UE Mengenlehre SoSe2024

lena.wallner@tuwien.ac.at

Session 4

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1) Assume that GCH holds. Show that for infinite cardinals κ and λ,

$$\kappa^{\lambda} = \begin{cases} \lambda^+ & \text{if } \kappa \leq \lambda, \\ \kappa^+ & \text{if } cof(\kappa) \leq \lambda < \kappa \text{ and} \\ \kappa & \text{if } \lambda < cof(\kappa). \end{cases}$$

2) Show that

   (a) $(\aleph_n)^{\aleph_0} = 2^{\aleph_0} \cdot \aleph_n$ for each $n < \omega$ and

   (b) $\Pi_{n<\omega}\aleph_n = (\aleph_\omega)^{\aleph_0}$.

3) (a) Let α and β be limit ordinals and let $f : \alpha \to \beta$ be cofinal and strictly increasing. Prove that $cof(\alpha) = cof(\beta)$.

   (b) Show that $E^\kappa_\lambda := \{\alpha < \kappa \mid cof(\alpha) = \lambda\}$ is stationary for κ and λ regular cardinals with $\lambda < \kappa$.

4) (*) Let $S \subseteq \omega_1$ be stationary. Show that for every $\alpha < \omega_1$, there is a closed subset of S which has ordertype $\alpha + 1$.