DETERMINATION OF ORTHOTROPIC ELASTIC STIFFNESS OF WOOD BY ULTRASONIC WAVES

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OVERVIEW AND LITERATURE

Quasi-static mechanical testing is the most common experimental technique to determine elastic stiffness of materials. Problems arise in case of anisotropic materials, with small specimens, and with porous materials, where the determination of material stiffness can be strongly biased by inelastic deformations occurring in the material samples.

Wood is modelled as an elastic, anisotropic natural composite material with orthorombic symmetry, where the symmetry planes are defined by the 3 principal material directions - longitudinal (l), transversal (t), radial (r). Ultrasonic wave propagation allows for the direct measurement of all orthotropic elastic stiffness tensor components on one specimen by applying only negligibly small stresses to the material. Here normal and shear stiffnesses (i.e. the diagonal terms) of spruce are reported.

[1] **Zaoui, A.:** Continuum micromechanics: Survey. *Journal of Engineering Mechanics*, 128(8), 808, 2002.
[2] **Helbig, K.:** *Foundations of Anisotropy for Exploration Seismics.* Handbook of Geophysical Exploration, 22, Pergamon, Elsevier Science Ltd., Oxford, United Kingdom, 1994.
[3] **Carcione, J.M.:** *Wave fields in real media: wave propagation in anisotropic, anelastic and porous media.* Handbook of Geophysical Exploration, 31, Pergamon, Elsevier Science Ltd., Oxford, United Kingdom, 2001.
[4] **Kolsky, H.:** *Stress Waves in Solids.* Oxford University Press, London, United Kingdom, 1953.
[5] **Hearmon, R.F.S.:** The elastic constants of anisotropic materials. *Reviews of Modern Physics,* 18(3), 409, 1946.
[6] **Bucur, V. and Archer, R.R.:** Elastic constants for wood by an ultrasonic method. *Wood Science and Technology,* 18, 255-265, 1984.

HOW TO DEFINE A MATERIAL?

In continuum (micro)mechanics [1], elastic properties are related to a material volume (also called representative volume element RVE), which must be considerably larger than the inhomogeneities inside this material volume. Measurement of stiffness properties requires homogeneous stress and strain states in the RVE, so that the characteristic length of the RVE needs to be much smaller than the scale of the characteristic loading of the medium, i.e. the wavelength.

REQUIREMENT FOR

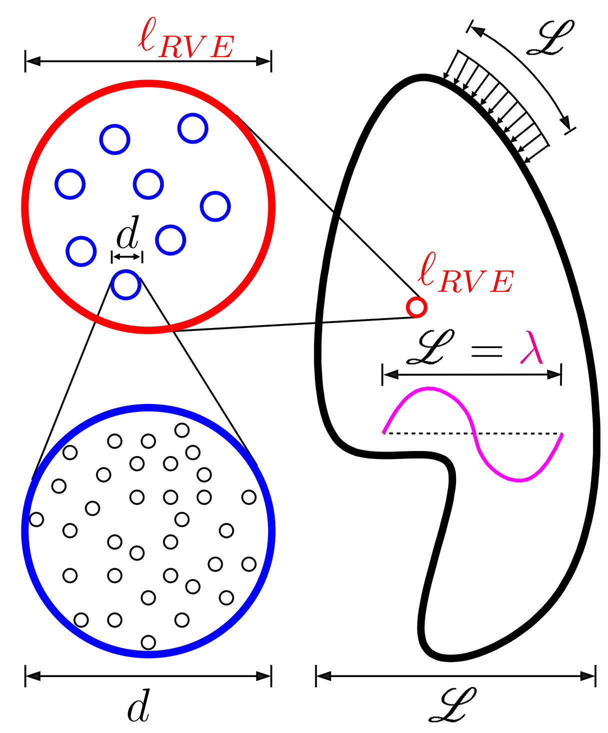
DEFINITION OF MATERIAL PROPERTIES: $d \ll \ell_{RVE}$

APPLICATION OF DIFFERENTIAL CALCULUS: $\ell_{RVE} \ll \mathscr{L}$

SEPARATION OF SCALES: $d \ll \ell_{RVE} \ll \mathscr{L}$

LOAD = WAVE

WAVELENGTH FOR MATERIAL CHARACTERIZATION: $\lambda \gg \ell_{RVE} \gg d$



d ... inhomogeneity [mm]
ℓ_{RVE} ... characteristic length of RVE [mm]
$\mathscr{L}$ ... characteristic length of structure or load [mm]
λ ... wavelength [mm]

HOW ARE WAVES AND STIFFNESS RELATED?

The ratio of wavelength to the characteristic length a [mm] of the sample surface where the transducer is applied determines whether a quasi-infinite medium (i.e. ultrasonic beam is laterally constrained) or a finite medium (i.e. beam propagates in 1-D media) is characterized [2,3,4].

UNBOUNDED INFINITE ELASTIC MEDIUM

$a \gg \ell_{RVE}$ OR $a > 2\lambda$

EQUATION OF MOTION: $\partial_j \sigma_{ij} = \rho\, \partial^2_{tt} u_i$

PLANE WAVE EQUATION: $u_i(x_i, t) = u_0\, p_i \exp(i\,(k_j\, x_j - \omega\, t))$

DEFINITION OF PHASE VELOCITY: $v_p = \omega / k = \lambda\, f$

GENERALIZED HOOKE'S LAW: $\sigma_{ij} = \mathbb{C}_{ijkl}\, \varepsilon_{kl}$

LINEARIZED STRAIN TENSOR: $\varepsilon_{ij} = (\partial_j u_i + \partial_i u_j)/2$

WAVEVECTOR: $k_j = k \cdot n_j = 2\,\pi/\lambda \cdot n_j$

KELVIN-CHRISTOFFEL EQUATION: $(\Gamma_{ik} - \rho\, v_p^2\, \delta_{ik})\, p_k = 0$

KELVIN-CHRISTOFFEL MATRIX: $\Gamma_{ik} = \mathbb{C}_{ijkl}\, n_j\, n_l$

DIAGONAL STIFFNESS TENSOR COMPONENTS: NORMAL $C_{iiii} = \rho\, v^2_{L|i}$; SHEAR $C_{ijij} = \rho\, v^2_{T|i,j}$; i, j ... symmetry directions

3 EIGENVALUES, 3 EIGENVECTORS: choose $\mathbb{C}_{ijkl}, n_i \Rightarrow (\rho\, v_p^2)^n, (p_i)^n$

ORTHOTROPIC STIFFNESS TENSOR: $\mathbb{C}_{ijkl} = f\,(3\, C_{iiii}, 3\, C_{ijij}, 3\, C_{iijj})$

INVERSION OF ORTHOTROPIC STIFFNESS TENSOR: $\mathbb{C}^{-1}_{ijkl} = \mathbb{D}_{ijkl}$

DIRECTION OF: n_i ... propagation, wavefront normal; p_i ... polarization, particle vibration

9 INDEPENDENT ENGINEERING CONSTANTS: $3\, E_i, 3\, G_{ij}, 3\, \nu_{ij}$

VELOCITY OF A [KM/S]:
$v_{L|i}$ ... longitudinal bulk wave
$v_{T|i,j}$ ... transversal bulk wave
$v_{E|i}$ ... extensional wave in a bar

$a \ll \lambda$

YOUNG'S MODULI: $E_i = \rho\, v^2_{E|i}$

1-D EQUATION: $(E - \rho\, v_p^2)\, u_x = 0$

HOOKE'S LAW: $\sigma_{xx} = E\, \varepsilon_{xx}$

STRAIN: $\varepsilon_{xx} = \partial_x u_x$

EQUATION OF MOTION: $\partial_x \sigma_{xx} = \rho\, \partial^2_{tt} u_x$

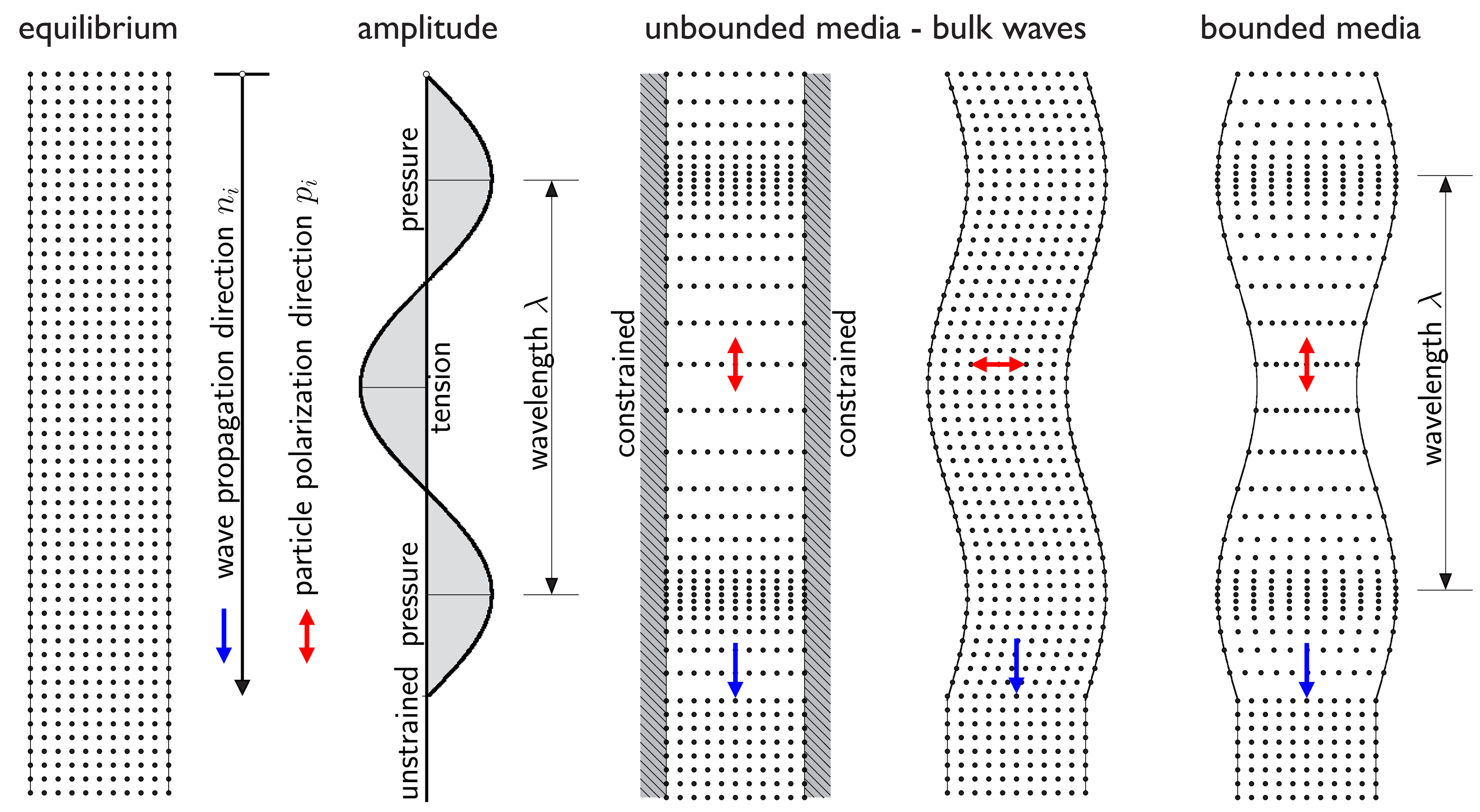
1-D WAVE EQUATION: $u_x(x, t) = a \exp(i\, k\, (x - v_p\, t))$

BOUNDED FINITE ELASTIC MEDIUM (E.G. BAR)

$\mathbb{C}_{ijkl}$ ... stiffness tensor [GPa]
$\mathbb{D}_{ijkl}$ ... compliance tensor [GPa]
σ_{ij} ... stress tensor [GPa]
G_{ij} ... shear moduli [GPa]
ν_{ij} ... Poisson's ratios [-]
u_i ... deformation vector [mm]
u_0 ... amplitude (max. def.)
k ... wavenumber [1/mm]
ω ... angular frequency [MHz]
f ... frequency [MHz]
t ... time [μ]
δ_{ij} ... Kronecker delta

HOW DO WAVES PROPAGATE?

Ultrasonic waves propagate in any solid and are the result of the transfer of a disturbance from one particle (i.e. material volume) to its neighbors. The corresponding strain rate related to these material volumes is sufficiently low as to be considered as quasi-static, and the resulting stresses are small enough such that linear elasticity is valid.



longitudinal (L) or compression or dilatational wave with velocity $v_{L|i}$; $p_i\, n_i = 1$

transversal (T) or shear or equivoluminal wave with velocity $v_{T|i,j}$; $p_i\, n_i = 0$

extensional (E) or bar wave with velocity $v_{E|i}$; $p_i\, n_i = 1$

In non-symmetry planes *quasi*-longitudinal (QL) and *quasi*-transversal (QT) waves propagate.

QL- AND QT-WAVES IF: $p_i\, n_i \neq 1$ $\quad p_i\, n_i \neq 0$

DEFINTION OF VELOCITY INDICES:
v_i ... i = propagation and polarization direction
$v_{i,j}$ ... i = propagation, j = polarization direction

SYMMETRY OF TRANSVERSAL WAVE:
$v_{i,j} \approx v_{j,i}$ ... wood: not perfect orthotropic
... up to 30% difference

The velocity of the ultrasonic puls, i.e. the group velocity (=velocity of the wave packet), is measured. This velocity is only equal to the phase velocity in isotropic materials and in symmetry planes of anisotropic materials.

HOW ARE ULTRASONIC WAVES GENERATED?

PULSER: Emits electrical square-pulse (100 - 400 Volt). Sets zero trigger for oscilloscope.

OSCILLOSCOPE: Displays received signal (bandwidth 600 MHz, 10 Gigasamples/s). Access to time of flight t_s [µs].

GROUP VELOCITY: $v = \dfrac{\ell_s}{t_s}$ **WAVELENGTH**: $\lambda = \dfrac{v}{f}$

SENDING TRANSDUCER: Piezoelectric element transforms electrical into mechanical signal.

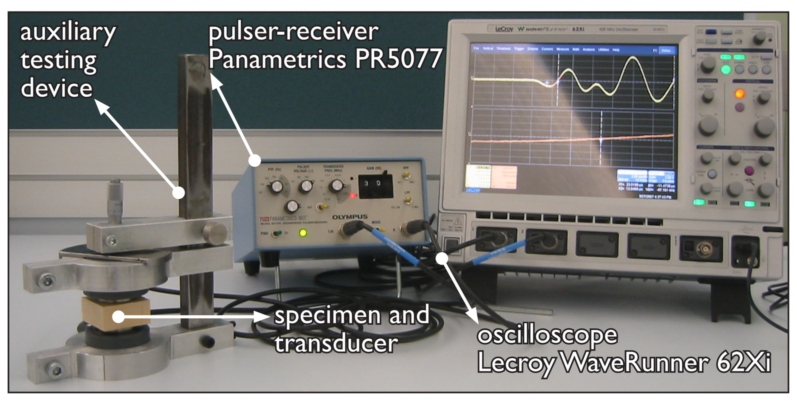
RECEIVER: Amplifies signal (bandwidth 0.1 - 35 MHz, voltage gain up to 59 dB).

PIEZOELECTRIC ELEMENTS:
- Tailored for certain frequency f [MHz] (the higher the frequency, the smaller the elements)
- Depending on cut and orientation a L- or T-wave is transmitted

SPECIMEN: Defines travel distance ℓ_s [mm]. Signal is attenuated and dispersed. Coupling medium: honey.

RECEIVING TRANSDUCER: Piezoelectric element transforms mechanical into electrical signal.

TRANSMISSION THROUGH METHOD

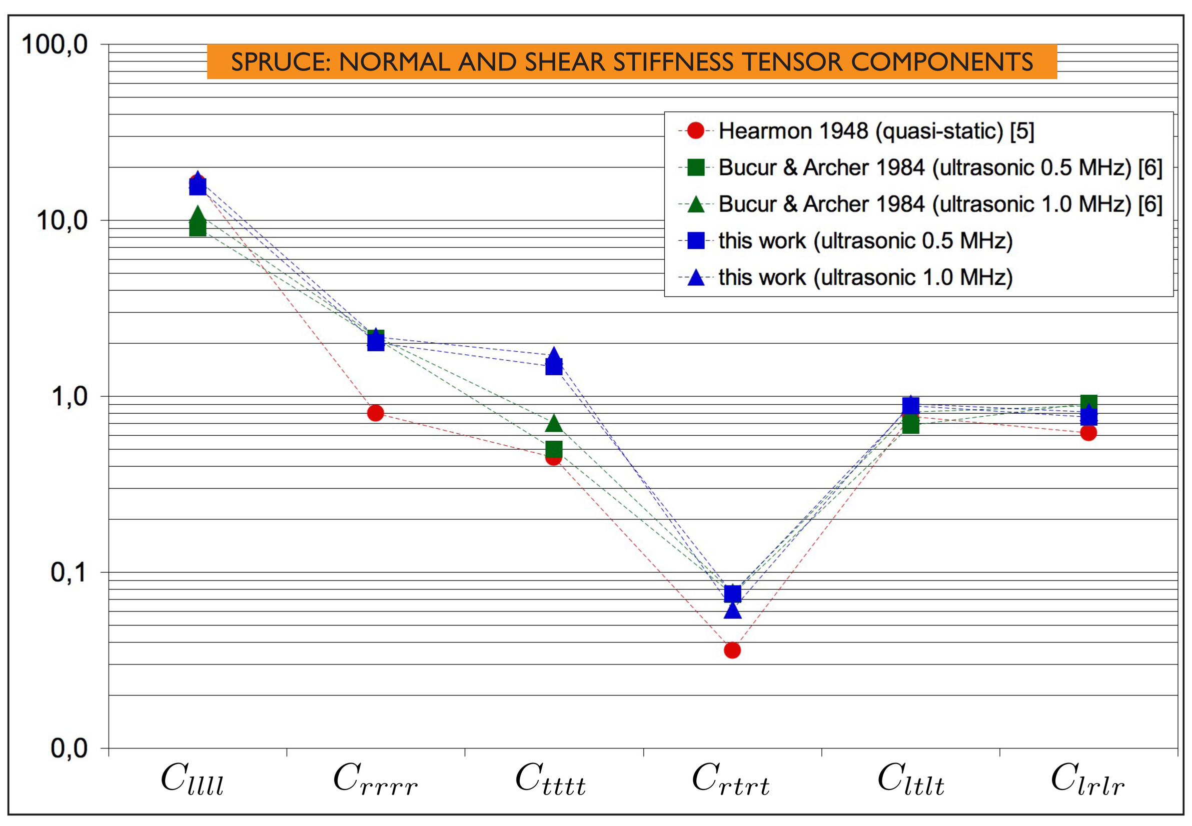


RESULTS

3 cuboid-shaped specimens were cut along the symmetry plane of the material, oriented in the longitudinal, radial, and transversal direction, respectively. Waves (0.5, 1.0 MHz) were sent through the heights of these specimens.

ρ ... apparent mass density: 0.41 – 0.44 g/cm^3

ρ_s ... mass density of solid phase: ≈ 1.4 g/cm^3 (cell wall)



POROSITY: $\Phi = 100\, \dfrac{\rho_s - \rho}{\rho_s} \approx 70\%$

INHOMOGENEITY: $d = 30\ \mu m$, avg. wood cell diameter

MATERIAL VOLUME: $\ell_{RVE} \geq 0.15$ mm ($\ll \lambda = 1 - 10$ mm)