stopping criteria

notation/assumptions:

- $\mathbf{A}$ diagonalizable with $\mathbf{T}^{-1}\mathbf{A}\mathbf{T} = \mathbf{D}$.
- $\mathbf{x} \in \mathbb{R}^n$ with $\|\mathbf{x}\|_2 = 1$
- Rayleigh quotient $R(\mathbf{x}) = \mathbf{x}^\top \mathbf{A}\mathbf{x} = \frac{\mathbf{x}^\top \mathbf{A}\mathbf{x}}{\|\mathbf{x}\|_2^2}$.
- $\mathbf{r} := \mathbf{A}\mathbf{x} - R(\mathbf{x})\mathbf{x}$.

eigenvalue error estimate:

- $\min_i |\lambda_i - R(\mathbf{x})| \leq \mathrm{cond}_2(\mathbf{T})\|\mathbf{r}\|_2$
- $\min_i |\lambda_i - R(\mathbf{x})| \leq \|\mathbf{r}\|_2$ if A symmetric
- $\min_i |\lambda_i - R(\mathbf{x})| \leq C\|\mathbf{r}\|_2^2$ if $\mathbf{A}$ symmetric and $R(\mathbf{x})$ sufficiently close to a simple EV

Examples (standard vector iteration); (initial vector: $\mathbf{x}_0 = (1,1,1)^\top$)

$$\mathbf{A}_1 = \begin{pmatrix} 10 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

m	$\lvert\lambda_{max} - R(\mathbf{x}_m)\rvert$	$\dfrac{\lvert\lambda_{max} - R(\mathbf{x}_m)\rvert}{\lVert\mathbf{r}_m\rVert_2^2}$
1	5.78	2.5_{-1}
2	4.6_{-2}	1.09_{-1}
3	3.6_{-4}	1.08_{-1}
4	2.8_{-6}	1.08_{-1}
5	2.1_{-8}	1.08_{-1}
6	1.7_{-10}	1.08_{-1}
7	1.3_{-12}	1.08_{-1}
8	1.0_{-14}	0.9_{-1}

$$\mathbf{A}_2 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 4.5 \end{pmatrix}, \quad \mathrm{cond}_2(T) \approx 26.5.$$

m	$\lvert\lambda_{max} - R(\mathbf{x}_m)\rvert$	$\dfrac{\lvert\lambda_{max} - R(\mathbf{x}_m)\rvert}{\lVert\mathbf{r}_m\rVert_2}$
1	2.0	1.1
10	5.3_{-2}	8.2
20	2.2_{-1}	10.8
30	1.5_{-2}	11.6
40	4.6_{-3}	11.8
50	1.4_{-3}	11.9
60	4.3_{-4}	11.9

vector iteration for EVP with EV that are not separated (in absolute value)

$$\mathbf{A}_3 = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 0.1 \end{pmatrix},$$

$$c = \cos(\pi/3), \quad s = \sin(\pi/3),$$

$$\lambda = 0.5 \pm 0.5\sqrt{3}\,\mathbf{i}, \quad \lambda = 0.1$$

ℓ	$\widetilde{\lambda}_\ell$	$\Vert\mathbf{A}\mathbf{x}_\ell - \widetilde{\lambda}_\ell \mathbf{x}_\ell\Vert_2$
0	0.36666666666667	0.73181661333667
1	0.49800995024876	0.86432674164283
2	0.49998000099995	0.86600837240466
3	0.49999980000010	0.86602523346615
4	0.49999999800000	0.86602540208126
5	0.49999999998000	0.86602540376741
6	0.49999999999980	0.86602540378427
7	0.50000000000000	0.86602540378444
8	0.50000000000000	0.86602540378444
9	0.50000000000000	0.86602540378444
10	0.50000000000000	0.86602540378444

$$\mathbf{x}_0 = (1, 1, 1)^\top / \sqrt{3}$$