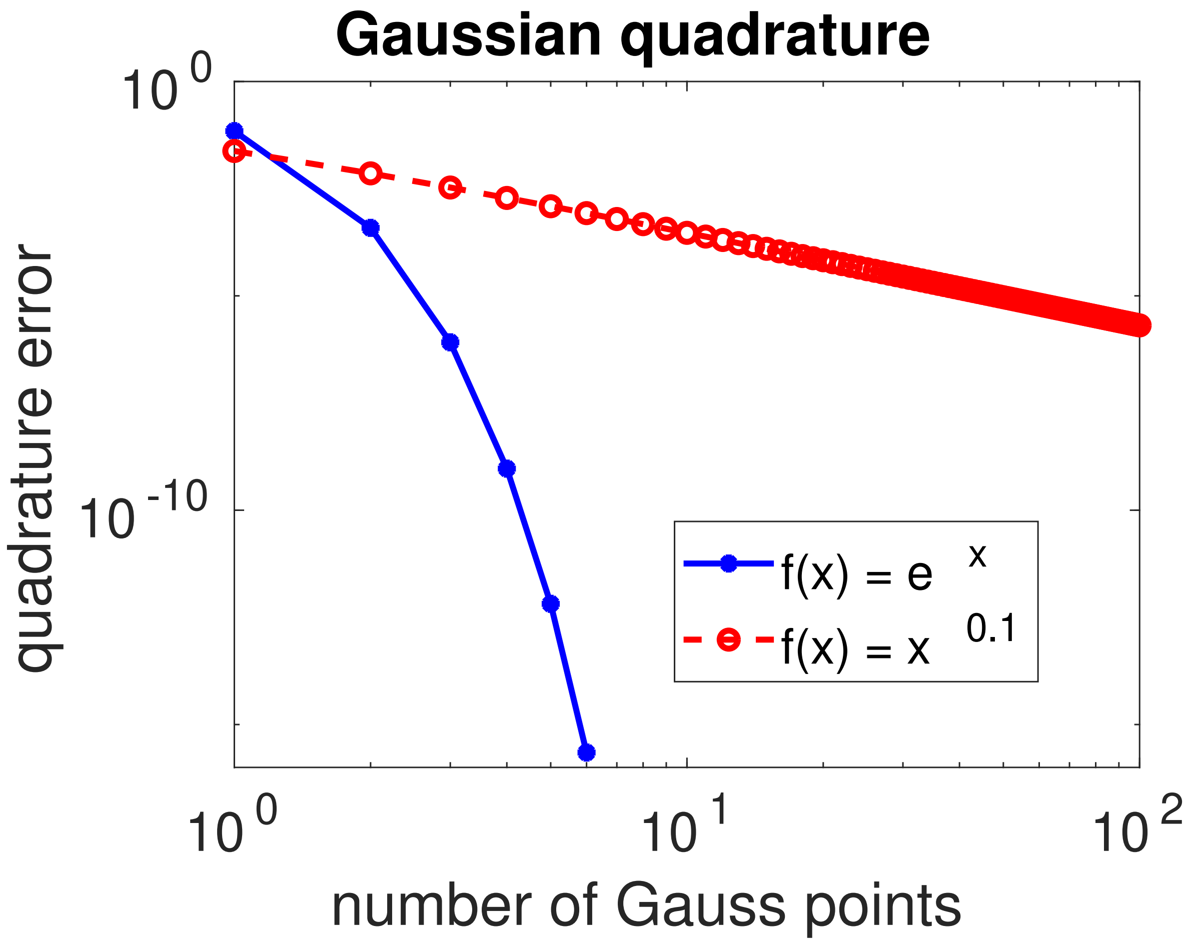
classical Gaussian quadrature for $\int_0^1 e^x\,dx$ and $\int_0^1 x^{0.1}\,dx$

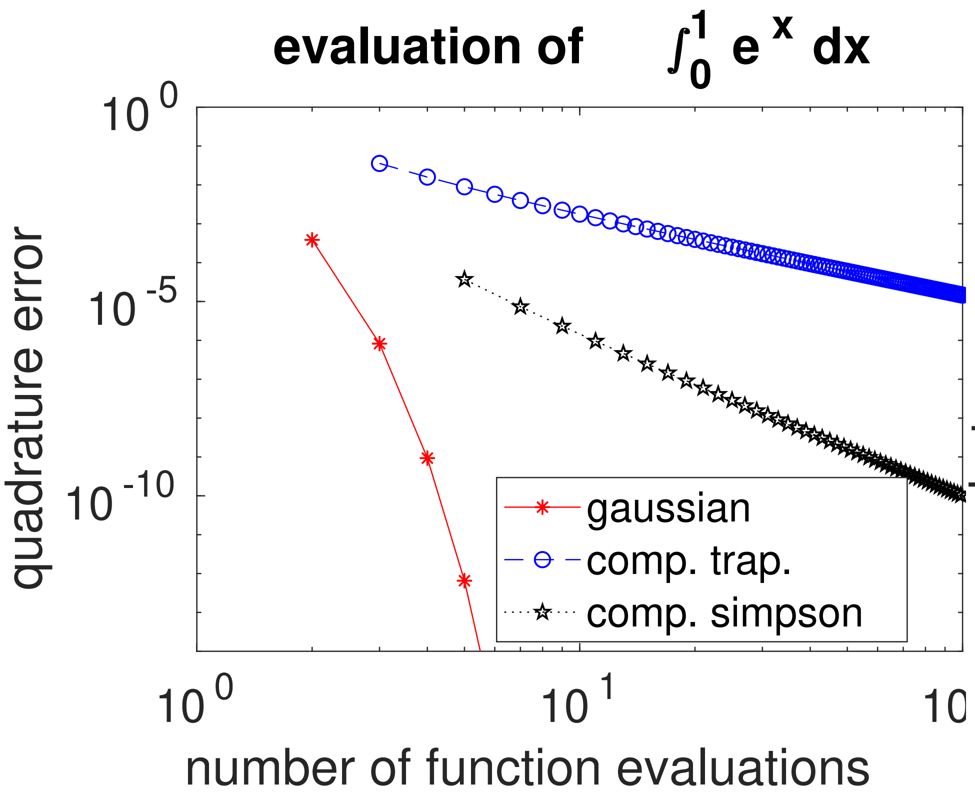


Exercise: Show:

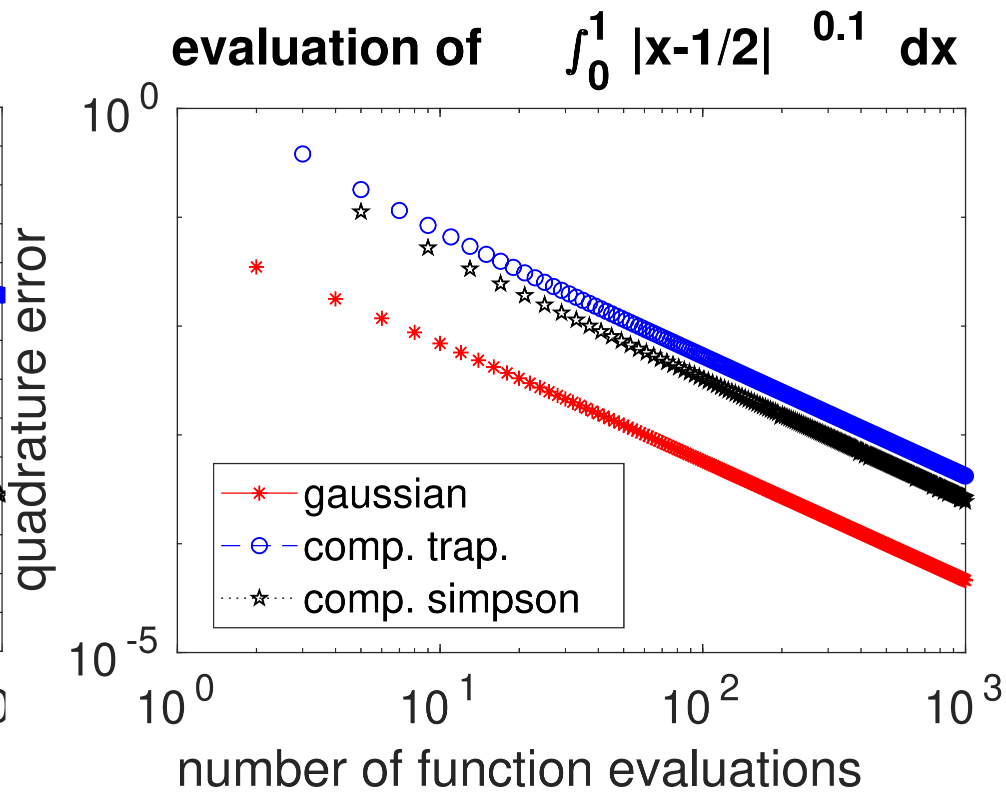
$$\left|\int_0^1 e^x\,dx - Q_n(e^x)\right| \le \frac{1}{(2n+1)!}\left(\frac{e}{2}\right)^{2(n+1)}$$

Comparison of different quadrature rules

smooth integrand



non-smooth integrand



How to evaluate $\int_0^1 x^\alpha f(x)\,dx$ for smooth f?

1. composite rules such as composite trapezoidal or Simpson rule:
   use partitions Δ that condense the quadrature points near $x = 0$
   $\rightarrow$ see adaptive quadrature

2. Gaussian rules with weight: create Gaussian rules $Q_n(f) \approx \int_0^1 x^\alpha f(x)\,dx$. (not discussed in class!)

3. composite Gaussian rules on partitions Δ that are suitably refined near $x = 0$.

Example: composite Gaussian rule for $\int_0^1 f(x)\,dx$

Define the partition Δ by the knots 0, q^L, $q^{L-1}, \ldots, q, 1$.