ADAPTIVE GALERKIN FEM FOR NON-AFFINE LINEAR PARAMETRIC PDES

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ABSTRACT

Adaptive stochastic Galerkin FEM (ASGFEM) with residual based a posteriori error estimation have shown to exhibit optimal convergence in practice for some standard parametric linear PDEs. However, their implementation is rather involved and requires significant effort when different types of PDEs should be tackled. Motivated by recent results with empirical low-rank tensor regression in the framework of statistical learning, we propose a non-intrusive reconstruction method that only uses samples of the solution and yields the Galerkin projection with high probability [2]. The a posteriori error control involves all discretisation parameters, determining the deterministic error components and the statistical error. Moreover, for the sum of error and estimator, the empirical ASGFEM can be shown to converge.

To realize the error estimator, a sufficiently accurate tensor representation of the coefficient is required, which easily becomes challenging for instance when it is defined as an exponential function of a Gaussian field. We consider this common case and recall that it corresponds to the solution of a differential equation, which can be computed by means of a Petrov-Galerkin method [1].

As an alternative for the Darcy equation, a deterministic sparse adaptive log-transformed discretisation in mixed and primal form is touched upon briefly.

References

[1] Martin Eigel, Nando Farchmin, Sebastian Heidenreich, Philipp Trunschke *Efficient approximation of high-dimensional exponentials by tensornetworks*, arxiv:2105.09064 (2021).

[2] Martin Eigel, Nando Farchmin, Sebastian Heidenreich, Philipp Trunschke *Adaptive non-intrusive reconstruction of solutions to high-dimensional parametric PDEs*, arXiv 2112.01285 (2021).

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