DETERMINATION OF MATERIAL ELASTICITY BY ULTRASONIC WAVES

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OVERVIEW AND LITERATURE

Quasi-static mechanical testing is the most common experimental technique to determine elastic stiffness of materials. Problems arise in case of anisotropic materials, with small specimens and with porous materials, where the determination of material stiffness can be strongly biased by inelastic deformations occurring in the material samples.

In order to determine all elastic stiffness tensor components, several specimens need to be manufactured, and even then the measurement of Poisson's ratios is a highly delicate task. Ultrasonic wave propagation allows for the direct measurement of all elastic tensor components on one specimen by applying only negligibly small stresses to the material.

[1] **Zaoui, A.:** Continuum micromechanics: Survey. *Journal of Engineering Mechanics*, 128(8), 808-816, 2002.

[2] **Helbig, K.:** *Foundations of Anisotropy for Exploration Seismics*. Handbook of Geophysical Exploration, 22, Pergamon, Elsevier Science Ltd., Oxford, United Kingdom, 1994.

[3] **Carcione, J.M.:** *Wave fields in real media: wave propagation in anisotropic, anelastic and porous media*. Handbook of Geophysical Exploration, 31, Pergamon, Elsevier Science Ltd., Oxford, United Kingdom, 2001.

[4] **Kolsky, H.:** *Stress Waves in Solids*. Oxford University Press, London, United Kingdom, 1953.

HOW TO DEFINE A MATERIAL?

In continuum (micro)mechanics [1], elastic properties are related to a material volume (also called representative volume element RVE), which must be considerably larger than the inhomogeneities inside this material volume, and it must be subjected to homogeneous stress and strain states. Hence, the characteristic length of the RVE needs to be much smaller than the scale of the characteristic loading of the medium, i.e. the wavelength.

REQUIREMENT FOR

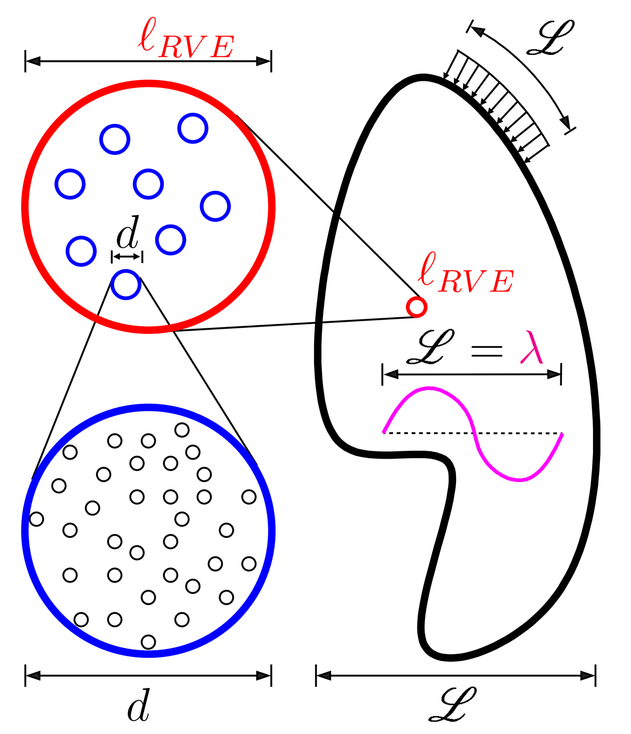
DEFINITION OF MATERIAL PROPERTIES: $d \ll \ell_{RVE}$

APPLICATION OF DIFFERENTIAL CALCULUS: $\ell_{RVE} \ll \mathscr{L}$

SEPARATION OF SCALES: $d \ll \ell_{RVE} \ll \mathscr{L}$

LOAD = WAVE

WAVELENGTH FOR MATERIAL CHARACTERIZATION: $\lambda \gg \ell_{RVE} \gg d$



d ... inhomogeneity [mm]

ℓ_{RVE} ... characteristic length of RVE [mm]

$\mathscr{L}$ ... characteristic length of structure or load [mm]

λ ... wavelength [mm]

HOW ARE WAVES AND STIFFNESS RELATED?

The ratio of wavelength to the characteristic length a [mm] of the sample surface where the transducer is applied determines whether a quasi-infinite medium (i.e. ultrasonic beam is laterally constrained) or a finite medium (i.e. beam propagates in 1-D media) is characterized [2,3,4].

UNBOUNDED INFINITE ELASTIC MEDIUM

$a \gg \ell_{RVE}$ OR $a > 2\lambda$

EQUATION OF MOTION: $\partial_j \sigma_{ij} = \rho\, \partial^2_{tt} u_i$

PLANE WAVE EQUATION: $u_i(x_i, t) = u_0\, p_i \exp(i\,(k_j\, x_j - \omega\, t))$

DEFINITION OF PHASE VELOCITY: $v_p = \omega / k = \lambda\, f$

GENERALIZED HOOKE'S LAW: $\sigma_{ij} = \mathbb{C}_{ijkl}\, \varepsilon_{kl}$

LINEARIZED STRAIN TENSOR: $\varepsilon_{ij} = (\partial_j u_i + \partial_i u_j)/2$

WAVEVECTOR: $k_j = k \cdot n_j = 2\,\pi / \lambda \cdot n_j$

KELVIN-CHRISTOFFEL EQUATION: $(\Gamma_{ik} - \rho\, v_p^2\, \delta_{ik})\, p_k = 0$

KELVIN-CHRISTOFFEL MATRIX: $\Gamma_{ik} = \mathbb{C}_{ijkl}\, n_j\, n_l$

ISOTROPIC STIFFNESS TENSOR COMPONENTS: $C_{1111} = \rho\, v_L^2$ $\quad C_{1212} = \rho\, v_T^2$

3 EIGENVALUES, 3 EIGENVECTORS: choose $\mathbb{C}_{ijkl}, n_i \Rightarrow (\rho\, v_p^2)^n, (p_i)^n$

LAME'S CONSTANTS: $\lambda = \rho\,(v_L^2 - 2\,v_T^2)$ $\quad \mu = G = \rho\, v_T^2$

ISOTROPIC STIFFNESS TENSOR: $\mathbb{C}_{ijkl} = \lambda\, \delta_{ij}\, \delta_{kl} + \mu\,(\delta_{ik}\, \delta_{jl} + \delta_{il}\, \delta_{jk})$

YOUNG'S MODULUS: $E = \rho\, v_T^2\, \dfrac{3\,v_L^2 - 4\,v_T^2}{v_L^2 - v_T^2}$

POISSON'S RATIO: $\nu = \dfrac{v_L^2/2 - v_T^2}{v_L^2 - v_T^2}$

DIRECTION OF

n_i ... propagation, wavefront normal

p_i ... polarization, particle vibration

BOUNDED FINITE ELASTIC MEDIUM (E.G. BAR)

$a \ll \lambda$

YOUNG'S MODULUS: $E = \rho\, v_E^2$

1-D EQUATION: $(E - \rho\, v_p^2)\, u_x = 0$

HOOKE'S LAW: $\sigma_{xx} = E\, \varepsilon_{xx}$

STRAIN: $\varepsilon_{xx} = \partial_x u_x$

EQUATION OF MOTION: $\partial_x \sigma_{xx} = \rho\, \partial^2_{tt} u_x$

1-D WAVE EQUATION: $u_x(x, t) = a \exp(i\, k\,(x - v_p\, t))$

VELOCITY OF A [KM/S]

v_L ... longitudinal bulk wave

v_T ... transversal bulk wave

v_E ... extensional wave in a bar

$\mathbb{C}_{ijkl}$ ... stiffness tensor [GPa]

σ_{ij} ... stress tensor [GPa]

u_i ... deformation vector [mm]

u_0 ... amplitude [mm] (max. deformation)

k ... wavenumber [1/mm]

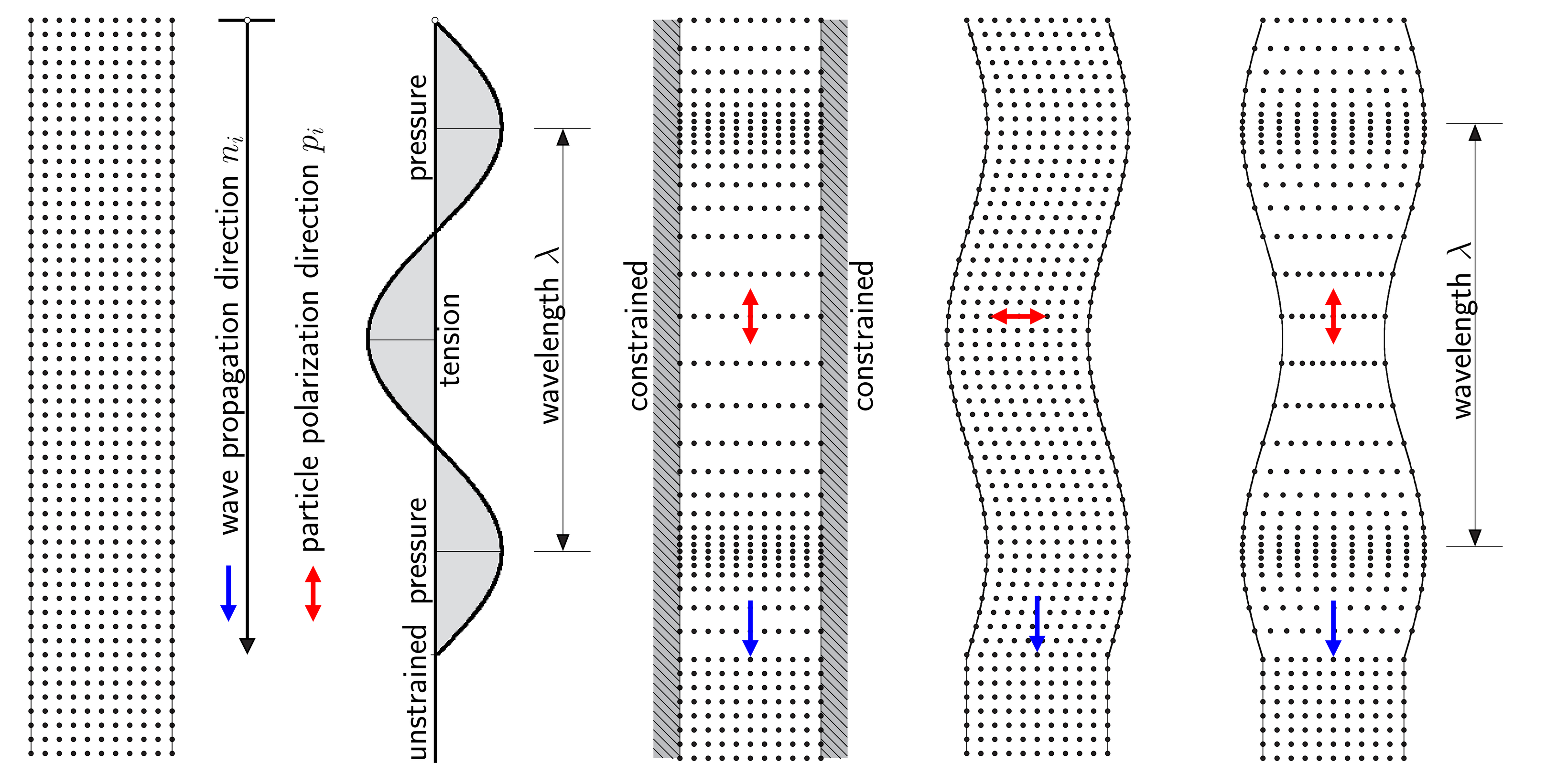
f ... frequency [MHz]

t ... time [μs]

δ_{ij} ... Kronecker delta

HOW DO WAVES PROPAGATE?

Ultrasonic waves propagate in any solid and are the result of the transfer of a disturbance from one particle (i.e. material volume) to its neighbors. The corresponding strain rate related to zthese material volumes is sufficiently low as to be considered as quasi-static and the resulting stresses are small enough such that linear elasticity is valid.



In non-symmetry planes *quasi*-longitudinal (QL) and *quasi*-transversal (QT) waves propagate.

QL- AND QT-WAVES IF: $p_i\, n_i \neq 1$ $\quad p_i\, n_i \neq 0$

longitudinal (L) or compression or dilatational wave with velocity v_L, $p_i\, n_i = 1$

transversal (T) or shear or equivoluminal wave with velocity v_T, $p_i\, n_i = 0$

extensional (E) or bar wave with velocity v_E, $p_i\, n_i = 1$

RELATIONSHIPS BETWEEN PHASE VELOCITIES

$$\frac{v_L^2}{v_E^2} = \frac{1 - \nu}{(1 + \nu)\,(1 - 2\,\nu)} = \frac{C_{1111}}{E} \qquad v_T^2 = \frac{E}{\rho}\,\frac{1}{2\,(1 + \nu)} = \frac{C_{1212}}{\rho} \qquad v_E^2 = \frac{E}{\rho}$$

The velocity of the ultrasonic puls, i.e. the group velocity (=velocity of the wave packet), is measured. This velocity is only equal to the phase velocity in isotropic materials and in symmetry planes of anisotropic materials.

HOW ARE ULTRASONIC WAVES GENERATED?

PULSER: Emits electrical square-pulse (100 - 400 Volt). Sets zero trigger for oscilloscope.

OSCILLOSCOPE: Displays received signal (bandwidth 600 MHz, 10 Gigasamples/s). Access to time of flight t_s [μs].

GROUP VELOCITY: $v = \dfrac{\ell_s}{t_s}$ $\quad$ **WAVELENGTH:** $\lambda = \dfrac{v}{f}$

SENDING TRANSDUCER: Piezoelectric element transforms electrical into mechanical signal.

RECEIVER: Amplifies signal (bandwidth 0.1 - 35 MHz, voltage gain up to 59 dB).

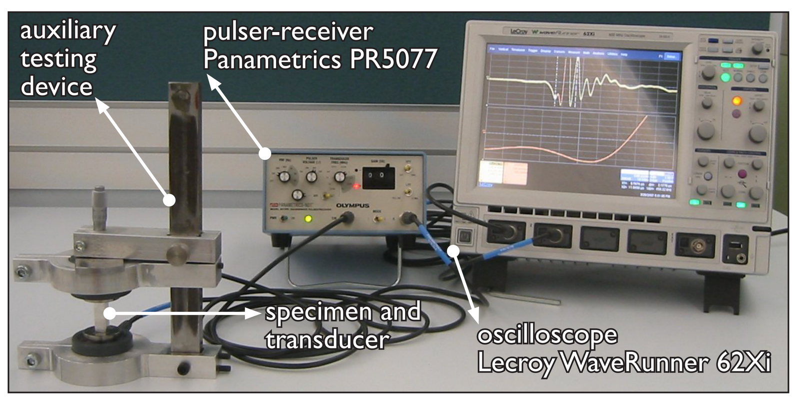
PIEZOELECTRIC ELEMENTS

- Tailored for certain frequency f [MHz] (the higher frequency, the smaller the elements)
- Depending on cut and orientation a L- or T-wave is transmitted

SPECIMEN: Defines travel distance ℓ_s [mm]. Signal is attenuated and dispersed. Coupling medium: honey.

RECEIVING TRANSDUCER: Piezoelectric element transforms mechanical into electrical signal.

TRANSMISSION THROUGH METHOD



RESULTS

7 materials with different microstructures und porosities (pore sizes range from 20 to 500 μm) were tested. Depending on the geometry of the specimen, bulk wave or extensional wave propagation occured. In the case of bulk wave propagation, pulses with equal wavelength, i.e. different frequencies for longitudinal and transversal waves, were combined.

ρ ... apparent mass density [g/cm^3]

ρ_s ... mass density of solid phase [g/cm^3]

POROSITY: $\Phi = 100\, \dfrac{\rho_s - \rho}{\rho_s}$