_\ell^\star, u^\star)$
5. tail summability $\quad \mathrm{H}_\ell^k \lesssim q_{\mathrm{lin}}^{|\ell,k,j|-|\ell',k',j'|} \, \mathrm{H}_{\ell'}^{k',j'} \iff \sum_{|\ell,k,j|>|\ell',k',j'|} \mathrm{H}_\ell^{k,j} \lesssim \mathrm{H}_{\ell'}^{k',j'}$

Corollary (unconditional convergence)

$$|||u^\star - u_\ell^k||| \lesssim \mathrm{H}_\ell^k \lesssim q_{\mathrm{lin}}^{|\ell,k,j|} \, \mathrm{H}_0^0 \longrightarrow 0 \quad \text{as} \quad |\ell,k,j| \longrightarrow \infty$$

Gantner, Haberl, Praetorius, Schimanko: *Math. Comp.*, 90 (2021)

Bringmann, Feischl, Miraçi, Praetorius, Streitberger: arXiv: 2311.15738 (2024)

Full R-linear convergence yields rates = complexity

- $\mathfrak{R}(\alpha) := \sup_{(\ell,k,j)\in\mathcal{Q}} (\#\mathcal{T}_\ell)^\alpha \mathrm{H}_\ell^{k,j} < \infty$ rate α wrt. $\dim \mathcal{X}_\ell \eqsim \#\mathcal{T}_\ell$ is achievable
- $\widehat{\mathfrak{R}}(\alpha) := \sup_{(\ell,k,j)\in\mathcal{Q}} \Big(\sum_{\substack{(\ell',k',j')\in\mathcal{Q} \\ |\ell',k',j'|\le|\ell,k,j|}} \#\mathcal{T}_{\ell'} \Big)^\alpha \mathrm{H}_\ell^{k,j} < \infty$ rate α wrt. total cost/runtime is achievable

Proposition

Suppose full R-linear convergence of $\mathrm{H}_\ell^{k,j}$, then there holds $\mathfrak{R}(\alpha) \le \widehat{\mathfrak{R}}(\alpha) \le \frac{C_{\mathrm{lin}}}{(1-q_{\mathrm{lin}}^{1/\alpha})^\alpha} \mathfrak{R}(\alpha)$.

- Proof: $\#\mathcal{T}_{\ell'} \le \mathfrak{R}(\alpha)^{\frac{1}{\alpha}} (\mathrm{H}_{\ell'}^{k',j'})^{-\frac{1}{\alpha}}$ $\forall\, (\ell',k',j') \in \mathcal{Q}$, sum and use geometric series:

$$\Big(\sum_{\substack{(\ell',k',j')\in\mathcal{Q} \\ |\ell',k',j'|\le|\ell,k,j|}} \#\mathcal{T}_{\ell'} \Big)^\alpha \le \mathfrak{R}(\alpha)^{\frac{1}{\alpha}} \sum_{\substack{(\ell',k',j')\in\mathcal{Q} \\ |\ell',k',j'|\le|\ell,k,j|}} (\mathrm{H}_{\ell'}^{k',j'})^{-\frac{1}{\alpha}} \le \mathfrak{R}(\alpha)^{\frac{1}{\alpha}} C_{\mathrm{lin}}^{\frac{1}{\alpha}} \Big(\sum_{\substack{(\ell',k',j')\in\mathcal{Q} \\ |\ell',k',j'|\le|\ell,k,j|}} q_{\mathrm{lin}}^{(|\ell,k,j|-|\ell',k',j'|)/\alpha} \Big) (\mathrm{H}_\ell^{k,j})^{-\frac{1}{\alpha}}$$

Main results II: optimal complexity of AFEM

Theorem (optimal convergence with respect to overall computational cost)

Consider arbitrary $\alpha > 0$ and $\|u^\star\|_{\mathbb{A}_\alpha} := \sup_{N \geq \#\mathcal{T}_0} N^\alpha \big[\min_{\#\mathcal{T}_{\rm opt} \leq N} \eta_{\rm opt}(u^\star_{\rm opt}) \big] < \infty$.

Let full R-linear convergence hold. For sufficiently small $0 < \theta < 1$ and $\lambda_{\rm lin} > 0$

$$\Longrightarrow \quad \|u^\star\|_{\mathbb{A}_\alpha} \lesssim \sup_{(\ell,k,j)\in\mathcal{Q}} \Big(\sum_{\substack{(\ell',k',j')\in\mathcal{Q} \\ |\ell',k',j'| \leq |\ell,k,j|}} \#\mathcal{T}_{\ell'} \Big)^\alpha \mathrm{H}_\ell^{k,j} \lesssim \max\big\{ \|u^\star\|_{\mathbb{A}_\alpha}, \mathrm{H}_0^0) \big\}$$

if $u^\star$ *can be approximated at rate* α *over dofs and there holds full R-linear convergence,*
then AFEM approximates $u^\star$ *at rate* α *with respect to overall computational cost.*

1. $0 < \theta \ll 1 \quad \Longrightarrow \quad$ *optimal rates* for AFEM with exact solver
2. $0 < \lambda_{\rm lin} \ll \theta \quad \Longrightarrow \quad u_\ell^{K,J} \approx u_\ell^\star$ and Dörfler marking is equivalent

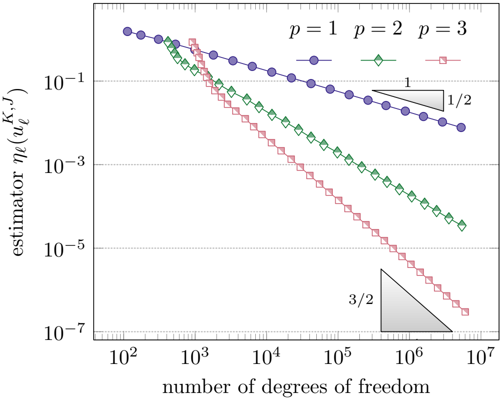
Stevenson: *Found. Comput. Math.*, 7 (2007)
Carstensen, Feischl, Page, Praetorius: *Comput. Math. Appl.*, 67 (2014)
Gantner, Haberl, Praetorius, Schimanko: *Math. Comp.*, 90 (2021)

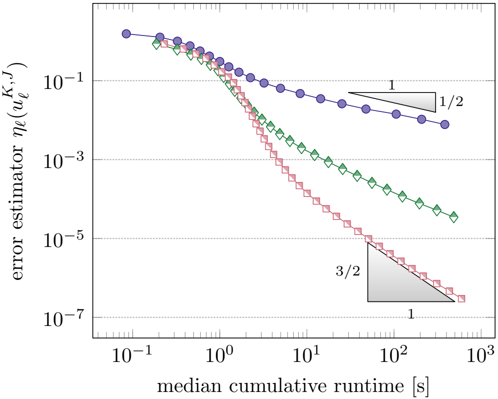
Numerical experiments

Optimality of AILFEM

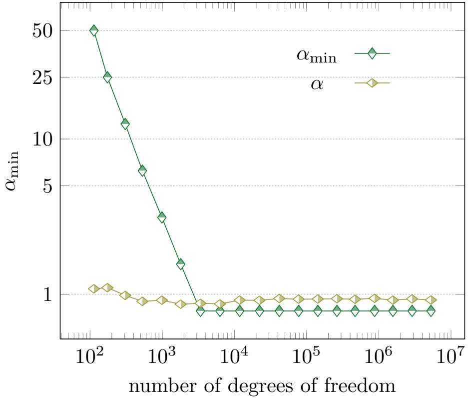
Test case: L-shaped domain and coefficient $\mu(t) = 1 + \exp(-t)$

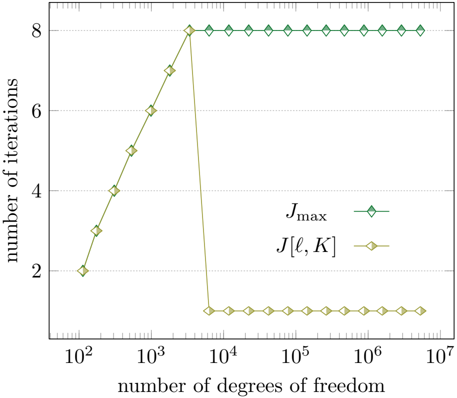
Parameters: $\theta = 0.5$ and $\lambda_{\text{lin}} = 0.7$





Parameter-less algebraic stopping criterion





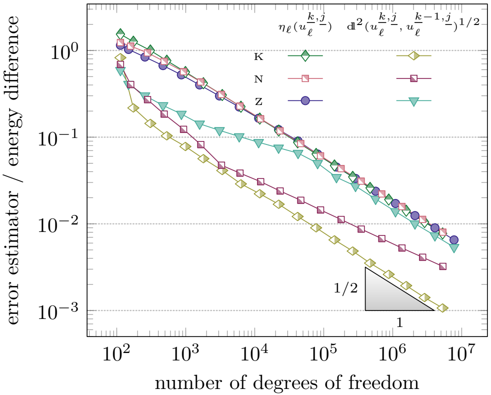
Study of remaining parameters: θ and λ_{lin}

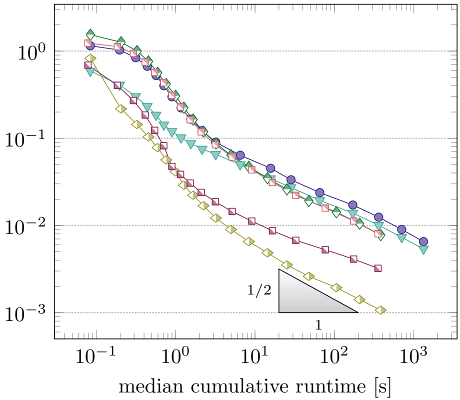
estimator-weighted cumulative time

λ_{lin} \ θ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.348	0.248	0.209	0.203	0.210	0.207	0.213	0.252	0.320
0.2	0.371	0.247	0.209	0.205	0.194	0.200	0.220	0.231	0.334
0.3	0.348	0.247	0.209	0.189	0.200	0.202	0.220	0.230	0.327
0.4	0.349	0.248	0.209	0.188	0.193	0.190	0.220	0.229	0.324
0.5	0.348	0.247	0.209	0.189	0.205	0.201	0.202	0.249	0.326
0.6	0.349	0.247	0.209	0.188	0.184	0.193	0.225	0.233	0.303
0.7	0.348	0.247	0.209	0.191	0.194	0.198	0.205	0.248	0.321
0.8	0.348	0.247	0.209	0.187	0.172	0.179	0.205	0.232	0.324
0.9	0.347	0.246	0.208	0.190	0.174	0.190	0.220	0.232	0.304

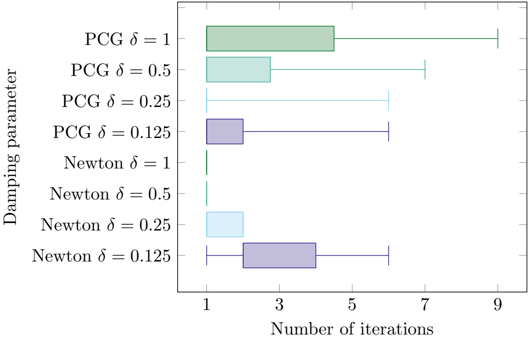
Other iteration alternatives: linearization

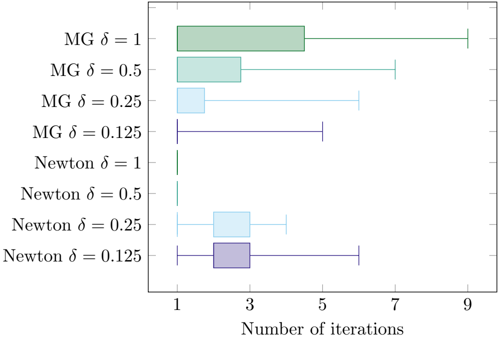
N: Newton (damped $\delta = 1/2$)   K: Kačanov   Z: Zarantonello (damped $\delta = 1/6$)





Other iteration alternatives: algebra





Take home messages

1. analysis of adaptive algorithms should focus rather on rates wrt. *complexity/time* than dofs
2. *linear complexity* and *contraction* of the iterative solvers is crucial
3. *nested* iterations and new *parameter-free* algebraic stopping criterion enforcing norm-energy equivalence
4. *reliability* via a posteriori error estimators is ensured
5. full R-linear convergence
    - gives *contraction* regardless of algorithmic step
    - holds for *arbitrary* adaptivity parameters
    - provides the equivalence *rates* = *complexity*
6. *optimal complexity* is ensured for sufficiently small parameters

Thank you for your attention!

Miraçi, Praetorius, Streitberger

Unconditional full linear convergence and optimal complexity of adaptive iteratively linearized FEM for nonlinear PDEs

Preprint, arXiv: 2401.17778 (2024)

slides