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Session 9

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Definition. Let $\lambda \in Lim$. We denote the set of all non-trivial elementary embeddings $V_\lambda \to V_\lambda$ by $\mathcal{E}_\lambda$. For $j, k \in \mathcal{E}_\lambda$, we set $j * k := \bigcup_{\alpha<\lambda} j(k \upharpoonright V_\alpha)$. The **critical sequence of** j is $\langle \kappa_n \mid n < \omega \rangle$ where $\kappa_0 := crit(j)$ and $\kappa_{n+1} := j(\kappa_n)$ for each $n < \omega$.

1) Let $j, k \in \mathcal{E}_\lambda$.
   - (a) Show that $j * k, j \circ k \in \mathcal{E}_\lambda$ and compute $crit(j * k)$ and $crit(j \circ k)$.
   - (b) Verify that $(j * k) \circ j = j \circ k$.
   - (c) Set $j_0 := j$ and $j_{n+1} := j_n * j$ for each $n < \omega$. Let $\langle \kappa_n \mid n < \omega \rangle$ be the critical sequence of j. Compute that $\kappa_2 < crit(j_2 * j_1) < \kappa_3$.
2) Fix $j \in \mathcal{E}_\lambda$. Let $\langle \kappa_n \mid n < \omega \rangle$ be the critical sequence of j. You may assume that $sup\{\kappa_n \mid n < \omega\} = \lambda$ (this follows from Theorem 9.38 from the lecture notes).
   - (a) Show that $V_{\kappa_0} \preceq V_\lambda$.
   - (b) Prove that $V_\lambda \models ZFC+$ "$\exists$ a proper class of measurable cardinals".