NUMERICAL SOLUTION OF 2D LINEAR FREDHOLM INTEGRAL EQUATION SYSTEMS OF SECOND KIND BY RADIAL BASIS FUNCTIONS

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ABSTRACT

The theory of integral equations has close contacts with many different areas of mathematics. Many problems in the fields of differential equations can be recast as integral equations and they are encountered in various fields of science and numerous applications as elasticity, plasticity, heat and mass transfer, oscillatory theory, fluid dynamics among others.

A 2D linear second kind Fredholm system can be expressed in the form

$$\mathbf{f}(x,y)=\mathbf{u}(x,y)-\int_{\Omega}\mathbf{k}(x,y,t,s)\mathbf{u}(t,s)dtds,\ \ (x,y)\in\Omega, \tag{1}$$

where $\Omega=(0,1)\times(0,1)\subset\mathbb{R}^2$ is the domain, the vector-valued function $\mathbf{f}$ is given by $\mathbf{f}(x,y)=(f_1(x,y),\ldots,f_m(x,y))^T\in\mathbb{R}^m$, the matrix-valued function $\mathbf{k}$ is given by $\mathbf{k}(x,y,t,s)=(k_{ij}(x,y,t,s))_{1\le i,j\le m}\in\mathbb{R}^{m,m}$ and $\mathbf{u}$ is the unknown vector-valued function of the system.

It is known that if $\mathbf{k}$ and $\mathbf{f}$ are continuous (1) has a unique solution.

We propose a new numerical method to approximate the solution of this type of 2D Fredholm integral systems. This method is based on the minimization of a suitable quadratic functional in a finite-dimensional space generated by a radial basis functions set.

We study its convergence under adequate hypotheses and we present some numerical examples in order to show the validity of the method. Our proposed technique gives an acceptable accuracy with small use of the data, resulting also in a low computational cost.

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