Gaussian elimination

$$\begin{array}{lllll} & 1x_1 & +2x_2 & +3x_3 & = 10 \\ \text{row - } \frac{4}{1}\cdot 1.\text{ row:} & 4x_1 & +5x_2 & +6x_3 & = 11 \\ \text{row - } \frac{7}{1}\cdot 1.\text{ row:} & 7x_1 & +8x_2 & +10x_3 & = 70 \end{array}$$

$$L^{(1)} = \begin{pmatrix} 1 & & \\ -\frac{4}{1} & 1 & \\ -\frac{7}{1} & & 1 \end{pmatrix}$$

$$\begin{array}{lllll} & 1x_1 & +2x_2 & +3x_3 & = 10 \\ & & -3x_2 & -6x_3 & = -29 \\ \text{row - } \frac{-6}{-3}\cdot 2.\text{ row:} & & -6x_2 & -11x_3 & = 0 \end{array}$$

$$L^{(2)} = \begin{pmatrix} 1 & & \\ & 1 & \\ & -\frac{-6}{-3} & 1 \end{pmatrix}$$

$$\begin{array}{llll} 1x_1 & +2x_2 & +3x_3 & = 10 \\ & -3x_2 & -6x_3 & = -29 \\ & & +1x_3 & = 58 \end{array}$$

back substitution yields: $x_3 = 58$, etc.

---

observation: $$LU = \begin{pmatrix} 1 & & \\ +\frac{4}{1} & 1 & \\ +\frac{7}{1} & +\frac{-6}{-3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ & -3 & -6 \\ & & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix}$$

Gaussian elimination

$$\boxed{\begin{array}{ccccccccc} a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & = & b_2 \\ \vdots & & & & & & & \vdots & \\ a_{n1}x_1 & + & a_{n2}x_2 & + & \cdots & + & a_{nn}x_n & = & b_n \end{array}}$$

$\Downarrow \quad l_{i1} := \dfrac{a_{i1}}{a_{11}},$ subtract $l_{i1} \times$ 1^{st} equation from ith equation

$$\boxed{\begin{array}{ccccccccc} a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ & & a_{22}^{(2)}x_2 & + & \cdots & + & a_{2n}^{(2)}x_n & = & b_2^{(2)} \\ & & \vdots & & & & & \vdots & \\ & & a_{n2}^{(2)}x_2 & + & \cdots & + & a_{nn}^{(2)}x_n & = & b_n^{(2)} \end{array}}$$

$\Downarrow \quad l_{i2} := \dfrac{a_{i2}^{(2)}}{a_{22}^{(2)}},$ subtract $l_{i2} \times$ 2^{st} equation from ith equation

$$
\begin{array}{ccccccccccc}
a_{11}x_1 & + & a_{12}x_2 & + & a_{13}x_3 & + & \cdots & + & a_{1n}x_n & = & b_1 \\
 & & a_{22}^{(2)}x_2 & + & a_{23}^{(2)}x_3 & + & \cdots & + & a_{2n}^{(2)}x_n & = & b_2^{(2)} \\
 & & & & a_{33}^{(3)}x_3 & + & \cdots & + & a_{3n}^{(3)}x_n & = & b_3^{(3)} \\
 & & & & \vdots & & & & & \vdots & \\
 & & & & a_{n3}^{(3)}x_3 & & \cdots & & a_{nn}^{(3)}x_n & = & b_n^{(3)}
\end{array}
$$

$$\Downarrow$$

$$\vdots$$

$$\Downarrow$$

$$
\begin{array}{ccccccccccc}
a_{11}x_1 & + & a_{12}x_2 & + & a_{13}x_3 & + & \cdots & + & a_{1n}x_n & = & b_1 \\
 & & a_{22}^{(2)}x_2 & + & a_{23}^{(2)}x_3 & + & \cdots & + & a_{2n}^{(2)}x_n & = & b_2^{(2)} \\
 & & & & a_{33}^{(3)}x_3 & + & \cdots & + & a_{3n}^{(3)}x_n & = & b_3^{(3)} \\
 & & & & & \ddots & & & \vdots & \vdots & \\
 & & & & & & & & a_{nn}^{(n)}x_n & = & b_n^{(n-1)}
\end{array}
$$