A GLOBALIZATION FRAMEWORK FOR SEMISMOOTH NEWTON METHODS

FLORIAN MANNEL*

ABSTRACT

Semismooth Newton methods are a well-established tool for solving nonsmooth equations, including those that arise as first order optimality conditions of optimal control problems. Unfortunately, however, they are only locally convergent, in general. In this talk we present a framework that allows to globalize semismooth Newton methods in Banach spaces, provided a globally convergent base method exists. This yields globalizations for many situations in which none have previously existed.

The main idea of the framework is well-known from solving smooth equations: The semismooth Newton step is employed if it is of sufficient quality in some sense; otherwise, the base method is used to compute a step. The novelty is the criterion that is used to decide whether the semismooth Newton step is accepted. In fact, the criterion also results in new globalizations of Newton's method for smooth equations.

Every algorithm within the framework has the following properties:

- It is globally convergent under the same assumptions as the base method.
- If its iterates have a regular cluster point, then they converge to this point and the convergence speed is that of the semismooth Newton method. In finite dimensions, this includes finite convergence if the equations are piecewise linear.

We present the idea of the framework and its convergence properties, we show a few of the algorithms that it covers, and we provide numerical results for the application of the framework to optimal control problems.

* University of Lübeck, mannel@mic.uni-luebeck.de