BI.discrete & BI.computable

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Rate optimal adaptive FEM with inexact solver for nonlinear operators

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joint work with

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Der Wissenschaftsfonds.

Introduction

Adaptivity – What else?

Overall aim

- compute $u_h \approx u^\star$ with $|\!|\!| u^\star - u_h |\!|\!| \leq$ tolerance
- subject to minimal computational costs

- out of reach at the moment
- but clearly requires
  - adaptivity — to resolve singularities
  - effective solver — to avoid unnecessary costs
  - data compression — e.g., for dense BEM matrices

Tasks / Questions / Outline

- design adaptive algorithm which also steers (iterative) solver?
- convergence of adaptive strategy?
- optimal convergence rates w.r.t. degrees of freedom?
- new: optimal convergences rates w.r.t. computational work / time?

Cohen, Dahmen, DeVore: SIAM J. Numer. Anal. 70 (2001)

Ern, Vohralik: SIAM J. Sci. Comput. 35 (2013)

Gantner, Haberl, Praetorius, Stiftner: IMA J. Numer. Anal. 38 (2018)

Führer, Haberl, Praetorius, Schimanko: Numer. Math., 141 (2019)

Gantner, Haberl, Praetorius, Schimanko: work in progress 2019

Quasi-optimal computational costs?

$$\|u^\star\|_{\mathbb{A}_s} := \sup_{N>0} \Big(N^s \min_{\mathcal{T}_{\rm opt}\in\mathbb{T}_N} \big[\, \|\!| u^\star - u^\star_{\rm opt} |\!\| + \eta_{\rm opt}(u^\star_{\rm opt}) \,\big] \Big)$$

$$\simeq \sup_\ell (\#\mathcal{T}_\ell)^s \big[\, \|\!| u^\star - u^\star_\ell |\!\| + \eta_\ell(u^\star_\ell) \,\big]$$

$$\simeq \sup_{(\ell,n)} (\#\mathcal{T}_\ell)^s \big[\, \|\!| u^\star - u^n_\ell |\!\| + \eta_\ell(u^n_\ell) \,\big]$$

$$\simeq \sup_{(\ell,n)} \Big(\sum_{(\ell',n')\le(\ell,n)} \#\mathcal{T}_{\ell'} \Big)^s \big[\, \|\!| u^\star - u^n_\ell |\!\| + \eta_\ell(u^n_\ell) \,\big]$$

- goal: thorough proof of these equivalences for adaptive algorithm!

Almost optimal computational costs!

new: optimal

$$\|u^\star\|_{\mathbb{A}_s} \simeq \sup_{(\ell,n)\in\mathcal{Q}} \Big(\sum_{\substack{(\ell',n')\in\mathcal{Q}\\(\ell',n')\le(\ell,n)}} \#\mathcal{T}_{\ell'} \Big)^s \big[\, |||u^\star - u_\ell^n||| + \eta_\ell(u_\ell^n) \,\big]$$

known: almost optimal

$$\|u^\star\|_{\mathbb{A}_s} := \sup_{N>0} \Big(N^s \min_{\mathcal{T}_{\rm opt}\in\mathbb{T}_N} \big[\, |||u^\star - u^\star_{\rm opt}||| + \eta_{\rm opt}(u^\star_{\rm opt}) \,\big] \Big) < \infty$$

$$\Longrightarrow \quad \sup_{(\ell,n)\in\mathcal{Q}} \Big(\sum_{\substack{(\ell',n')\in\mathcal{Q}\\(\ell',n')\le(\ell,n)}} \#\mathcal{T}_{\ell'} \Big)^{s-\varepsilon} \big[\, |||u^\star - u_\ell^n||| + \eta_\ell(u_\ell^n) \,\big] < \infty \qquad \forall \varepsilon > 0$$

Gantner, Haberl, Praetorius, Stiftner: IMA J. Numer. Anal. 38 (2018)

Führer, Haberl, Praetorius, Schimanko: Numer. Math., 141 (2019)

Model Problem

Model problem

$$
\begin{aligned}
-\mathrm{div}\,\mathcal{A}(\nabla u^\star) + f(u^\star) &= 0 \quad \text{in } \Omega \\
u^\star &= 0 \quad \text{on } \Gamma = \partial\Omega
\end{aligned}
$$

Example for strongly monotone nonlinearities

- nonlinear material laws $\boldsymbol{M} = \chi(|\boldsymbol{H}|)\,\boldsymbol{H}$ in magnetostatics
- together with $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$
- e.g., $\mathcal{A}(\nabla u) = \Big(1 + \dfrac{1}{1 + |\nabla u|}\Big)\,\nabla u$

Zeidler: Nonlinear functional analysis and its applications, part II/B

Abstract setting

- $\mathcal{H}$ separable Hilbert space with norm $\|\cdot\|$
- $\boldsymbol{A} : \mathcal{H} \to \mathcal{H}^*$ is

(O1) strongly monotone $\alpha \|u - v\|^2 \le \langle \boldsymbol{A}u - \boldsymbol{A}v \,, u - v\rangle$

(O2) Lipschitz continuous $\|\boldsymbol{A}u - \boldsymbol{A}v\| \le L \, \|u - v\|$

Main Theorem on Strongly Monotone Operators (Zarantonello '60)

- exists unique $u^\star \in \mathcal{H}$ s.t. $\langle \boldsymbol{A}u^\star \,, v\rangle = 0$ for all $v \in \mathcal{H}$

Corollary

- exists unique $u_h^\star \in \mathcal{X}_h$ s.t. $\langle \boldsymbol{A}u_h^\star \,, v_h\rangle = 0$ for all $v_h \in \mathcal{X}_h$
- $\|u^\star - u_h^\star\| \le \dfrac{L}{\alpha} \min\limits_{u_h \in \mathcal{X}_h} \|u^\star - u_h\|$.

Zarantonello: U.S. Army Math. Res. Center, Tech. Report 160 (1960)

Zeidler: Nonlinear functional analysis and its applications, part II/B

Zarantonello iteration

- constructive proof by Banach fixpoint theorem
- $I_{\mathcal{H}} : \mathcal{H} \to \mathcal{H}^*$ Riesz mapping $\quad \langle I_{\mathcal{H}} u \,, v \rangle = (u \,,\, v)_{\mathcal{H}}$
- $\Phi(u) := u - \dfrac{\alpha}{L^2}\, I_{\mathcal{H}}^{-1} \boldsymbol{A} u,$

$\Longrightarrow \quad \|\Phi(u) - \Phi(v)\| \le q\, \|u - v\| \quad$ with $\quad q := \Big(1 - \dfrac{\alpha^2}{L^2}\Big)^{1/2}$

1. Φ has unique fixpoint $u^\star \in \mathcal{H}$ $\quad (\iff \quad \boldsymbol{A} u^\star = 0 \ )$
2. $u^0 \in \mathcal{H}$ arbitrary, $\quad u^n := \Phi(u^{n-1})$

$\Longrightarrow \quad \|u^\star - u^n\| \le \dfrac{q}{1-q}\, \|u^n - u^{n-1}\| \le \dfrac{q^n}{1-q}\, \|u^1 - u^0\|$

Zarantonello: U.S. Army Math. Res. Center, Tech. Report 160 (1960)

Zeidler: Nonlinear functional analysis and its applications, part II/B

Discrete Zarantonello iteration

- $u_h^0 \in \mathcal{X}_h$ arbitrary, $u_h^n := \Phi_h(u_h^{n-1})$

$$\Longrightarrow \quad |||u_h^\star - u_h^n||| \le \frac{q}{1-q}\,|||u_h^n - u_h^{n-1}||| \le \frac{q^n}{1-q}\,|||u_h^1 - u_h^0|||$$

Discrete Zarantonello iteration

- solve $\quad (u_h^n\,, v_h)_{\mathcal{H}} = (u_h^{n-1}\,, v_h)_{\mathcal{H}} + \frac{\alpha}{L^2}\,\langle \boldsymbol{A} u_h^{n-1}\,, v_h\rangle \quad$ for all $v_h \in \mathcal{X}_h$

- i.e., each step of Zarantonello iteration solves Laplace problem

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Congreve, Wihler: J. Comput. Appl. Math. 311 (2017)

Adaptive Algorithm

A posteriori error control

- $|||u^\star - u_\ell^n||| \le |||u^\star - u_\ell^\star||| + |||u_\ell^\star - u_\ell^n|||$
- estimator: $|||u^\star - u_\ell^\star||| \lesssim \eta_\ell(u_\ell^\star) \lesssim \eta_\ell(u_\ell^n) + |||u_\ell^\star - u_\ell^n|||$
- contraction: $|||u_\ell^\star - u_\ell^n||| \lesssim |||u_\ell^n - u_\ell^{n-1}|||$

$$\Longrightarrow \quad \big[\, |||u^\star - u_\ell^n||| + \eta_\ell(u_\ell^n) \,\big] \lesssim \eta_\ell(u_\ell^n) + |||u_\ell^n - u_\ell^{n-1}|||$$

- adaptive algorithm should equilibrate RHS

Congreve, Wihler: J. Comput. Appl. Math. 311 (2017)

Adaptive algorithm

- initial mesh $\mathcal{T}_0$ with initial guess $u_0^0 := 0$
- adaptivity parameters $0 < \theta \leq 1, \quad \lambda > 0$

For all $\ell := 0, 1, 2, \ldots$ iterate:

1. SOLVE+ESTIMATE REPEAT for $n = 1, 2, 3, \ldots$
   - compute $u_\ell^n := \Phi_\ell(u_\ell^{n-1})$
   - compute indicators $\eta_\ell(T, u_\ell^n)$ for all $T \in \mathcal{T}_\ell$

   UNTIL $\|u_\ell^n - u_\ell^{n-1}\| \leq \lambda\, \eta_\ell(u_\ell^n)$
2. MARK find (minimal) set $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ s.t.
$$\theta \sum_{T \in \mathcal{T}_\ell} \eta_\ell(T, u_\ell^n)^2 \leq \sum_{T \in \mathcal{M}_\ell} \eta_\ell(T, u_\ell^n)^2$$
3. REFINE refine (at least) all $T \in \mathcal{M}_\ell$ to obtain $\mathcal{T}_{\ell+1}$
4. define $\underline{n} := \underline{n}(\ell) := n$ and $u_{\ell+1}^0 := u_\ell^n$ (nested iteration)

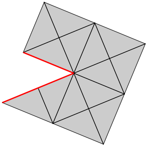
Experiments

- mixed BVP

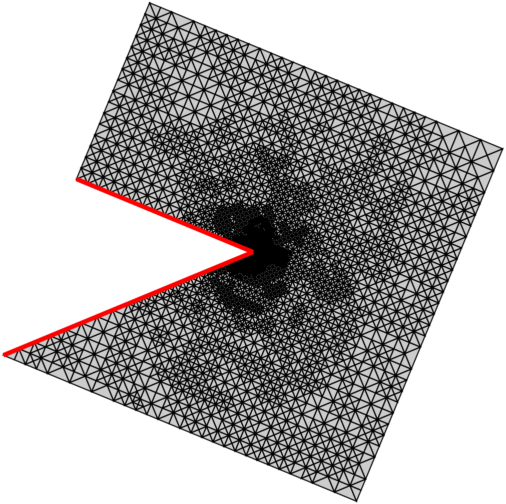
$$\begin{aligned} -\mathrm{div}\,\big(\mu(|\nabla u^\star|)\,\nabla u^\star\big) &= f \quad \text{in } \Omega \\ \mu(|\nabla u^\star|)\,\nabla u^\star \cdot \boldsymbol{n} &= g \quad \text{on } \Gamma_N \\ u^\star &= 0 \quad \text{on } \Gamma_D \end{aligned}$$

- $\mu(t) := 2 + \dfrac{1}{1+t} \quad \Longrightarrow \quad \alpha = 2, \quad L = 3 \quad \text{w.r.t.} \quad |\!|\!| \cdot |\!|\!| = \|\nabla(\cdot)\|_{L^2(\Omega)}$

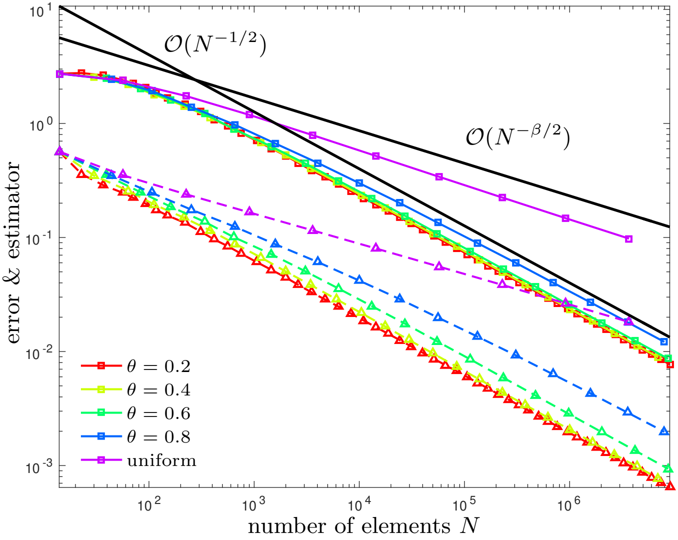
- prescribed singular solution $u(x) = r^\beta\ \cos(\beta\varphi)$
- $\eta_\ell(T, u_\ell) = h_T^2\,\|f + \mathrm{div}\,\big(\mu(|\nabla u_\ell|)\,\nabla u_\ell\big)\|_{L^2(T)}^2$
  $+ h_T\,\|g - \mu(|\nabla u_\ell|)\,\nabla u_\ell \cdot \boldsymbol{n}\|_{L^2(\partial T\cap\Gamma_N)}^2$
  $+ h_T\,\|\,[\,\mu(|\nabla u_\ell|)\,\nabla u_\ell \cdot \boldsymbol{n}\,]\,\|_{L^2(\partial T\cap\Omega)}^2$



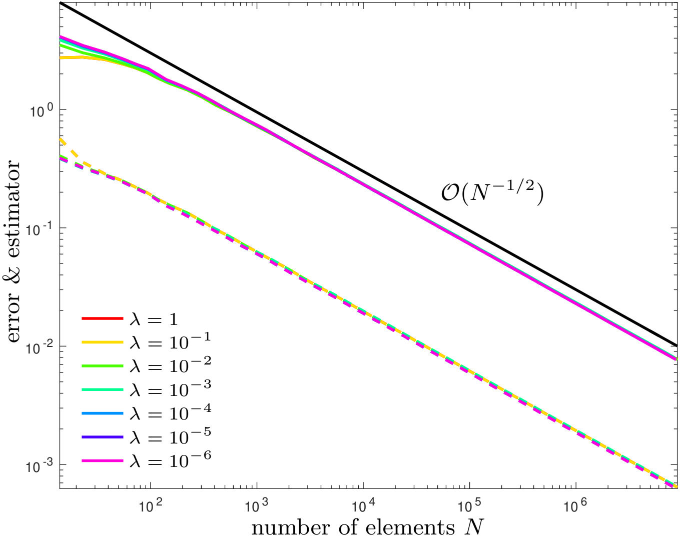
Adaptive mesh $\#\mathcal{T}_{20} = 7.657$



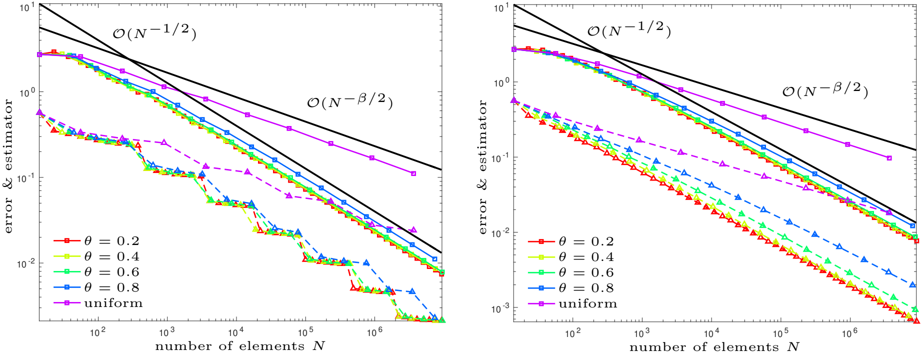
Dependence on θ for $\lambda = 10^{-2}$



Dependence on λ for $\theta = 0.2$

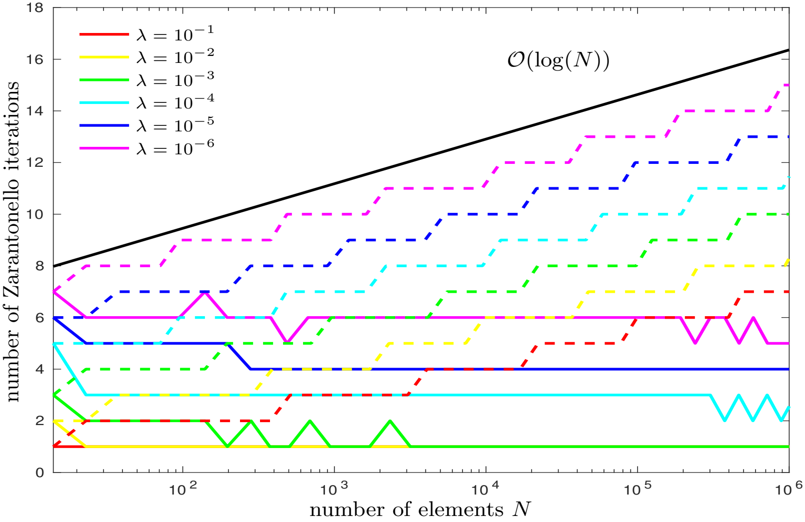


$u_\ell^0 := 0$ vs. nested iteration 1/2



- $\lambda = 10^{-2}$
- $u_0^0 := 0$ (left) vs. nested iteration (right)

$u_\ell^0 := 0$ vs. Nested iteration 2/2



- known: $\underline{n}(\ell) = \mathcal{O}[\log(\#\mathcal{T}_\ell)]$ vs. $\underline{n}(\ell) = \mathcal{O}[1 + \log(1 + \eta_{\ell-1}/\eta_\ell)]$

Convergence

Axioms of Adaptivity: (A1) stability & (A2) reduction

(A1) stability on non-refined elements

- $\exists C_{\rm stab} > 0 \quad \forall \mathcal{T}_H \quad \forall \mathcal{T}_h \in \texttt{refine}(\mathcal{T}_H) \quad \forall v_H \in \mathcal{X}_H \quad \forall v_h \in \mathcal{X}_h$

$$\Big| \Big(\sum_{T \in \mathcal{T}_H \cap \mathcal{T}_h} \eta_h(T, v_h)^2 \Big)^{1/2} - \Big(\sum_{T \in \mathcal{T}_H \cap \mathcal{T}_h} \eta_H(T, v_H)^2 \Big)^{1/2} \Big| \le C_{\rm stab} \, \| v_h - v_H \|$$

(A2) reduction on refined elements

- $\exists 0 < q_{\rm red} < 1 \quad \forall \mathcal{T}_H \quad \forall \mathcal{T}_h \in \texttt{refine}(\mathcal{T}_H) \quad \forall v_H \in \mathcal{X}_H \subseteq \mathcal{X}_h$

$$\Big(\sum_{T \in \mathcal{T}_h \backslash \mathcal{T}_H} \eta_h(T, v_H)^2 \Big)^{1/2} \le q_{\rm red} \Big(\sum_{T \in \mathcal{T}_H \backslash \mathcal{T}_h} \eta_H(T, v_H)^2 \Big)^{1/2}$$

Diening, Kreuzer: SIAM J. Numer. Anal. 46-2 (2008)

Cascon, Kreuzer, Nochetto, Siebert: SIAM J. Numer. Anal. 46-5 (2008)

Garau, Morin, Zuppa: Numer. Math. Theory Methods Appl. 5 (2012)

Axioms of Adaptivity: (A3) discrete reliability

(A3) discrete reliability

- $\exists C_{\rm rel} > 0 \quad \forall \mathcal{T}_H \quad \forall \mathcal{T}_h \in \texttt{refine}(\mathcal{T}_H)$

$$\|u_h^\star - u_H^\star\| \le C_{\rm rel}\Big(\sum_{T \in \mathcal{T}_H \setminus \mathcal{T}_h} \eta_H(T, u_H^\star)^2\Big)^{1/2}$$

- discrete reliability for (non-computable) discrete solution $u_H^\star$
- discrete reliability $\Longrightarrow$ reliability
- stability (A1) + reliability (A3) + contraction

$\Longrightarrow \quad \|u^\star - u_\ell^n\| + \eta_\ell(u_\ell^n) \lesssim \eta_\ell(u_\ell^n) + \|u_\ell^n - u_\ell^{n-1}\|.$

Stevenson: Found. Comput. Math. 7 (2007)

Garau, Morin, Zuppa: Numer. Math. Theory Methods Appl. 5 (2012)

Convergence 1/2: What if mesh is never refined?

Proposition (Gantner, Haberl, P., Stiftner '18)

- given $\ell \in \mathbb{N}_0$, $\lambda > 0$
- suppose that REPEAT does not terminate

$\Longrightarrow$ $u^\star = u_\ell^\star$ and $\eta_\ell(u^\star) = 0$,

$$|||u^\star - u_\ell^n||| + \eta_\ell(u_\ell^n) \lesssim q^n \quad \text{for all } n \geq 0$$

- $\eta_\ell(u_\ell^n) + |||u_\ell^n - u_\ell^{n-1}||| \lesssim |||u_\ell^n - u_\ell^{n-1}||| \lesssim |||u_\ell^\star - u_\ell^{n-1}||| \lesssim q^n$
- $|||u^\star - u_\ell^\star||| \lesssim \eta_\ell(u_\ell^\star) = \lim_{n\to\infty} \eta_\ell(u_\ell^n) = 0$

Gantner, Haberl, Praetorius, Stiftner: IMA J. Numer. Anal. 38 (2018)

Convergence 2/2: $|||u - u_\ell^n||| \lesssim \eta_\ell(u_\ell^n) \simeq \eta_\ell(u_\ell^\star) \xrightarrow{\ell\to\infty} 0$

- $0 < \lambda_\star := \Big(C_{\rm stab}\,\dfrac{q}{1-q}\Big)^{-1} \qquad 0 < \theta \le 1, \qquad 0 < \lambda < \lambda_\star\theta$

Equivalence

$\Longrightarrow \quad (1-\lambda/\lambda_\star)\,\eta_\ell(u_\ell^n) \le \eta_\ell(u_\ell^\star) \le (1+\lambda/\lambda_\star)\,\eta_\ell(u_\ell^n)$

- $0 < \theta' := \dfrac{\theta - \lambda/\lambda_\star}{1+\lambda/\lambda_\star} < \theta \le 1$

Dörfler for $\eta_\ell(u_\ell^\star)$

$$\Longrightarrow \quad \theta\eta_\ell(u_\ell^n)^2 \le \sum_{T\in\mathcal{M}_\ell} \eta_\ell(T,u_\ell^n)^2 \quad \Longrightarrow \quad \theta'\,\eta_\ell(u_\ell^\star)^2 \le \sum_{T\in\mathcal{M}_\ell} \eta_\ell(T,u_\ell^\star)^2$$

$$\Longrightarrow \quad \eta_{\ell+1}(u_{\ell+1}^\star)^2 \le q\,\eta_\ell(u_\ell^\star)^2 + C\,|||u_{\ell+1}^\star - u_\ell^\star|||^2$$

Gantner, Haberl, Praetorius, Stiftner: IMA J. Numer. Anal. 38 (2018)

Main results

Axioms of Adaptivity: (A4) quasi-orthogonality

(O3) $\boldsymbol{A}$ has potential $\mathcal{J} : \mathcal{H} \to \mathbb{K}$ $\qquad \langle \boldsymbol{A}u\,,v\rangle = \lim_{r\to 0} \dfrac{\mathcal{J}(u+rv) - \mathcal{J}(u)}{r}$

Lemma

- $\mathcal{X}_h \subseteq \mathcal{H}$ closed subspace
- $v_h \in \mathcal{X}_h$

$$\Longrightarrow \quad \frac{\alpha}{2}\,|||v_h - u_h^\star|||^2 \le \mathcal{J}(v_h) - \mathcal{J}(u_h^\star) \le \frac{L}{2}\,|||v_h - u_h^\star|||^2$$

- i.e., Galerkin equation equivalent to minimization of $\mathcal{J}$

$$\Longrightarrow \sum_{k=\ell}^{N} |||u_{k+1}^\star - u_k^\star|||^2 \simeq \mathcal{J}(u_\ell^\star) - \mathcal{J}(u_{N+1}^\star) \le \mathcal{J}(u_\ell^\star) - \mathcal{J}(u^\star) \simeq |||u^\star - u_\ell^\star|||^2$$

Zeidler: Nonlinear functional analysis and its applications, part II/B

Linear convergence (first)

Theorem (Gantner, Haberl, P., Stiftner '18)

- $0 < \theta \le 1, \quad 0 < \lambda < \lambda_\star \theta$
- suppose that REPEAT terminates for all $\ell \in \mathbb{N}$

$\Longrightarrow \quad \eta_{\ell+k}(u^n_{\ell+k}) \lesssim q^k \, \eta_\ell(u^n_\ell) \quad$ for all $\ell, k \ge 0$

- $\Delta^\star_{\ell+1} \le q \, \Delta^\star_\ell \quad$ with $\quad (\Delta^\star_h)^2 := \mathcal{J}(u^\star_h) - \mathcal{J}(u^\star) + \gamma \, \eta_h(u^\star_h)^2$
- note: $\Delta^\star_h \simeq \eta_h(u^\star_h) \simeq \eta_h(u^n_h) \simeq \|u^\star - u^n_h\| + \eta_h(u^n_h) := \Lambda^n_h$

Gantner, Haberl, Praetorius, Stiftner: IMA J. Numer. Anal. 38 (2018)

Key observation: Full linear convergence

- $\mathcal{Q} := \{(\ell, n) \in \mathbb{N}_0 \times \mathbb{N}_0 \,:\, (\ell, n) \text{ used in algorithm}\}$
- $|(\ell, n)| \quad \rightsquigarrow \quad$ number of overall solver steps until u_ℓ^n

Theorem (Gantner, Haberl, P., Schimanko '19++)

- $0 < \theta \leq 1, \quad 0 < \lambda < \lambda_\star \theta$
- not needed that `REPEAT` terminates
- $\Lambda_\ell^n := |\!|\!| u^\star - u_\ell^n |\!|\!| + \eta_\ell(u_\ell^n) \quad$ for all $\quad (\ell, n) \in \mathcal{Q}$

$\Longrightarrow \quad \Lambda_{\ell'}^{n'} \lesssim q^{|(\ell',n')| - |(\ell,n)|} \, \Lambda_\ell^n \quad$ for all $\quad (\ell', n') > (\ell, n)$

- proof: exploits stopping criterion + contraction
- linear convergence $\iff \displaystyle\sum_{(\ell',n')>(\ell,n)} \Lambda_{\ell'}^{n'} \lesssim \Lambda_\ell^n$ for all $(\ell', n') \in \mathcal{Q}$
- note: PCG allows even arbitrary $\lambda > 0$

Führer, Haberl, Praetorius, Schimanko: Numer. Math., 141 (2019)

Optimal convergence rates

Theorem (Gantner, Haberl, P., Stiftner '18 / & Führer '19)

- $0 < \theta < 1, \quad 0 < \lambda < \lambda_\star \theta$
- $\theta'' := \dfrac{\theta + \lambda/\lambda_\star}{1 - \lambda/\lambda_\star} < \theta_\star := (1 + C_{\mathrm{stab}}^2 C_{\mathrm{rel}}^2)^{-1}$
- use NVB for mesh-refinement
- $s > 0$ arbitrary

$$\Longrightarrow \quad \sup_{(\ell,n)\in\mathcal{Q}} (\#\mathcal{T}_\ell)^s \Lambda_\ell^n \simeq \sup_{(\ell,0)\in\mathcal{Q}} (\#\mathcal{T}_\ell)^s \Lambda_\ell^\star \overset{\checkmark}{\simeq} \underbrace{\sup_{N>0} \big(N^s \min_{\mathcal{T}_{\mathrm{opt}}\in\mathbb{T}_N} \Lambda_{\mathrm{opt}}^\star\big)}_{=:\, \|u^\star\|_{\mathbb{A}_s}}$$

- simple observation: $\Lambda_\ell^n \lesssim \Lambda_\ell^0 \lesssim \Lambda_{\ell-1}^n \simeq \Lambda_{\ell-1}^\star$

Gantner, Haberl, Praetorius, Stiftner: IMA J. Numer. Anal. 38 (2018)

Führer, Haberl, Praetorius, Schimanko: Numer. Math., 141 (2019)

Quasi-optimal computational costs

new: Theorem (Gantner, Haberl, P., Schimanko '19++)

- same assumptions as for optimal rates
- $s > 0$ arbitrary

$$\Longrightarrow \quad \sup_{(\ell,n)\in\mathcal{Q}} \Big(\sum_{(\ell',n')\le(\ell,n)} \#\mathcal{T}_{\ell'} \Big)^s \Lambda_\ell^n \simeq \sup_{(\ell,n)\in\mathcal{Q}} (\#\mathcal{T}_\ell)^s \Lambda_\ell^n \overset{\checkmark}{\simeq} \|u^\star\|_{\mathbb{A}_s}$$

- suppose: linear costs $\mathcal{O}(\#\mathcal{T})$ for
  - SOLVE: linear systems (e.g., one step of Zarantonello iteration)
  - ESTIMATE: refinement indicators (for one particular discrete function)
  - MARK: marking elements for refinement
  - REFINE: generate locally refined mesh

$\Longrightarrow \mathcal{O}\Big(\sum_{(\ell',n')\le(\ell,n)} \#\mathcal{T}_{\ell'} \Big)$ effort to compute Λ_ℓ^n ($\Longleftrightarrow$ comp. time)

Führer, Haberl, Praetorius, Schimanko: Numer. Math., 141 (2019)

Conclusions

Conclusions

- *Axioms of Adaptivity* for strongly monotone model problem
  - lowest-order FEM
  - axioms proved for scalar nonlinearity $\mathcal{A}(\nabla u) = \mu(|\nabla u|)\,\nabla u$
  - with simple Zarantonello iterative solver
- done: linear convergence with optimal algebraic convergence rates
  - new: quasi-optimal computational costs
  - key: full linear convergence
- current work: include PCG solver into adaptive algorithm
- future work: other estimators / Newton solvers / stopping criteria

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Garau, Morin, Zuppa: Numer. Math. Theory Methods Appl. 5 (2012)

Ern, Vohralik: SIAM J. Sci. Comput. 35 (2013)

Congreve, Wihler: J. Comput. Appl. Math. 311 (2017)

CMAM 2020 – SAVE THE DATE!

CMAM 2020

July 13–17, 2020

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The conference is organized under the aegis of the journal Computational Methods in Applied Mathematics (CMAM) and will be focused on various aspects of mathematical modeling and numerical analysis. Its scope coincides with the scope of the journal: It aims at fostering cooperation between researchers working in the area of theoretical numerical analysis and applications to modeling, simulation, and scientific computing

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Philipp Grohs (University of Vienna)
Barbara Kaltenbacher (University of Klagenfurt)
Dalibor Lukas (Technical University of Ostrava)
Andreas Veeser (University of Milan)
Thomas Wihler (University of Bern)

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Conference website

https://www.asc.tuwien.ac.at/cmam2020/

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- http://www.asc.tuwien.ac.at/cmam2020

Thanks for listening!

Gregor Gantner, Alexander Haberl, Dirk Praetorius, Bernhard Stiftner:
Rate optimal adaptive FEM with inexact solver for nonlinear operators,
IMA Journal on Numerical Analysis, 38 (2018). open access

Thomas Führer, Alexander Haberl, Dirk Praetorius, Stefan Schimanko:
Adaptive BEM with inexact PCG solver yields almost optimal computational costs,
Numerische Mathematik, 141 (2019). open access

Gregor Gantner, Alexander Haberl, Dirk Praetorius, Stefan Schimanko:
Rate optimality of AFEM with respect to overall computational costs,
work in progress 2019.