

numerical differentiation with symmetric difference quotients

$$D_{sym}(h) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

h	h^2	$m = 0$	$m = 1$	$m = 2$	$m = 3$
h_0	h_0^2	$D_{sym}(h_0) = D_{00}$	D_{01}	D_{02}	D_{03}
h_1	h_1^2	$D_{sym}(h_1) = D_{10}$	D_{11}	D_{12}	D_{13}
h_2	h_2^2	$D_{sym}(h_2) = D_{20}$	D_{21}	D_{22}	D_{23}
h_3	h_3^2	$D_{sym}(h_3) = D_{30}$	D_{31}	D_{32}	D_{33}
h_4	h_4^2	$D_{sym}(h_4) = D_{40}$	D_{41}	D_{42}	\vdots
h_5	h_5^2	$D_{sym}(h_5) = D_{50}$	D_{51}	\vdots	
h_6	h_6^2	$D_{sym}(h_6) = D_{60}$	\vdots		
\vdots	\vdots	\vdots			

$$D_{i0} = D_{sym}(h_i)$$

$$D_{ij} = D_{(i+1)(j-1)} - \frac{h_{i+j}^2}{h_{i+j}^2 - h_i^2} [D_{(i+1)(j-1)} - D_{i(j-1)}], \quad j \geq 1$$

numerical differ. with sym. difference quotients, $f(x) = e^x, x_0 = 0$

Neville scheme

h	h^2	$m = 0$	$m = 1$	$m = 2$	$m = 3$
1	1	1.17520119364380	0.99785375010206	1.00000315726189	0.99999999931903
2^{-1}	2^{-2}	1.04219061098749	0.99986881931440	1.00000004866189	0.9999999999737
2^{-2}	2^{-4}	1.01044926723267	0.99999184682767	1.00000000075775	0.9999999999999
2^{-3}	2^{-6}	1.00260620192892	0.99999949113712	1.00000000001183	1.0000000000000
2^{-4}	2^{-8}	1.00065116883507	0.9999996820716	1.00000000000018	1.0000000000001
2^{-6}	2^{-10}	1.00016276836414	0.9999999801312	1.00000000000001	0.9999999999997
2^{-6}	2^{-12}	1.00004069060087	0.9999999987583	0.9999999999997	
2^{-7}	2^{-14}	1.00001017255709	0.9999999999221		
2^{-8}	2^{-16}	1.00000254313343			

error and reduction factor:

h	h^2	$m = 0$	ρ	$m = 1$	ρ	$m = 2$	ρ	$m = 3$	ρ
1	1	1.752 ₋₁	—	-2.146 ₋₃	—	3.157 ₋₆	—	-6.81 ₋₁₀	—
2^{-1}	2^{-2}	4.219 ₋₂	0.25	-1.312 ₋₄	0.0611	4.866 ₋₈	0.0154	-2.63 ₋₁₂	0.0039
2^{-2}	2^{-4}	1.045 ₋₂	0.25	-8.153 ₋₆	0.0622	7.578 ₋₁₀	0.0156	-1.0 ₋₁₄	0.0038
2^{-3}	2^{-6}	2.606 ₋₃	0.25	-5.089 ₋₇	0.0624	1.183 ₋₁₁	0.0156	0.0	??
2^{-4}	2^{-8}	6.51 ₋₄	0.25	-3.179 ₋₈	0.0625	1.8 ₋₁₃	0.0152	1.0 ₋₁₄	??
2^{-5}	2^{-10}	1.628 ₋₄	0.25	-1.987 ₋₉	0.0625	1.0 ₋₁₄	??	-3.0 ₋₁₄	??
2^{-6}	2^{-12}	4.069 ₋₅	0.25	-1.242 ₋₁₀	0.0625	-3.0 ₋₁₄	??		
2^{-7}	2^{-14}	1.017 ₋₅	0.25	-7.79 ₋₁₂	0.0627				
2^{-8}	2^{-16}	2.543 ₋₆	0.25						
			0.25		0.0625		0.015625		.00390625