Newton-Cotes formulas

n	weights	$Q(f) - \int_0^1 f(x)\,dx$	name
1	$\frac{1}{2} \quad \frac{1}{2}$	$\frac{1}{12}h^3 f^{(2)}(\xi)$	trapezoidal rule
2	$\frac{1}{6} \quad \frac{4}{6} \quad \frac{1}{6}$	$\frac{1}{90}h^5 f^{(4)}(\xi)$	Simpson rule
3	$\frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8}$	$\frac{3}{80}h^5 f^{(4)}(\xi)$	Simpson's 3/8 rule
4	$\frac{7}{90} \quad \frac{32}{90} \quad \frac{12}{90} \quad \frac{32}{90} \quad \frac{7}{90}$	$\frac{8}{945}h^7 f^{(6)}(\xi)$	Milne rule
5	$\frac{19}{288} \quad \frac{75}{288} \quad \frac{50}{288} \quad \frac{50}{288} \quad \frac{75}{288} \quad \frac{19}{288}$	$\frac{275}{12096}h^7 f^{(6)}(\xi)$	—
6	$\frac{41}{840} \quad \frac{216}{840} \quad \frac{27}{840} \quad \frac{272}{840} \quad \frac{27}{840} \quad \frac{216}{840} \quad \frac{41}{840}$	$\frac{9}{1400}h^9 f^{(8)}(\xi)$	Weddle rule

knots: $x_i = \frac{i}{n}, \quad i - 0, \ldots, n, \qquad h = \frac{1}{n}$

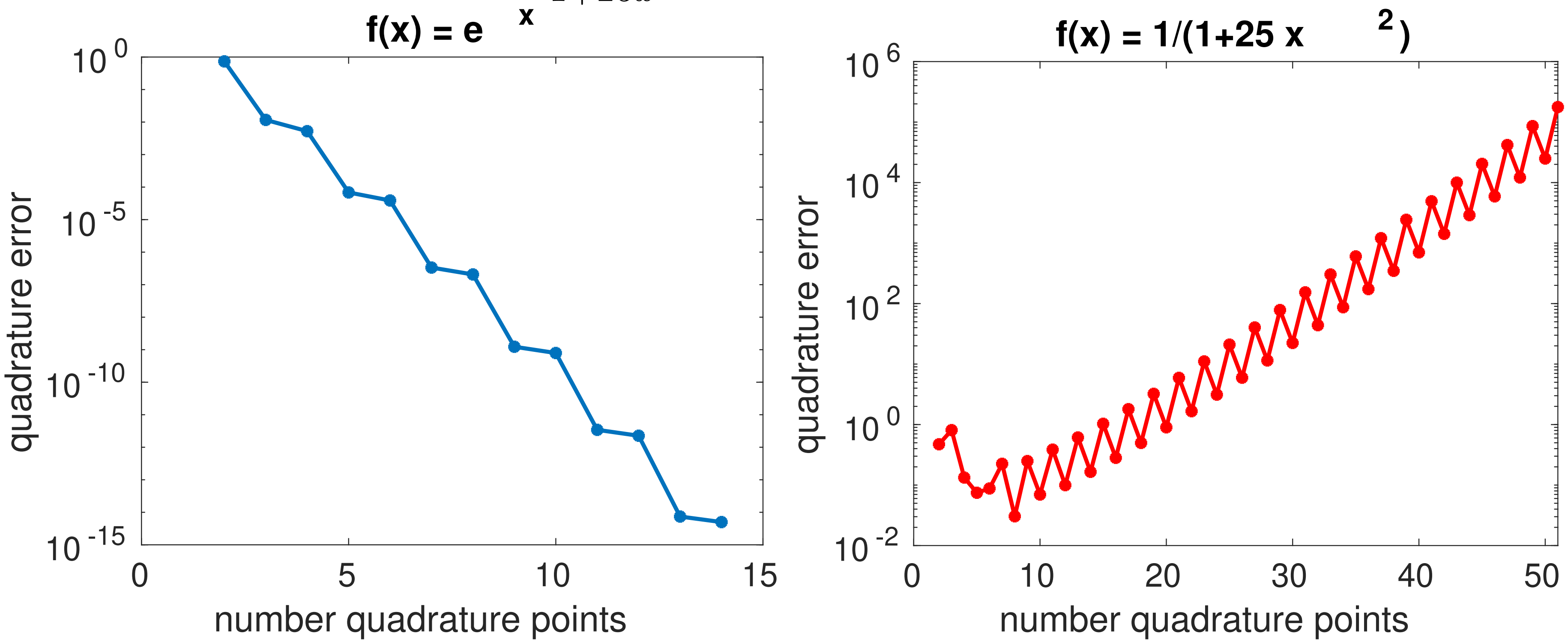
There holds: $$\int_0^1 f(x)\,dx = Q(f) \qquad \begin{cases} \forall f \in \mathcal{P}_n & \text{if } n \text{ is odd} \\ \forall f \in \mathcal{P}_{n+1} & \text{if } n \text{ is even} \end{cases}$$

For $n > 6$ the Newton-Cotes formulas have positive and negative weights $\to$ these are not used in practice (instead: Gauss rules)

Example: Newton-Cotes formulas for $n \to \infty$:

left: approximation of $\int_{-1}^{1} e^x \, dx$ using Newton-Cotes formulas

right: approximation of $\int_{-1}^{1} \frac{1}{1+25x^2} \, dx$ using Newton-Cotes formulas



observation: Newton-Cotes formulas do not work for $f(x) = 1/(1 + 25x^2)$ for $n \to \infty$.

Reasons: they are based on polyomial interpolatoin in uniformly distributed knots.