A decoupled, convergent and fully linear algorithm for the Landau–Lifshitz–Gilbert equation with magnetoelastic effects

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We consider the coupled system of the Landau–Lifshitz–Gilbert equation and Newton's second law to describe magnetic processes in ferromagnetic materials including magnetoelastic effects in the small-strain regime [4,1]. For this nonlinear system of time-dependent partial differential equations, we present a decoupled integrator based on first-order finite elements in space and an implicit one-step method in time [3]. We prove unconditional convergence of the sequence of discrete approximations towards a weak solution of the system as the mesh size and the time-step size go to zero. Compared to previous numerical works on this problem [2], for our method, we prove a discrete energy law that mimics that of the continuous problem and, passing to the limit, yields an energy inequality satisfied by weak solutions. Moreover, we do not employ a nodal projection to impose the unit-length constraint on the discrete magnetisation, so that the stability of the method does not require weakly acute meshes. Furthermore, our integrator and its analysis hold for a more general setting, including body forces and traction, as well as a more general representation of the magnetostrain. Numerical experiments underpin the theory and showcase the applicability of the scheme for the simulation of the dynamical processes involving magnetoelastic materials at submicrometer length scales.

References

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