

Haar null convex sets

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Haar null sets were introduced by J.P.R. Christensen in 1972 to extend the notion of sets with zero Haar measure to nonlocally compact Polish groups. In 2013, U.B. Darji defined Haar meagre sets, an analogue of Haar null sets in the context of Baire categories. We show that a closed convex set C in a separable Banach space X is Haar null if and only if C is Haar meagre. This result extends the well known fact that closed and convex sets in Euclidean spaces have zero Lebesgue measure if and only if their interior is empty and a more general theorem due to E. Matoušková, which asserts that closed and convex sets in separable, reflexive Banach spaces are Haar null if and only if they have empty interior. We also see some further characterisations of closed and convex Haar null sets and some of their consequences.