SPACE-TIME DISCONTINUOUS GALERKIN METHODS FOR WAVES

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ABSTRACT

We consider variational space-time discretizations of linear first-order hyperbolic systems. Based on the theory for symmetric Friedrichs systems we establish an analytic framework for weak and strong solutions, and we prove inf-sup stability of the continuous variational setting [1].

Discrete inf-sup stability is obtained for a space-time method with discontinuous Galerkin elements with upwind flux in space and time. The discretization is adaptive with independent choice of polynomial degrees p in time and q in space for every space-time cell. The discretization is fully implicit, and the overall linear problem is solved with a parallel Krylov method using a multigrid preconditioner based on a subspace hierarchy. The adaptivity is controlled by a dual weighted residual error estimator with respect to a given linear error functional.

The method is evaluated for a benchmark configuration in geophysics, the full waveform inversion to identify the subsurface material distribution by seismograms. Here we consider p-adaptive approximations of the forward problem based on a dual-primal error estimator with respect to a goal functional corresponding to seismic measurements [2].

Then, we also consider the approximation of weak solutions with minimal regularity requirements, and we prove convergence in the DG semi-norm. This applies to discontinuous Riemann problems, where we also compare the approximation results in different norms [3].

References

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