FINITE ELEMENT ANALYSIS FOR A THREE-STEP TWO-LEVEL APPROXIMATIONS OF THE BOUSSINESQ SYSTEM OF EQUATIONS

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ABSTRACT

This talk presents an error analysis of a three-step two-level Galerkin finite element method applied to the two-dimensional transient Boussinesq system of equations. In **Step I**, the problem is discretized in the spatial direction by employing a finite element method on a coarser mesh $\mathcal{T}_H$ with mesh size H. Then, in **Step II**, the nonlinear system is linearized around the coarser grid solution, similar to Newton's type iteration, and the resulting linear system is solved on a finer mesh $\mathcal{T}_h$ with mesh size h. In **Step III**, a correction is accomplished by solving a linear problem on the finer mesh and an updated final approximate solution is obtained. Optimal error estimates in $L^\infty(\mathbf{L}^2)$-norm, when $h = \mathcal{O}(H^{2-\delta})$, in $L^\infty(\mathbf{H}^1)$-norm, when $h = \mathcal{O}(H^{4-\delta})$ for the velocity and in $L^\infty(L^2)$-norm, when $h = \mathcal{O}(H^{2-\delta})$, in $L^\infty(H^1)$-norm, when $h = \mathcal{O}(H^{4-\delta})$ for the temperature and, in $L^\infty(L^2)$-norm, when $h = \mathcal{O}(H^{4-\delta})$ for the pressure, $\delta > 0$ arbitrarily small, are derived. Then, a complete discretization is achieved by applying backward Euler method in the time direction and fully discrete error estimates are established. Finally, the talk is concluded by providing numerical results and verifying the theoretical findings.

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