OPTIMALITY OF ADAPTIVE GALERKIN METHODS FOR RANDOM ELLIPTIC PDES

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ABSTRACT

We consider elliptic PDEs depending on infinitely many parameters entering into a parametrized series expansion of the diffusion coefficient. The variational formulation of this affinely parameterized elliptic PDEs on a domain $D \subset \mathbb{R}^d$ reads: $u(y) \in V := H^1_0(D)$ such that

$$\int_D \Big(\bar{a} + \sum_{\mu=1}^{\infty} y_\mu \psi_\mu\Big) \nabla u(y) \cdot \nabla v \, \mathrm{d}x = f(v), \ v \in V, \quad \text{for } y \in Y := [-1,1]^{\mathbb{N}},$$

where $f \in V'$, $\psi_\mu \in L_\infty(D)$ for $\mu \in \mathbb{N}$ and $\bar{a} \in L_\infty(D)$ are such that the problem is elliptic uniformly in y.

The focus of this talk is on adaptive finite element algorithms for computing sparse Legendre approximations with respect to y of the solutions $u(y)$, where each Legendre coefficient is a function in V. When the functions ψ_μ have a suitable multilevel structure, one obtains improved convergence results for such Legendre expansions [1].

In [2] it was shown that by combining an adaptive operator application for the parametric expansion with *independent* adaptive wavelet schemes for the spatial discretizations, one can achieve optimal convergence rates at an optimal computational cost, under natural assumptions. In this talk, we revisit these results and discuss how to achieve similar optimal results for schemes with adaptive spatial finite element discretizations.

References

[1] Markus Bachmayr, Albert Cohen, and Giovanni Migliorati, *Sparse polynomial approximation of parametric elliptic PDEs. Part I: affine coefficients*, ESAIM. Mathematical Modelling and Numerical Analysis **51** (2017), no. 1, 321–339.

[2] Markus Bachmayr and Igor Voulis, *An adaptive stochastic Galerkin method based on multilevel expansions of random fields: Convergence and optimality*, arXiv Preprint arXiv:2109:09136, 2021.

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