A FRAMEWORK FOR IMPLEMENTING GENERAL HIGHER ORDER VIRTUAL ELEMENT SPACES

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ABSTRACT

First introduced to solve second order elliptic problems, the virtual element method has gained a lot of attention due to both the versatility of the method and its ability to easily handle general polygonal meshes. As such, it has been applied to a wide range of problems; for example, higher order continuity spaces have been developed for the approximation of polyharmonic problems as well as the construction of pointwise divergence-free spaces for the Stokes problem. However, the implementation of VEM is not so straightforward and we are yet to see many major software packages providing access to VEM spaces.

In this talk we focus on a generic implementation that fits easily into existing software frameworks and leads to beneficial additional structure within the VEM spaces. We implemented our approach within the open source DUNE framework [1]. The software is implemented in `C++` but also provides a high level Python frontend.

In order to describe the generic VEM implementation, we define the projection operators, necessary for the virtual element discretization, in a way that; (a) does not depend on the variational form of the underlying problem, and (b) are computable using only the degrees of freedom. Following [2], we describe this general method and how it can be used to implement suitable spaces for second and forth order nonlinear problems and for compatible vector valued spaces.

References

[1] Bastian, P., Blatt, M., Dedner, A., Engwer, C., Klöfkorn, R., Kornhuber, R., Ohlberger, M., and Sander, O. A generic grid interface for parallel and adaptive scientific computing. Part II: Implementation and tests in DUNE. *Computing 82*, 2 (2008), 121–138.

[2] Dedner, A., and Hodson, A. Robust nonconforming virtual element methods for general fourth-order problems with varying coefficients. *IMA J. Numer. Anal.* (2021).

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