ROBUST NUMERICAL METHODS FOR FRACTIONAL DIFFUSION PROBLEMS

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Fractional diffusion operators appear naturally in many areas in mathematics, physics, ect. The most important property of the related b.v. problems is that they are non-local. Let us consider the fractional power of a self-adjoint elliptic operator introduced through its spectral decomposition. It is also self-adjoint but non-local. Such problems are computationally expensive. Several different techniques have been proposed to localize the nonlocal operator, thus increasing the space dimension of the computational domain. An alternative approach has been developed in [1, 2], see also the survey [3]. Let $\mathcal{A}$ be a SPD sparse matrix arising from FEM or FDM discretization of the initial (local) problem. Based on the best uniform rational approximations (BURA) of degree k of z^α, $0 \leq z \leq 1$, a class of efficient solution methods for algebraic systems involving $\mathcal{A}^\alpha$, $0 < \alpha < 1$, is proposed and analysed. Robust error estimates with respect to the condition number $\kappa(\mathcal{A})$ are derived, showing the exponential convergence rate of the BURA methods with respect to the degree of rational approximation k. Although the fractional power of $\mathcal{A}$ is a dense matrix, the algorithm has complexity of order $O(N \log^2 N)$, where N is the number of unknowns. At this point, we assume that some solver of optimal complexity (say multigrid or multilevel) is used for the auxiliary systems with matrices $\mathcal{A} + d_j\mathcal{I}$, $d_j \geq 0$, $j = 1, \ldots, k$. The presented (up to 3D) numerical tests are focussed on problems with low regularity of the solutions, including cases of adaptive mesh refinement. The comparative analysis demonstrates the advantages of the BURA methods, supported by a unified theoretical explanation. Some recent results about BURA based preconditioning of coupled problems are presented at the end [4].

REFERENCES

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