evaluation of a recursion: $x_{k+1} = 7/3x_k - 4/3x_{k-1}$

exact solution for initial value $x_0 = 1, \quad x_1 = 1 + \varepsilon$:

$$x_k = 1 + \frac{\varepsilon}{\lambda - 1}\left(\lambda^k - 1\right), \qquad \lambda = \frac{4}{3}$$

MATLAB:

$x_0 = 1,\ x_1 = 1$

$x_{10} =$	1.000000000000028
$x_{20} =$	1.000000000000620
$x_{30} =$	1.000000000011237
$x_{40} =$	1.000000000199907
$x_{50} =$	1.000000003550380
$x_{60} =$	1.000000063047287
$x_{70} =$	1.000001119577209
$x_{80} =$	1.000019881146851
$x_{90} =$	1.000353043971939
$x_{100} =$	1.006269258344517

MAPLE: (Digits:=500)

$x_1 = 1,\ x_2 = 1 + 10^{-16}$

$x_{10} =$	1.000000000000005
$x_{20} =$	1.000000000000094
$x_{30} =$	1.000000000001680
$x_{40} =$	1.000000000029831
$x_{50} =$	1.000000000529734
$x_{60} =$	1.000000009406876
$x_{70} =$	1.000000167044744
$x_{80} =$	1.000002966334913
$x_{90} =$	1.000052675364496
$x_{100} =$	1.000935394723062

observation: $\varphi : x_1 \mapsto x_{100}$ is ill conditioned since $\lambda^{100} \approx 3 \cdot 10^{12}$

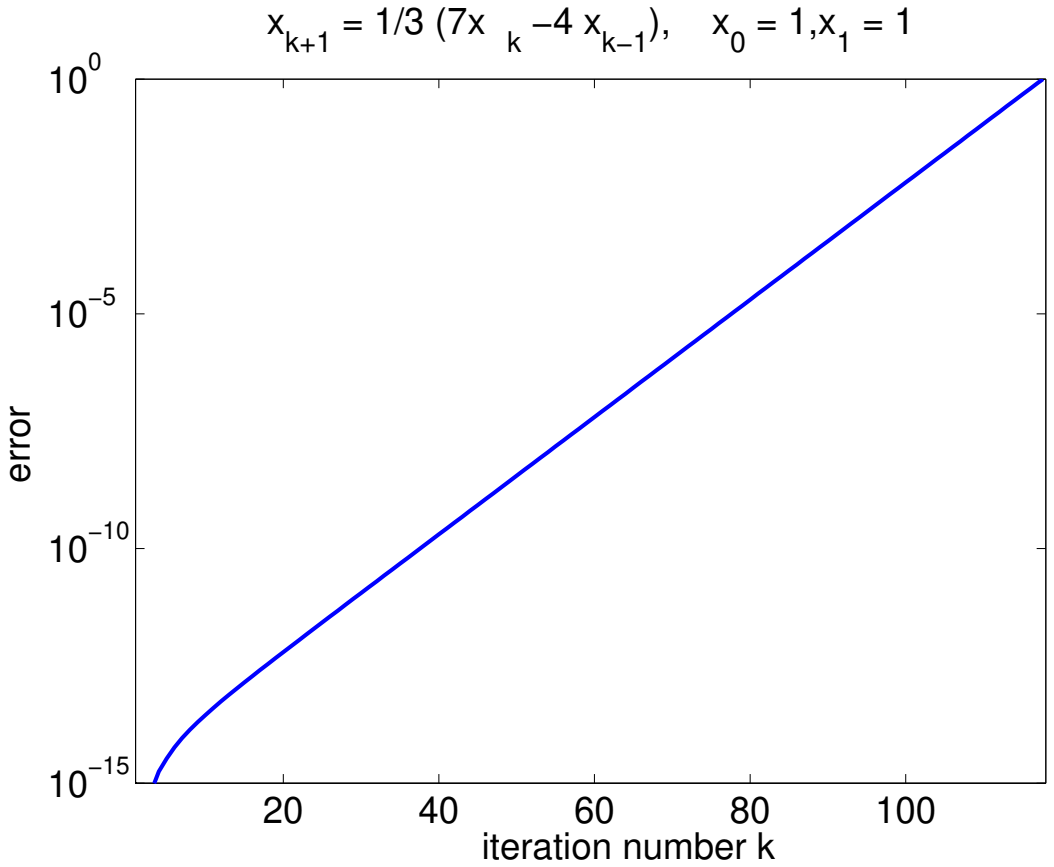
problem is ill conditioned $\Longrightarrow$ unavoidable rounding errors can be dramatically amplified!

evaluation of a recursion

$$x_{k+1} = \frac{1}{3}\left(7x_k - 4x_{k-1}\right), \qquad k = 1, 2, \ldots,$$

initial values: $x_0 = 1$, $x_1 = 1$

exact solution: $x_k = 1$ for all k



$$\begin{aligned}
x_0 &= 1.000000000000000 \\
x_{10} &= 1.000000000000028 \\
x_{20} &= 1.000000000000620 \\
x_{30} &= 1.000000000011237 \\
x_{40} &= 1.000000000199907 \\
x_{50} &= 1.000000003550380 \\
x_{60} &= 1.000000063047287 \\
x_{70} &= 1.000001119577209 \\
x_{80} &= 1.000019881146851 \\
x_{90} &= 1.000353043971939 \\
x_{100} &= 1.006269258344517
\end{aligned}$$