A hierarchy model for semiconductors and plasmas

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1 Introduction

It is known that the evolution of the particles for plasmas or semiconductor devices can be described by Euler-Poisson equations in which appear some small parameters, like the mass of electrons, the relaxation time and the Debye length. In order to perform numerical simulations of the hyperbolic equations, a lot of computing power and special algorithms are needed. In some situations, however, the model equations are often replaced by simpler equations, like drift-diffusion models. Starting from the Euler-Poisson equations and let the small parameters tend to zero in these equations we obtain a hierarchy of hydrodynamic and quasi-hydrodynamic models.

More specifically, let us consider an ensemble of electrons with density n_e and charge $q_e = -1$ and positively charged particles (holes for semiconductors, ions for plasmas) with density n_i and charge $q_i = 1$. Denote by $j_e = j_e(t,x)$ (respectively, j_i) the scaled current density of the electrons (respectively, ions) and by $\phi = \phi(t,x)$ the scaled electric potential at time $t > 0$ and position $x \in I\!\!R^d$. These variables satisfy the following scaled Euler-Poisson system **(HD-EI)** :

$$\partial_t n_\alpha + \mathrm{div} j_\alpha = 0, \tag{1.1}$$

$$m_\alpha \partial_t j_\alpha + m_\alpha \mathrm{div}(j_\alpha \otimes j_\alpha / n_\alpha) + \nabla p_\alpha(n_\alpha) = -q_\alpha n_\alpha \nabla\phi - m_\alpha j_\alpha/\tau_\alpha, \tag{1.2}$$

$$-\lambda^2 \Delta\phi = n_i - n_e, \tag{1.3}$$

where $\alpha = e, i$ and $(t,x) \in (0,+\infty) \times I\!\!R^d$. This system is complemented by initial conditions for (n_α, j_α) and by boundary conditions for ϕ :

$$t = 0: \ n_\alpha = n_{\alpha 0}(x), \ j_\alpha = j_{\alpha 0}(x), \ \ x \in I\!\!R^d, \ \ \alpha = e, i, \tag{1.4}$$

$$\lim_{|x|\to\infty} \phi(t,x) = 0, \quad \text{a.e. } \; t \in (0,+\infty) \times I\!\!R^d. \tag{1.5}$$

Here, $j_\alpha \otimes j_\alpha$ denotes the tensor product with components $j_{\alpha,j} j_{\alpha,k}$ for $j,k = 1,\ldots,d$. The small physical constants are the scaled particles mass m_α $(\alpha = e,i)$, the relaxation time $\tau_\alpha >$ and the Debye length $\lambda > 0$. They are independent each other. For $\alpha = e,i$, we take the pressure functions of the form

$$p_\alpha(n_\alpha) = a_\alpha^2 n_\alpha^{\gamma_\alpha}, \;\; n_\alpha \geq 0, \tag{1.6}$$

and define the enthalpy function h_α by

$$h'_\alpha(s) = p'_\alpha(s)/s, \;\; s > 0, \;\; h_\alpha(1) = 0,$$

where $\gamma_\alpha \geq 1$ and $a_\alpha > 0$ are constants. In one space dimension, the existence of global weak solutions to the equations (HD-EI) has been investigated by many authors. Two frameworks may be applied. The isothermal case $\gamma_i = \gamma_e = 1$ is usually treated by the fractional step Glimm scheme. The existence of weak solutions can be proved for arbitrarily large data in $BV(I\!\!R)$, see for example [22, 2]. This is mainly due to the property that for the homogeneous system the total variation of the quantity $\log n_\alpha(t,.)$ is non-increasing in time in the Glimm scheme. The electric field $E = -\nabla\phi$ can also be controlled if the initial data are supposed to be constants outside a fixed bounded interval.

In the cases $\gamma_i > 1$ and $\gamma_e > 1$, the solutions are constructed by the fractional step Lax-Friedrichs scheme or Godunov scheme [24, 18, 20]. The convergence of the schemes can be proved with the compensated compactness argument developed in [23, 6, 1, 17, 16]. More precisely, let $d = 1$ and $Q_T = (0,T) \times I\!\!R$ with $T > 0$. For $\alpha = e,i$, assume $\gamma_\alpha > 1$ and $0 \leq n_{\alpha 0}$, $J_{\alpha 0}/n_{\alpha 0} \in L^\infty(I\!\!R)$ with compact support. Then the problem (1.1)-(1.6) has a global entropy solution $(n_\alpha, j_\alpha, \phi)$ satisfying $0 \leq n_\alpha \in L^\infty(Q_T)$, $j_\alpha \in L^\infty(Q_T)$ and for any pair of smooth entropy-flux (η,ψ) with η convex $(\alpha = e,i)$:

$$\partial_t \eta(n_\alpha, j_\alpha) + \partial_x \psi(n_\alpha, j_\alpha) + (q_\alpha n_\alpha \partial_x \phi + m_\alpha j_\alpha/\tau_\alpha)\partial_{j_\alpha}\eta(n_\alpha, j_\alpha) \leq 0 \;\text{ in } D'(Q_T). \tag{1.7}$$

After a new scaling of time in the hydrodynamic model (HD-EI) the zero-relaxation-time limit $\tau_\alpha \to 0$ $(\alpha = e,i)$ leads to the drift-diffusion equations **(DD-EI)** (see section 2 or [12, 14]) :

$$m_e \partial_t n_e - \operatorname{div}(\nabla p_e(n_e) - n_e \nabla\phi) = 0, \tag{1.8}$$

$$m_i \partial_t n_i - \operatorname{div}(\nabla p_i(n_i) + n_i \nabla\phi) = 0, \tag{1.9}$$

$$-\lambda^2 \Delta\phi = n_i - n_e \, . \tag{1.10}$$

Since these equations are of parabolic-elliptic type, we consider them in an open and bounded domain $\Omega \subset I\!\!R^d$ with initial and boundary conditions $(\alpha = e,i)$:

$$n_\alpha(0) = n_{I,\alpha} \;\text{ in } \Omega, \tag{1.11}$$

$$n_\alpha = n_{D,\alpha}, \quad \phi = \phi_D \ \text{ on } \Sigma_D, \tag{1.12}$$

$$\nabla p_\alpha(n_\alpha) \cdot \nu = \nabla \phi \cdot \nu = 0 \ \text{ on } \Sigma_N. \tag{1.13}$$

Here we assume that the boundary $\partial\Omega$ of the domain consists of two disjoint sets Γ_D and Γ_N with $\text{meas}_{d-1}(\Gamma_D) > 0$,

$$\Omega_T = (0,T) \times \Omega, \ \ \Sigma_D = \ (0,T) \times \Gamma_D, \ \ \Sigma_N = \ (0,T) \times \Gamma_N,$$

and the function ν is the normal unit vector of $\partial\Omega$ which is assumed to exist almost everywhere.

For the given data, we impose the following assumptions for $\alpha = e, i$:

(A1) $n_{I,\alpha} \in L^\infty(\Omega)$, $n_{D,\alpha} \in C^0(\Sigma_D) \cap H^1(\Sigma_D)$, $\phi_D \in H^1(0,T; H^1(\Gamma_D)) \cap L^\infty(\Sigma_D)$.

(A2) There exist two constants $\underline{n}, \overline{n} > 0$ such that $0 < \underline{n} \le n_{I,\alpha} \le \overline{n}$ in Ω and $0 < \underline{n} \le n_{D,\alpha} \le \overline{n}$ on Σ_D.

(A3) $p_\alpha \in C^1([0,+\infty))$ is strictly increasing on $(0,+\infty)$.

Under the assumptions (A1) and (A3) it has been proved in [11] that there exists a solution (n_e, n_i, ϕ) to the problem (1.8)-(1.13) satisfying

$$n_\alpha \in H^1(0,T; V^*) \cap L^\infty(\Omega_T) \cap L^2(0,T; H^1(\Omega)), \ \ \phi \in L^\infty(0,T; H^1(\Omega)),$$

where V^* is the dual space of

$$V = \{u \in H^1(\Omega) : u = 0 \text{ on } \Gamma_D\}.$$

Moreover, under the assumptions (A1)-(A3), $\partial\Omega = \Gamma_D$, and some regularity conditions on the boundary functions, the solution is unique.

For the systems (HD-EI) and (DD-EI), we are mainly interested in three limits, i.e. the zero-relaxation-time limit $\tau_\alpha \to 0$, the zero-electron-mass limit $m_e \to 0$, and the zero-Debye-length limit (or the quasi-neutral limit) $\lambda \to 0$. We refer to [13, 15] for physical explanations of these limits. In the next section, we explain how to derive the formal hierarchy models from (HD-EI) and (DD-EI). Then we give in the last section some proved results of the mathematical justifications of these limits.

2 Formal derivation of hierarchy models

(1). Zero-electron-mass limit (HD-EI) $\longrightarrow$ (HD-I)

Usually, the ions are heavy compared to the electrons, i.e. $m_i \gg m_e$. Therefore, fix $m_i > 0$ and let $m_e \to 0$ in the equations (HD-EI), we obtain from (1.2) :

$$\nabla p_e(n_e) = -q_e n_e \nabla \phi,$$

or in particular

$$\phi = h_e(n_e).$$

Let $f_\alpha\,(\alpha = e, i)$ be the inverse function of h_α. Then

$$n_e = f_e(\phi).$$

Hence, for the isothermal fluid of electrons $\gamma_e = 1$, we obtain the well known Maxwell-Boltzmann relation :

$$n_e = f_e(\phi) = \exp(\frac{\phi}{a_e^2}).$$

Finally, the formal zero-electron-mass limit $m_e \to 0$ of (HD-EI) is governed by a nonlinear Euler-Poisson equations for ions **(HD-I)** :

$$\partial_t n_i + \mathrm{div} j_i = 0, \tag{2.1}$$

$$m_i \partial_t j_i + m_i \mathrm{div}(j_i \otimes j_i/n_i) + \nabla p_i(n_i) = -n_i \nabla \phi - m_i j_i/\tau_i, \tag{2.2}$$

$$-\lambda^2 \Delta \phi = n_i - f_e(\phi). \tag{2.3}$$

We recover (n_e, j_e) by the following equations :

$$n_e = f_e(\phi), \quad \partial_t n_e + \mathrm{div} j_e = 0.$$

A further discussion of this limit can be found in [9].

(2). Zero-Debye-length limit (HD-EI) $\longrightarrow$ (HD-EI-QN)

Perform the zero-Debye-length limit $\lambda \to 0$ in the model (HD-EI), we obtain system **(HD-EI-QN)** :

$$n := n_i = n_e. \tag{2.4}$$

$$\partial_t n + \mathrm{div} j_\alpha = 0, \tag{2.5}$$

$$m_\alpha \partial_t j_\alpha + m_\alpha \mathrm{div}(j_\alpha \otimes j_\alpha/n) + \nabla p_\alpha(n) = -q_\alpha n \nabla \phi - m_\alpha j_\alpha/\tau_\alpha, \tag{2.6}$$

for $\alpha = e, i$. From (2.5), we have

$$\mathrm{div}(j_i - j_e) = 0.$$

In one space dimension, if j_i and j_e vanish at infinity, we have $j := j_i = j_e$. Hence, adding the equations (2.6) for $\alpha = e$ and $\alpha = i$, we obtain a decoupled system for (n, j) and ϕ :

$$\partial_t n + \partial_x j = 0, \tag{2.7}$$

$$(m_e+m_i)\partial_t j+\partial_x((m_e+m_i)j^2/n+p_e(n)+p_i(n)) = -(m_e/\tau_e+m_i/\tau_i)j, \tag{2.8}$$

$$n\partial_x\phi = m_e\partial_t j+\partial_x(m_e j^2/n+p_e(n))+m_e j/\tau_e. \tag{2.9}$$

This limit has been studied by Cordier et al. in [3] for the traveling wave solutions and rigorously proved by Cordier and Grenier in [4] for local smooth solutions in the isothermal case.

(3). Zero-relaxation-time limit (HD-EI) $\longrightarrow$ (DD-EI)

Let us introduce a scaling of time $s=\tau t$ and define

$$N_\alpha(s,x)=n_\alpha(\frac{s}{\tau},x),\ \ J_\alpha(s,x)=\frac{1}{\tau}j_\alpha(\frac{s}{\tau},x),\ \ \Phi(s,x)=\phi(\frac{s}{\tau},x), \tag{2.10}$$

where, for simplicity, we have taken $\tau=\tau_e=\tau_i$. Otherwise, we take $\tau=\max(\tau_e,\tau_i)$. Denote again s by t. Then the equations (HD-EI) become $(\alpha=e,i)$:

$$\partial_t N_\alpha+\mathrm{div}J_\alpha=0, \tag{2.11}$$

$$\tau^2 m_\alpha\partial_t J_\alpha+\tau^2 m_\alpha\mathrm{div}(J_\alpha\otimes J_\alpha/N_\alpha)+\nabla p_\alpha(N_\alpha)=-q_\alpha N_\alpha\nabla\Phi-m_\alpha J_\alpha, \tag{2.12}$$

$$-\lambda^2\Delta\Phi=N_i-N_e, \tag{2.13}$$

and the initial boundary conditions (1.4)-(1.5) are now

$$N_\alpha^\tau(0,x)=n_0(x),\ \ J_\alpha^\tau(0,x)=\frac{1}{\tau}j_0(x),\ \ x\in I\!R^d, \tag{2.14}$$

$$\lim_{|x|\to\infty}\Phi^\tau(t,x)=0,\quad \text{a.e.}\ \ t\in(0,+\infty). \tag{2.15}$$

Let $\tau\to 0$, we have from (2.12) that

$$m_\alpha J_\alpha=-q_\alpha N_\alpha\nabla\Phi-\nabla p_\alpha(N_\alpha),$$

together (2.11) and (2.13), we obtain the drift-diffusion equations (DD-EI). In one space dimension and for the pressure functions given by (1.6) with $\gamma_\alpha>1\,(\alpha=e,i)$, this limit has been rigorously justified in [12, 14]. The result is stated in Theorem 1. In the isothermal case, we refer to [10] for the proof.

(4). Zero-electron-mass limit (DD-EI) $\longrightarrow$ (DD-I)

Performing the limit $m_e\to 0$ in equation (1.8) gives

$$\mathrm{div}(n_e\nabla(h_e(n_e)-\phi))=\mathrm{div}(\nabla p_e(n_e)-n_e\nabla\phi)=0.$$

If $n_e>0$ and the Dirichlet boundary data are in equilibrium, i.e.

$$h_e(n_{D,e}) = \phi_D \ \text{ on } \Sigma_D,$$

then we deduce

$$h_e(n_e) = \phi$$

in whole domain Ω_T. Hence, we obtain again the relation $n_e = f_e(\phi)$, and the zero-electron-mass limit in (DD-EI) leads to a nonlinear drift-diffusion equations **(DD-I)** :

$$m_i\partial_t n_i - \text{div}(\nabla p_i(n_i) + n_i\nabla\phi) = 0, \tag{2.16}$$

$$-\lambda^2\Delta\phi = n_i - f_e(\phi), \ \ n_e = f_e(\phi). \tag{2.17}$$

The proof of this limit is shown in [13], see also Theorem 2.

(5). Zero-Debye-length limit (DD-EI) $\longrightarrow$ (DD-EI-QN)

Let $\lambda \to 0$ in (DD-EI), we obtain

$$n := n_e = n_i, \tag{2.18}$$

$$m_e\partial_t n - \text{div}(\nabla p_e(n) - n\nabla\phi) = 0,$$

$$m_i\partial_t n - \text{div}(\nabla p_i(n) + n\nabla\phi) = 0.$$

Adding and subtracting these last two equations, we obtain again a decoupled system **(DD-EI-QN)** :

$$(m_e + m_i)\partial_t n - \Delta(p_e(n) + p_i(n)) = 0, \tag{2.19}$$

$$-\text{div}((m_e + m_i)n\nabla\phi = \Delta(m_e p_i(n) - m_i p_e(n)). \tag{2.20}$$

If the initial data are in equilibrium, i.e. $n_{I,e} = n_{I,i}$, this limit can be proved rigorously even in the presence of boundary layers [15]. This result is presented in Theorem 4.

(6). Zero-mass limit (DD-EI) $\longrightarrow$ (NPE)

Similar to the zero-electron-mass limit, the formal limit as $m_e \to 0$ and $m_i \to 0$ in the system (DD-EI) gives the following nonlinear Poisson equation **(NPE)** :

$$-\lambda^2\Delta\phi = f_i(\beta - \phi) - f_e(\phi), \tag{2.21}$$

where β is a given function determined by the following problem [13] :

$$\text{div}(n_i\nabla\beta) = 0 \ \text{ in } \ \Omega_T, \ \ \beta = h_i(n_{D,i}) + \phi_D \ \text{ on } \Sigma_D, \ \ \nabla\beta.\nu = 0 \ \text{ on } \Sigma_N.$$

Obviously, (NPE) can also be obtained formally from the system (DD-I) by letting $m_i \to 0$ in equation (2.16)-(2.17).

The above six limits constitute one branch of hierarchy models. Another branch of models is obtained in the limits $\lambda \to 0$ and $\tau \to 0$ of (HD-I) or (DD-I). Indeed, for the

$$
\begin{tikzcd}
\text{(HD-EI-QN)} & \text{(HD-EI)} \arrow[l, "\lambda \to 0"'] \arrow[rr, "m_e \to 0"] \arrow[d, "\begin{array}{c}\tau_e \to 0\\ \tau_i \to 0\end{array}"'] & & \text{(HD-I)} \arrow[r, "\lambda \to 0"] \arrow[d, "\tau_i \to 0"] & \text{(HD-I-QN)} \\
\text{(DD-EI-QN)} & \text{(DD-EI)} \arrow[l, "\lambda \to 0"'] \arrow[rr, "m_e \to 0"] \arrow[dr, "\begin{array}{c}m_e \to 0\\ m_i \to 0\end{array}"'] & & \text{(DD-I)} \arrow[r, "\lambda \to 0"] \arrow[dl, "m_i \to 0"] & \text{(DD-I-QN)} \\
& & \text{(NPE)} & &
\end{tikzcd}
$$

Figure : A hierarchy model.

system (HD-I), the existence of global solutions has been proved in [5, 21] in a similar way than for the system (HD-EI). It is easy to see that the zero-Debye-length limit $\lambda \to 0$ of (HD-I) leads to system **(HD-I-QN)** :

$$n_i = f_e(\phi),$$

$$\partial_t n_i + \mathrm{div} j_i = 0,$$

$$m_i \partial_t j_i + m_i \mathrm{div}(j_i \otimes j_i / n_i) + \nabla p_i(n_i) = -n_i \nabla \phi - m_i j_i / \tau_i,$$

and the zero-relaxation-time limit $\tau_i \to 0$ of (HD-I) gives again (DD-I). Finally, let $\lambda \to 0$, we obtain system **(DD-I-QN)** :

$$m_i \partial_t n_i - \Delta(p_e(n_i) + p_i(n_i)) = 0,$$

$$\phi = h_e(n_i).$$

These three limits have also been studied in [4, 12, 15] respectively. One common feature of the models (HD-I) and (DD-I) is that the Poisson equation is nonlinear. In general it is more difficult to treat the limits in (HD-I) and (DD-I) than in (HD-EI) and (DD-EI). However, for the quasi-neutral limits $\lambda \to 0$ in the drift-diffusion equations, the result is better for (DD-I) than for (DD-EI), due to a L^∞ estimate for ϕ.

A summary of the above models and limits is presented in Figure.

3 Some results on the limits

In this section, we state the main results proved for these hierarchy models. We first consider the zero-relaxation-time limits $\tau \to 0$ of the hydrodynamic models in one space dimension. For simplicity, we give only the result for equations (HD-EI). A similar result for (HD-I) is also valid.

Theorem 1 *((HD-EI) $\longrightarrow$ (DD-EI), [12, 14]) Let $d = 1$ and $T > 0$. For $\alpha = e, i$, assume $0 \le n_{\alpha 0}, j_{\alpha 0} \in L^\infty(I\!R)$ with compact support and $\gamma_\alpha > 1$. Let $(n_\alpha^\tau, j_\alpha^\tau, \phi^\tau)$ be a weak entropy solution to (2.11)-(2.15). Then, as $\tau \to 0$, up to subsequences, $(n_\alpha^\tau, j_\alpha^\tau, \phi^\tau)$ converges to $(n_\alpha, j_\alpha, \phi)$, a solution of (DD-EI), in the following sense :*

$$n_\alpha^\tau \longrightarrow n_\alpha^\tau, \;\; \phi^\tau \longrightarrow \phi \;\; in \; L^p_{loc}(Q_T) \;\; strongly, \; for \; all \;\; p \in [1, +\infty),$$

$$j_\alpha^\tau \rightharpoonup j_\alpha \;\; in \;\; L^2(Q_T) \;\; weakly,$$

$$\tau^2 (j_\alpha^\tau)^2 / n_\alpha^\tau \longrightarrow 0 \;\; in \;\; L^2(Q_T) \;\; strongly.$$

The proof of this theorem consists of two steps. First, we establish, for all $p \ge 1$, uniform L^p estimate for $(n_\alpha^\tau)_{\tau>0}$ and L^2 estimate for $(j_\alpha^\tau)_{\tau>0}$ instead of L^∞ estimate for $(n_\alpha^\tau)_{\tau>0}$. This is achieved by using the entropy conditions (1.7). The key point is to choose a sequence of positive and convex entropies, belonging to the weak entropy family. Its construction is based on the characterization of convexity of the weak entropies proved in [17]. Next, we show the strong compactness of the sequence $(n_\alpha^\tau)_{\tau>0}$, which follows from the div-curl lemma and the monotonicity method.

The zero-relaxation-time limit has also been studied in [19]. However, the limit is obtained only under assumption of given uniform L^∞ estimate for n_α^τ. Moreover, the singular initial condition of j_α^τ is not taken into account in their work.

Now we consider the zero-electron-mass limit $m_e \to 0$ and the zero-mass limit of the drift-diffusion models (DD-EI). For simplicity, we use the notation $\delta = m_e$ in the limit (DD-EI) $\longrightarrow$ (DD-I) and $\delta = m_e = m_i$ in the limit (DD-EI) $\longrightarrow$ (NPE).

Theorem 2 *((DD-EI) $\longrightarrow$ (DD-I), [13]) Assume (A1)-(A3) and $\phi_D = h_e(n_{D,e})$. Let $(n_e^\delta, n_i^\delta, \phi^\delta)$ be a weak solution to (1.8)-(1.13). Then there exists a subsequence (not relabeled) converging, as $\delta \to 0$, to a solution (n_e, n_i, ϕ) of (2.16)-(2.17), in the following sense :*

$$n_e^\delta \longrightarrow f_e(\phi), \; n_i^\delta \longrightarrow n_i \;\; in \;\; L^2(\Omega_T) \;\; strongly,$$

$$n_i^\delta \rightharpoonup n_i \;\; in \;\; L^2(0, T; H^1(\Omega)) \cap H^1(0, T; V^*) \;\; weakly,$$

$$\phi^\delta \longrightarrow \phi \;\; in \;\; L^2(0, T; H^1(\Omega)) \;\; strongly.$$

Theorem 3 *((DD-EI) $\longrightarrow$ (NPE), [13]) Let the assumptions (A1)-(A3), and $\phi_D = h_e(n_{D,e})$, $\beta = h_i(n_{D,i}) + \phi_D \in I\!R$ on Σ_D. Let $(n_e^\delta, n_i^\delta, \phi^\delta)$ be a solution to (1.8)-(1.13). Then up to a subsequence, as $\delta \to 0$,*

$$n_e^\delta \longrightarrow f_e(\phi), \;\; n_i^\delta \longrightarrow f_i(\beta - \phi) \;\; in \;\; L^2(\Omega_T) \;\; strongly,$$

$$\phi^\delta \longrightarrow \phi \;\; in \;\; L^2(0,T;H^1(\Omega)) \;\; strongly,$$

where ϕ is a solution of the nonlinear Poisson equation (NPE) in Ω with boundary conditions :

$$\phi = \phi_D \;\; on \;\; \Gamma_D, \;\; \nabla\phi.\nu = 0 \;\; on \;\; \Gamma_N.$$

Finally, we consider the zero-Debye-length limits in (DD-EI). We have

Theorem 4 *((DD-EI) $\longrightarrow$ (DD-EI-QN), [15]) Let (A1)-(A3) hold and assume $n_{I,e} = n_{I,i}$. For $\alpha = e, i$, let $(n_\alpha^\lambda, \phi^\lambda)$ be a solution to (1.8)-(1.13). Furthermore, let $\omega \subset \Omega$ with $\overline{\omega} \subset \Omega$. Then there is a subsequence (not relabeled) such that, as $\lambda \to 0$,*

$$n_\alpha^\lambda \longrightarrow n \;\; in \;\; L^p((0,T)\times\omega) \;\; strongly, for\ all \;\; p \in [1,+\infty),$$

$$\phi^\lambda \rightharpoonup \phi \;\; in \;\; L^2(0,T;H^1(\omega)) \;\; weakly,$$

where (n,ϕ) solves (2.19)-(2.20) and $n(0) = n_{I,e}$ in the sense of $H^{-1}(\omega)$. If additionally $n_{D,e} = n_{D,i}$ holds on Σ_D, then the above convergence results are global and (n,ϕ) satisfies the Dirichlet boundary conditions (1.13).

The proof of these three results is based on an appropriate entropy estimate together with compactness by monotonicity and convexity arguments. The assumption (A2) implies, by maximum principle, that the densities n_α are strictly positive in Ω_T. This is essential to show a $L^2(0,T;(\Omega))$ estimate for $h_\alpha(n_\alpha) + q_\alpha\phi$ by the entropy inequality. Note that in Theorem 4, we don't need the assumption $n_{D,e} = n_{D,i}$. Therefore, the boundary layers may occur in the zero-Debye-length limit of (DD-EI).

The zero-electron-mass limit $m_e \to 0$ in the system (DD-EI) provides an existence result for the model (DD-I). The existence of solutions to the model (DD-I) under general assumptions on the initial and boundary conditions is given in [13]. The zero-Debye-length limit $\lambda \to 0$ of (DD-EI) can be made rigorously for more general pressure functions and for so-called vacuum solutions (for which $n_\alpha = 0$ may occur locally) [15]. For the related limits in the nonlinear drift-diffusion equation (DD-I), we refer to [13, 15] for detail. Finally, we mention the works of [7, 8] for the zero-Debye-length limit $\lambda \to 0$ of (DD-EI), where homogeneous Neumann boundary conditions are imposed. Hence, no boundary layers are expected.

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