Curved thin-film limits of chiral Dirichlet energies

Giovanni Di Fratta[1], Valeriy Slastikov[2]

[1] *Dipartimento di Matematica e Applicazioni "R. Caccioppoli", Università degli Studi di Napoli "Federico II", Via Cintia, Complesso Monte S. Angelo, 80126 Napoli, Italy*

[2] *School of Mathematics, University of Bristol Bristol BS8 1UG, United Kingdom*

Curved thin films are currently of great interest due to their capability to support spontaneous skyrmion solutions, i.e., chiral spin textures observable in a stable state even when no spin-orbit coupling mechanism, in the guise of Dzyaloshinskii-Moriya interaction (DMI), is considered. The evidence of these states sheds light on the role of the geometry in magnetism: chiral spin-textures can be stabilized by curvature effects only, in contrast to the planar case for which the DMI is required.

In this talk, after a brief overview of the existing literature on the micromagnetics of curved thin films, we investigate the more general problem of curved thin-film limit of a family of perturbed Dirichlet energies in the space of H^1 Sobolev maps defined in a tubular neighborhood of an $(n-1)$-dimensional submanifold N of $\mathbb{R}^n$ and with values in an $(m-1)$-dimensional submanifold M of $\mathbb{R}^m$. The perturbation K that we consider is represented by a matrix-valued function defined on M and with values in $\mathbb{R}^{m\times n}$.

Under natural regularity hypotheses on N, M, and K, we show that the family of these energies converges, in the sense of Γ-convergence, to an energy functional on N of an unexpected form, which is of particular interest in the theory of magnetic skyrmions. As a byproduct of our results, we get that in the curved thin-film limit, antisymmetric exchange interactions also manifest under an anisotropic term whose specific shape depends both on the curvature of the thin film and the curvature of the target manifold.

References

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