solitary wave solutions of the Korteweg-de Vries equation

The KdV equation is

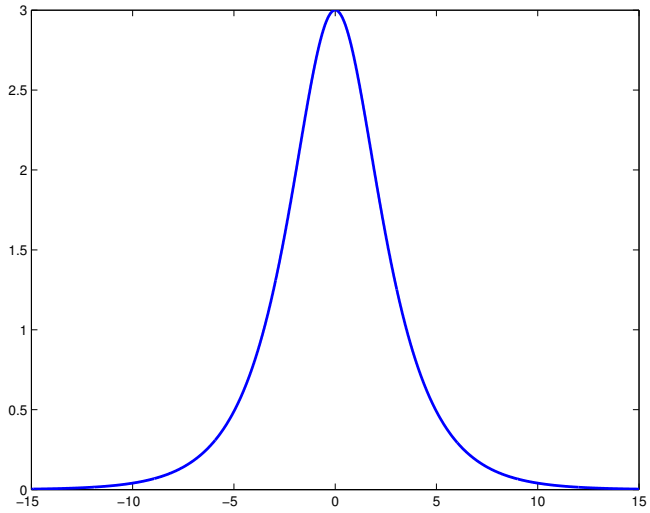
$$\eta_t + \eta\eta_x + \eta_{xxx} = 0 \quad x \in \mathbb{R}, \quad t > 0.$$

This nonlinear equation permits solutions of the form

$$\eta(x,t) = 3c\,\mathrm{sech}^2\left(\frac{\sqrt{c}}{2}(x - ct)\right),$$

for arbitrary wave speed $c > 0$.

- the equation is **nonlinear** and balances in just the right way nonlinear and dispersive effects to admit such a solution. Note: The nonlinear equation $\eta_t + \eta\eta_x = 0$ (Burgers equation) and the dispersive equation $\eta_t + \eta_{xxx} = 0$ both don't permit smooth travelling waves.
- The speed of propagation depends on the amplitude of the "wave"



- The KdV equation has a very rich mathematical structure. For example, it has an infinite number of conserved quantities