WAVENUMBER-EXPLICIT hp-FEM-BEM COUPLING FOR THE HELMHOLTZ EQUATION

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ABSTRACT

Helmholtz problems are naturally posed in full space. To represent the solution in full, we adopt a FEM-BEM coupling that connects a heterogeneous Helmholtz equation in a bounded computational domain to a homogeneous Helmholtz equation on its exterior. The coupling is effected with a mortar variable that is related to an impedance trace on the smooth coupling interface and a first-kind integral representation of the exterior Dirichlet-to-Impedance operator, [1, 2]. The formulation is thus a three-field formulation, and this particular choice of mortar variable ensures that the subblock corresponding to the volume unknowns is invertible. We discretize all three fields arising in the formulation with piecewise polynomials of degree $p/p-1$ on meshes of size h. For analytic interface and (piecewise) analytic coefficients (with smooth interfaces where the coefficients jump) we show quasi-optimality of the method under the scale resolution condition that a) kh/p is sufficiently small and b) $p/\log k$ is bounded away from zero. The proof of this result generalizes recent techniques developed in [3, 4].

References

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