

# Analytic approach to magnetisation reversal

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The response to an applied field  $\vec{H}$  of a relevant class of magnetic materials is often modelled assuming the system as an assembly of mesoscopic regions ("particles"), each described by an hysteresis transducer. This problem is commonly faced by means of numerical procedures: a time consuming approach, not very efficient when the so-obtained constitutive equation is implemented in codes used to design electrotechnical components.

We have developed a mathematical technique that analytically predicts the response, under an alternating quasi-static field, of an ensemble of non-interacting particles at  $T = 0$  K, each characterised by a local anisotropy axis with modulus  $K$ , and forming an angle  $\varphi$  with a reference direction defined by  $\vec{H}$  (Figure 1). Instead of trying to find the single particle solution, that is  $\gamma = \gamma_H(K, \varphi)$ : the equilibrium angle for the local polarisation  $\vec{J}_s$ , afterwards integrated over the whole system, we started from entire particle assembly, handling  $K$ ,  $\varphi$ , and  $\gamma$  as random variables with their associated probability density function (pdf). Statistical methods [1] are then exploited to retrieve  $p_H(\gamma)$ : the equilibrium angle pdf, ending with an exact expression for the hysteresis loops (and consequently for the energy losses) of the entire system, with the single particle behaviour emerging as a special case. This procedure is sketched in Figure 1, with the constitutive law  $J(H)$  tested against numeric results.

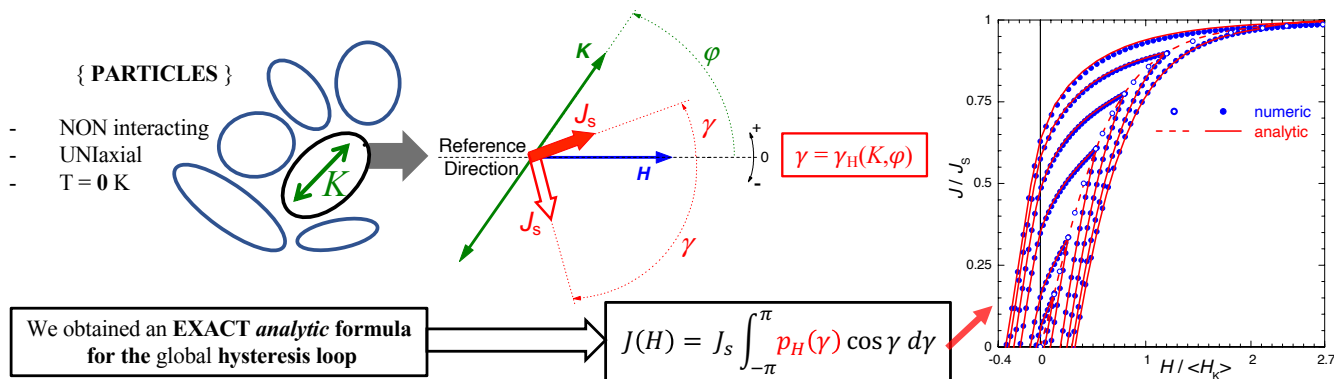


Figure 1: Scheme of the analytic-statistical approach leading to hysteresis loop prediction for a particle assembly (on the left). An example of the agreement between the loops analytically and numerically obtained is shown on the right

If the magnetisation reversal occurs through  $\vec{J}_s$  coherent rotations only, the Stoner-Wohlfarth (SW) model applies, and both reversible and irreversible processes of each particle are governed by the so called "astroid": a closed curve in the  $(H_{\parallel} = H \cos \varphi, H_{\perp} = H \sin \varphi)$  plane whose contour represents the border separating the  $(H_{\parallel}, H_{\perp})$  region corresponding to two minima of the particle Gibbs free energy (inside) from the one where one minimum only appears. When the analytic-statistical approach introduced above is applied to a SW particles ensemble, one obtains a prediction of the hysteresis loops that perfectly fits the one obtained via numerical procedure (right hand side of Figure 1). The proposed technique has been also exploited to simulate the behaviour of particles where the magnetisation reversal is accomplished by means of non-coherent rotations or even when the domain walls are present inside the particles, simply modifying the astroid border. These situations have been successfully tested against numerical procedures. It must be finally remarked that the statistic approach outlined works with any anisotropy distribution, so permitting one to easily predict the role played by the sample texture, (e.g. the effect of a macroscopic easy axis induced either by annealing or applied stress).

## References

[1] Athanasios Papoulis, *Probability, Random Variables, and Stochastic Processes* (McGraw-Hill, 1984).