

# Accuracy of the demagnetization tensor

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Computing the demagnetization field correctly is of importance in both micromagnetism and magnetostatic applications. The magnetic field,  $\mathbf{H}$ , from a geometrical object that is homogeneously magnetized can be expressed as

$$\mathbf{H}(\mathbf{r}) = -\nabla\phi_M(\mathbf{r}) = -\nabla \frac{1}{4\pi} \oint_{S'} \frac{\hat{\mathbf{n}}(\mathbf{r}') \cdot \mathbf{M}(\mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|} dS'$$

where  $\mathbf{r}$  is the radius vector to the point where the field is to be known,  $\mathbf{r}'$  and  $S'$  are the radius vector and the surface of the object generating the field,  $\hat{\mathbf{n}}$  is the surface normal and  $\mathbf{M}$  is the magnetization. This equation can be written as the product of a tensor, i.e. the demagnetization tensor,  $\mathbb{N}$ , and the magnetization as

$$\mathbf{H}(\mathbf{r}) = -\mathbb{N} \cdot \mathbf{M}$$

The demagnetization tensor is known for several geometrical objects or tiles, such as a prism [1], tetrahedron [2] and cylindrical tile [3,4], as shown in Fig. 1. The magnetic field generated by these tiles can be used for a fast and accurate calculation of the demagnetization field in micromagnetic simulations, as is done in the MagTense micromagnetic framework [5]. However, the demagnetization tensor has previously been found to be numerically inaccurate at distances far away, but potentially also close to, specific tiles [6].

Here we present an analysis of the accuracy of the demagnetization tensor compared to the dipolar field, as this is what the magnetic field from any kind of tile should approximate at distances far from the tile. We computed the error for 2452 points on a spherical surface for 121 spheres with radius varying uniformly in logspace between 0 and 10000 tile radii. For each of these 121 spheres, we computed the median error. Shown in Fig. 1 is the normalized fractional error of the field compared to the dipolar field away from the three different tiles, as computed using the four different expressions for the demagnetization tensor stated above.

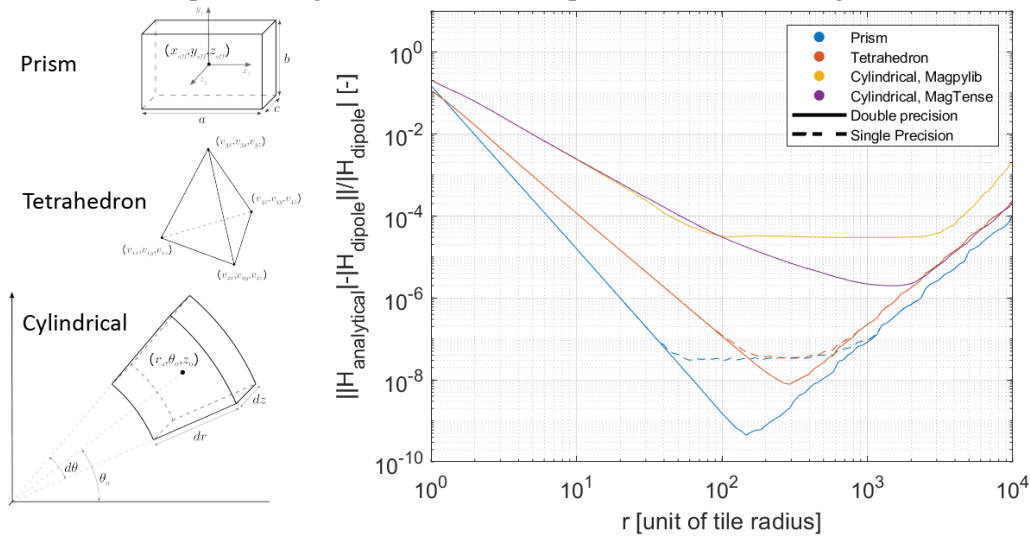


Figure 1: The geometries considered and the error as function of the distance,  $r$ , from the tile considered.

As can be seen from the figure, the demagnetization tensor correctly calculates the field out to several thousand tile radii, for both single and double precision computations.

## References

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