

# A LOW-RANK TENSOR PRODUCT FRAMEWORK FOR RADIATIVE TRANSFER IN SLAB GEOMETRY

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## ABSTRACT

In this talk we are going to present a low-rank tensor product framework for the approximation of the radiative transfer equation (RTE) in slab geometry. Classical numerical schemes in radiative transfer, such as the  $P_N$ - and  $S_N$ -approximations, exploit the tensor product structure of the underlying phase space, but they are affected by the so-called curse of dimensionality, which describes the exponential scaling of computational complexity with the physical dimension. In order to tackle the dimensionality issue, we propose to search for tensor product solutions in low-rank format. Using an appropriate variational formulation [1] allows us to recast the hyperbolic radiative transfer problem as a degenerate elliptic equation. Galerkin projection of this degenerate elliptic equation yields an equation for an operator that has a low-rank decomposition, which allows efficient application to objects with a low-rank decomposition [3]. To solve this equation, we consider a preconditioned Richardson iteration with rank control, which has been used in the context of high-dimensional elliptic equations in [2]. Due to the degeneracy of the elliptic equation, the construction of a preconditioner with a low-rank decomposition is challenging, and we discuss several approaches. We prove convergence of the resulting iterative scheme, and we show the potential of the proposed framework through some numerical experiments.

## REFERENCES

- [1] H. Egger, M. Schlottbom, *A mixed variational framework for the radiative transfer equation*, Math. Models Methods Appl. Sci. 22 (2012)
- [2] M. Bachmayr, R. Schneider, *Iterative methods based on soft thresholding of hierarchical tensors*, FoCM 17, 1037–1083 (2017)
- [3] L. Grasedyck, D. Kressner, C. Tobler *A literature review of low-rank tensor approximation techniques*, GAMM-Mitt. 36, 53-78 (2013)

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