Biharmonic obstacle problem: error bounds for approximate solutions

Darya Apushkinskaya

RUDN University, Moscow, Russia;
Saarland University, Saarbrücken, Germany

We discuss the bounds of the difference between the exact solution of the variational problem, associated with a free boundary obstacle problem for the biharmonic operator, and any function (approximation) from the energy class satisfying the prescribed boundary conditions and the restrictions stipulated by the obstacle.

Using the general theory developed for a wide class of convex variational problems we deduce the error identity. One part of this identity characterizes the deviation of the function (approximation) from the exact solution, whereas the other is a fully computed value (it depends only on the data of the problem and known functions). In real life computations, this identity can be used to control the accuracy of approximate solutions.

The measure of deviation from the exact solution used in the error identity contains four terms of different nature. Two of them are the norms of the difference between the exact solutions (of the direct and dual variational problems) and corresponding approximations. Two others are not representable as norms. These are nonlinear measures vanishing if the coincidence set defined by means of an approximate solution satisfies certain conditions (for example, coincides with the exact coincidence set).

The error identity is true for any admissible (conforming) approximations of the direct variable, but it imposes some restrictions on the dual variable. We show that these restrictions can be removed, but in this case the identity is replaced by an inequality. For any approximations of the direct and dual variational problems, the latter gives an explicitly computable majorant of the deviation from the exact solution.

The talk is based on results of [1] obtained in collaboration with Sergey I. Repin.

References

[1] Apushkinskaya D. E. and Repin S. I. *Biharmonic obstacle problem: guaranteed and computable error bounds for approximate solutions.* Computational Mathematics and Mathematical Physics. **60** (2020) No. 11, 1881–1897.