ADAPTIVE MULTILEVEL STOCHASTIC COLLOCATION FEM FOR PARAMETRIC PDES

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ABSTRACT

Sparse grid stochastic collocation representations of parametric uncertainty, in combination with finite element discretization in physical space, have emerged as an efficient alternative to Monte-Carlo strategies, particularly in the context of nonlinear PDE models or linear PDE problems that are nonlinear in the parameterization of the uncertainty.

We present an adaptive algorithm for computing stochastic collocation finite element approximations that employ individually tailored spatial discretizations across collocation points (multilevel stochastic collocation FEM) [1, 2]. In this algorithm, we employ hierarchical a posteriori estimates for reliable error control in computed approximations and use the associated error indicators to guide the adaptive refinement process; see [1]. We will illustrate the performance of the developed algorithm in numerical examples for elliptic PDE problems with non-affine parameterization of random inputs and with parameter-dependent local spatial features. In particular, we will demonstrate the effectivity and robustness of the proposed error estimation strategy and discuss the convergence properties of the generated adaptive multilevel approximations.

References

[1] A. Bespalov, D. J. Silvester, F. Xu. *Error estimation and adaptivity for stochastic collocation finite elements. Part I: single-level approximation*, Preprint, arXiv:2109.07320 (2021).

[2] A. Bespalov, D. J. Silvester. *Error estimation and adaptivity for stochastic collocation finite elements. Part II: multilevel approximation*, Preprint, arXiv:2202.08902 (2022).

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