A CONFORMING FEM FOR THE NAVIER–STOKES/DARCY/HEAT COUPLED SYSTEM

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ABSTRACT

In this paper, we propose and analyze a conforming finite element method for a mathematical model present, for instance, in desalination processes based on membrane distillation, consisting of the coupling of fluid flow with porous media flow subject to a heat source. The flows are governed by a natural convection model in the free fluid region, also known as Boussinesq model, and a Darcy-Heat coupled system in the porous media region. The coupled system is complemented by suitable boundary conditions on the exterior boundary and a set of transmission conditions on the interface given by mass conservation, balance of normal forces, the Beavers–Joseph–Saffman law, and the continuity of the heat flux and the fluid temperature. We consider the standard velocity-pressure-temperature variational formulation for the Boussinesq system, and a dual-mixed scheme coupled with a primal formulation for the Darcy and Heat equations, respectively, in the porous medium region. The latter yields the introduction of the trace of the porous medium pressure as a suitable Lagrange multiplier. As for the associated Galerkin scheme we employ Bernardi–Raugel and Raviart–Thomas elements for the velocities, piecewise constant elements for the pressures, continuous piecewise linear functions for the temperatures and continuous piecewise linear functions for the aforementioned the Lagrange multiplier. We prove well-posedness for both, the continuous and discrete schemes and derive the corresponding error estimates. Finally, we report some numerical examples confirming the predicted rates of convergence, and illustrating the performance of the method.

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