DISTRIBUTED SOLUTION OF LAPLACIAN EIGENVALUE PROBLEMS

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ABSTRACT

I discuss solution of large-scale eigenvalue problems in distributed computing environments where communication between tasks is expensive, such as a cluster of networked workstations running the HTCondor batch system. As a model problem, I consider computing eigenvalues on $(0, \Lambda)$ of the Dirichlet Laplacian discretized using conforming first order FEM: find $(\lambda, u) \in (0, \Lambda) \times V_h \setminus \{0\}$ satisfying

$$(\nabla u, \nabla v) = \lambda(u, v) \quad \text{for all } v \in V_h. \tag{1}$$

I believe that the method can be used to solve EVPs related to other elliptic PDEs as well. The talk is based on [1, 2].

I develop a Ritz method using method subspace constructed from local spaces associated to an overlapping domain decomposition. The partition of unity method is used to combine the local spaces into a conforming global space. Local space on subdomain ω is constructed to approximate the ω-restriction of eigenfunctions associated to $(0, \Lambda)$ with desired accuracy as follows: First, the ω-restrictions of these eigenfunctions are related to a boundary-to-interior (B2I) mapping from boundary of an extended domain $\partial\tilde{\omega}$ s.t. $\omega \subset \tilde{\omega}$ to ω. The local subspace is then obtained by computing the range of this B2I-mapping.

We give outline of error analysis and test the method by computing 200 lowest eigenvalues of 3D Dirichlet Laplacian using cluster of 25 workstations.

References

[1] A. Hannukainen, J. Malinen, and A. Ojalammi. *Efficient solution of symmetric eigenvalue problems from families of coupled systems*, SINUM, **57(4)** (2019), 1789–1814.

[2] A. Hannukainen, J. Malinen, and A. Ojalammi. *Distributed Solution of Laplacian Eigenvalue Problems*, SINUM, **60(1)** (2022), 76–103.

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