problem sheet 11

discussion: week of Monday, 10.1.2022

11.1. (**to be uploaded in TUWEL**) Consider the for $N \in \mathbb{N}$ the system of equations

$$\frac{-u_{i+1} + 2u_i - u_{i-1}}{h^2} + u_i^3 = 1, \qquad i = 1, \ldots, N-1.$$

Formulate Newton's method for its solution and program it in `matlab`/`python`. Estimate the error (in the $\|\cdot\|_2$-norm) by considering the difference of two consecutive iterates. Plot the error versus the iteration number.

Remark: the above system of equations results from the numerical approximation of the "boundary value problem"

$$-u''(x) + (u(x))^3 = 1, \qquad x \in (0,1), \qquad u(0) = u(1) = 0.$$

The values u_i are approximations to the true values $u(x_i)$ with $x_i = ih$, $i = 1, \ldots, N-1$.

11.2. Newton's method converges only for initial values close to the zero x^* of f. One possibility to address this difficulty is the so-called *continuation method*: One considers a function $H(x,s)$ with $H(x,1) = f(x)$ and for which a zero x_0 of $H(x,0)$ is known. One then selects points s_i, $i = 1, \ldots, N$ and employs Newton's method to compute the zero x_{i+1} of $H(x, s_{i+1})$, taking x_i as the initial value. Perform the method for

$$f(x) = \arctan x, \qquad H(x,s) = \arctan x - (1-s)\arctan 4$$

and initial value $x_0 = 4$. Select $s_i = i/10$, $i = 0, \ldots, 10$.

11.3. (steepest descent method) Program the *steepest descent method* to find the minimum of the function

$$f(x,y) = 4x^2 - 4xy + 2y^2.$$

For the stepsize control, use the Armijo rule with $\sigma = 0.9$ and $q = 0.5$. Start with the initial guess $(2,3)^\top$. The method should converge to the (global) minimum $(0,0)^\top$. Plot the error versus the iteration number. What convergence do you observe, i.e., linear or quadratic?