

1000 ways to spell Chebyshev

Pafnuty Lvovitch Chebyshev (1821–1894)

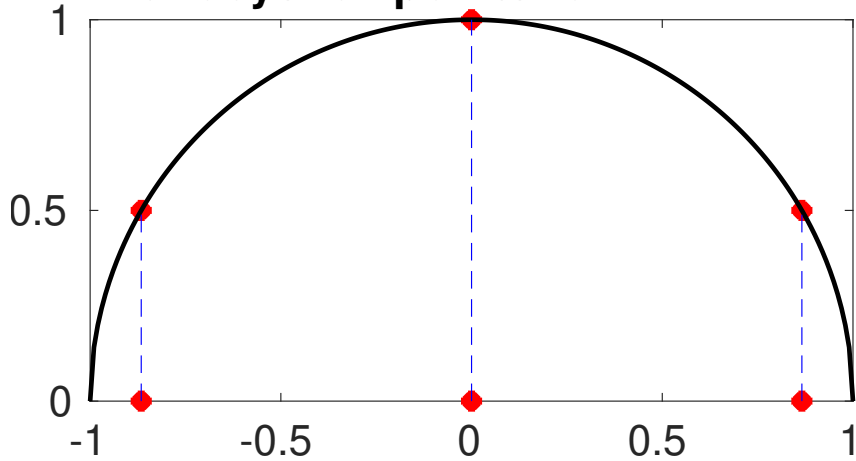
Pafnutii L'vovitsch Tschebysheff
Paphnutij L'vovič Tchebicheff
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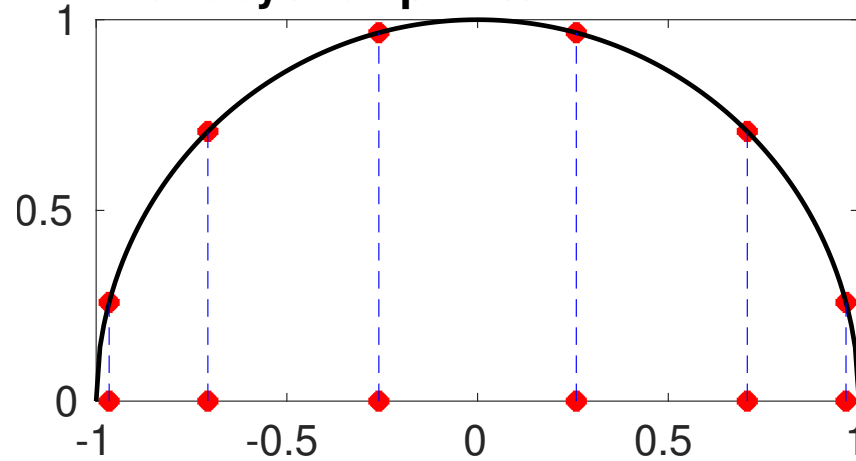
Chebyshev points

$$x_i = \cos\left(\frac{2i+1}{2n+2}\pi\right), \quad i = 0, \dots, n.$$

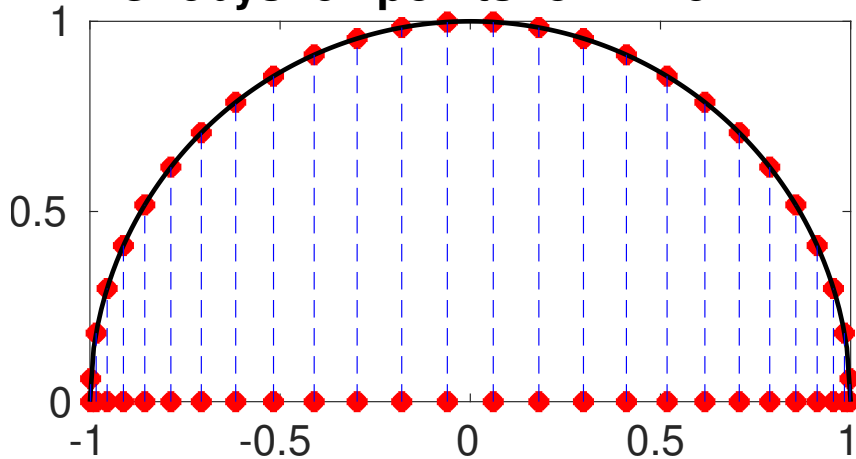
Chebyshev points for n=2



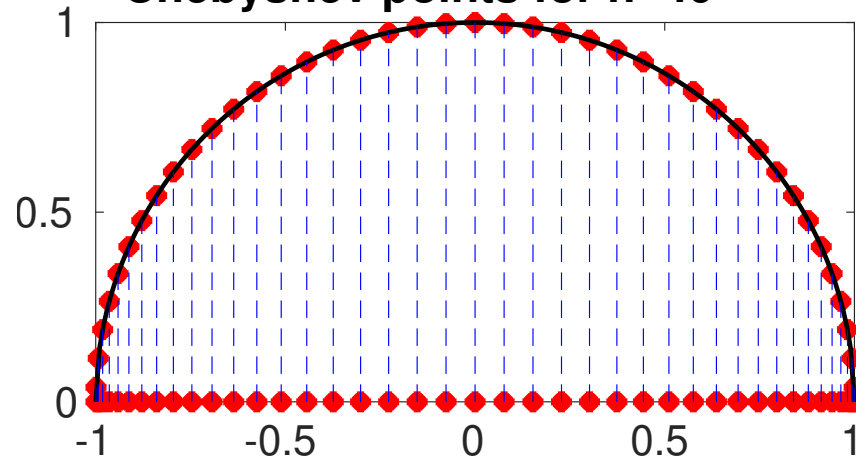
Chebyshev points for n=5



Chebyshev points for n=25



Chebyshev points for n=40



error estimates for Chebyshev interpolation

$$\|u - I_n^{Cheb} u\|_{C([-1,1])} \leq (1 + \Lambda_n) \inf_{v \in \mathbb{P}_n} \|u - v\|_{C([-1,1])}$$

The Lebesgue constant Λ_n satisfies

$$\frac{2}{\pi} \ln(n+1) + 0.53\dots \leq \Lambda_n \leq \frac{2}{\pi} \ln(n+1) + 1.$$

n	Λ_n
5	2.104
10	2.489
15	2.728
20	2.901

explicit formula: $\Lambda_n = \frac{1}{n+1} \sum_{i=0}^n \cot \frac{(2i+1)\pi}{4(n+1)} = \frac{2}{\pi} \ln(n+1) + \frac{2}{\pi} \left(\gamma + \ln \frac{8}{\pi} \right) + O\left(\frac{1}{n^2}\right), \quad \gamma = 0.57\dots$