APPLICATIONS OF A SPACE-TIME FIRST-ORDER SYSTEM LEAST-SQUARES FORMULATION FOR PARABOLIC PDES

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ABSTRACT

While the common space-time variational formulation of a parabolic equation results in a bilinear form that is non-coercive, [1] recently proved well-posedness of a space-time first-order system least-squares formulation of the heat equation. Least-squares formulations always correspond to a symmetric and coercive bilinear form. In particular, the Galerkin approximation from any conforming trial space exists and is a quasi-best approximation. Additionally, the least-squares functional automatically provides a reliable and efficient error estimator.

In [2], we have generalized the least-squares method of [1] to general second-order parabolic PDEs with possibly inhomogenoeus Dirichlet or Neumann boundary conditions. For homogeneous Dirichlet conditions, we present in this talk convergence of a standard adaptive finite element method driven by the least-squares estimator, which has also been demonstrated in [2]. The convergence analysis is applicable to a wide range of least-squares formulations for other PDEs, answering a long-standing open question in the literature. Moreover, we employ the space-time least-squares method for parameter-dependent problems as well as optimal control problems. In both cases, coercivity of the corresponding bilinear form plays a crucial role.

References

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