SAMPLE GEOMETRY DEPENDENCIES IN MEASUREMENTS OF ULTRASONIC WAVE VELOCITY

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OVERVIEW AND LITERATURE

Measuring the velocity of an ultrasonic wave and deducing the material stiffness from it requires basic knowledge of wave propagation in solids.

The theory applied to describe the relation between wave propagation and material stiffness is based on the concept of a plane elastic wave propagating in an infinite medium (this wave is called bulk wave). While the former assumption of a plane wave is justified for most experimental situations, this is not true for the latter assumption, which does not hold for specimens of all sizes and forms.

In an infinite (non-dispersive) medium, the measured (bulk) velocity is independent of the frequency of the wave, and depends exclusively on the mass density and stiffness of the investigated material. However, in finite samples, the ultrasonic wave velocity depends on geometrical parameters, including the frequency-governed wavelength.

[1] **Buckingham, E.:** On physically similar systems: illustrations of the use of dimensional equations. *Physical Review*, 4, 345, 1914.

[2] **Carcione, J.M.:** *Wave fields in real media: wave propagation in anisotropic, anelastic and porous media.* Handbook of Geophysical Exploration, 31, Pergamon, Elsevier Science Ltd., Oxford, United Kingdom, 2001.

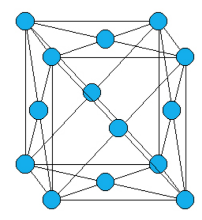
[3] **Redwood, M.:** Ultrasonic Waveguides – A physical approach. *Ultrasonics*, 1(2), 99, 1963.

MATERIAL

How is Aluminum structured?

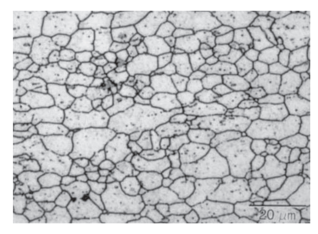
PURE ALUMINUM (SINGLE CRYSTAL)

- crystal structure: face centered cubic
- highest possible packed structure
- unit cell: lattice constant a=0.40494 nm
- cubic symmetry
- 3 elastic constants



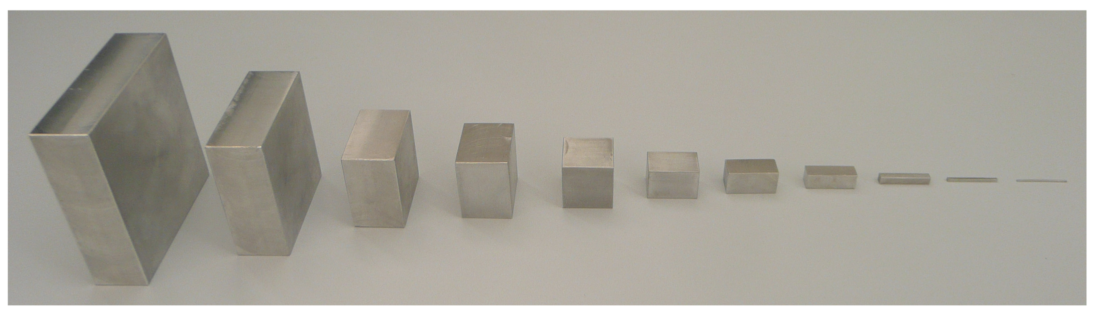
ALUMINUM ALLOY (POLYCRYSTAL)

- grain structure (small crystalls)
- isotropic orientation, distribution
- homogenization yields ⟶
- isotropic symmetry
- 2 elastic constants



Which specimens where used?

11 cuboid-shaped specimens made from aluminum alloy 5083 with a height of 30 mm and different cross-sectional dimensions (a=1, 2, 5, 10, 15, 20, 30, 40, 50, 75, 100 mm) were used.



ALUMINUM ALLOY 5083 - COMPOSITION

- highly resistant in extreme environments (sea water, chemicals)
- highest strength (due to magnesium) of non-heat treatable alloys

ρ [g/cm^3]	Al [%]	Mg [%]	Mn [%]	Si [%]	Fe [%]	Zn [%]	Ti [%]	Cr [%]	Cu [%]
2.656	balance	4.0 - 4.9	0.4 - 1.0	0.4	0.4	0.25	0.15	0.05 - 0.25	0.1

ULTRASOUND

How are ultrasonic waves generated?

PULSER
Emits electrical square-pulse (100 - 400 Volt).
Sets zero trigger for oscilloscope.

OSCILLOSCOPE
Displays received signal (bandwidth 600 MHz, 10 Gigasamples/s).
Access to time of flight t_s [μs].

GROUP VELOCITY $v = \dfrac{\ell_s}{t_s}$ **WAVELENGTH** $\lambda = \dfrac{v}{f}$

SENDING TRANSDUCER
Piezoelectric element transforms electrical into mechanical signal.

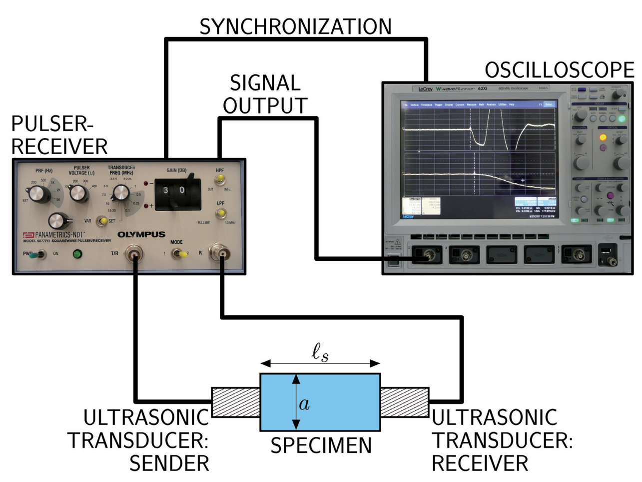
RECEIVER
Amplifies signal (bandwidth 0.1 - 35 MHz, voltage gain up to 59 dB).

PIEZOELECTRIC ELEMENTS
- Tailored for certain frequency f [MHz] (the higher the frequency, the smaller the elements)
- Depending on cut and orientation a L- or T-wave is transmitted

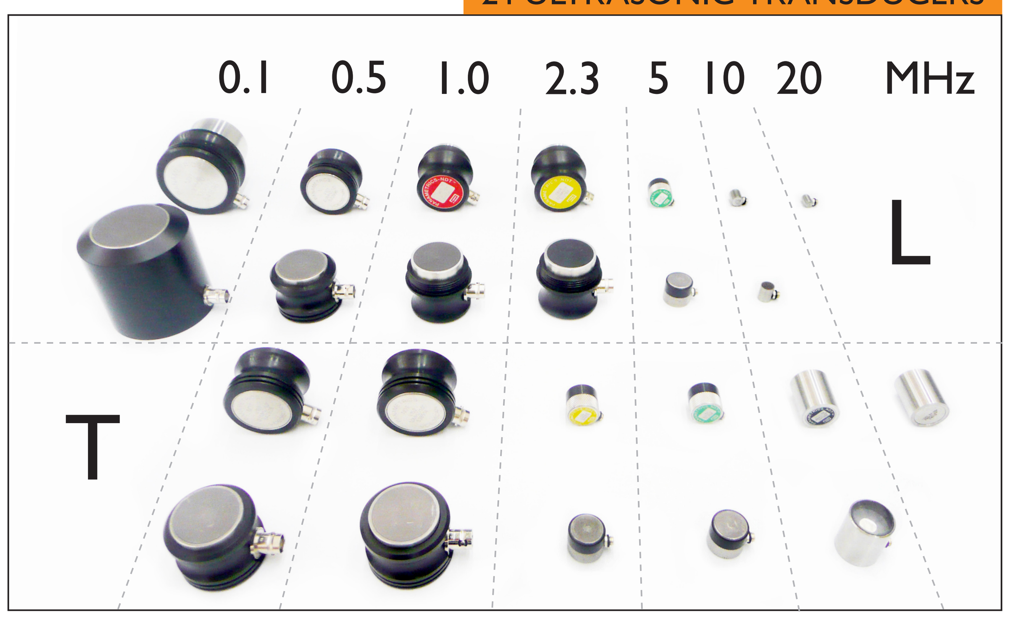
SPECIMEN
Defines travel distance ℓ_s [mm].
Signal is attenuated and dispersed.
Coupling medium: honey.

RECEIVING TRANSDUCER
Piezoelectric element transforms mechanical into electrical signal.

TRANSMISSION THROUGH METHOD



24 ULTRASONIC TRANSDUCERS



How are wave and stiffness related?

Combination of the conservation law of linear momentum, of the generalized Hooke's law, of the linearized strain tensor, and of the general plane wave solution for the displacements inside an infinite solid medium yields the elasticity tensor components as functions of the material mass density and the wave propagation velocity [2].

ISOTROPIC STIFFNESS TENSOR COMPONENTS

$$C_{1111} = \rho\, v_L^2 \qquad C_{1212} = \rho\, v_T^2$$

YOUNG'S MODULUS / POISSON'S RATIO

$$E = \rho\, v_T^2 \, \frac{3\, v_L^2 - 4\, v_T^2}{v_L^2 - v_T^2} \qquad \nu = \frac{v_L^2/2 - v_T^2}{v_L^2 - v_T^2}$$

DIMENSIONAL ANALYSIS

What is the aim of dimensional analysis?

The aim of dimensional analysis [1] is to extract relevant parameters of a given physical problem, i.e. the functional relation between one physical quantity (the dependent variable) and several other physical quantities (the independent variables), through the study of the dimensions of the all involved quantities, while making sure that the functional relationship does not depend on the units of measurements.

dependent quantity — independent quantities (N=5)

FUNCTIONAL RELATION

$$v_L = F(C_{1111}, \rho, h, a, \lambda)$$

dimensionally independent quantities (k=3) — dimensionally dependent quantities (N-k=2)

rank k=3: choose k=3 freely among N=5

N-k+1=3
$\{Q_i\} = \{a, \lambda, v_L\}$
express through k=3

$L, M, T \ldots$ abstract positiv numbers, describe transformation factors between systems of units.

SYSTEMS OF UNITS

L(ength) M(mass) T(ime)

DIMENSION OF PHYSICAL QUANITY

$$[Q_i] = L^{\alpha_i} M^{\beta_i} T^{\gamma_i}$$

divide dimensionally dep. through powers of dimensionally indep. quantities

DIMENSIONLESS FORM

$$[\Pi_i] = \frac{[Q_i]}{[C_{1111}]^{n_1^i}[\rho]^{n_2^i}[h]^{n_3^i}}$$

insert dimensions and solve equations

DIMENSIONAL EXPONENT MATRIX

	Q_i	v_L	ρ	C_{1111}	h	a	λ
L	α_i	1	-3	-1	1	1	1
M	β_i	0	1	1	0	0	0
T	γ_i	-1	0	-2	0	0	0

EXPONENTS

Q_i	a	λ	v_L
n_1^i	0	0	1/2
n_2^i	0	0	-1/2
n_3^i	1	1	0

DIMENSIONLESS FUNCTIONAL RELATION

$$\Pi_v = \mathcal{F}(\Pi_a, \Pi_\lambda) \qquad \frac{v_L}{\sqrt{C_{1111}/\rho}} = \mathcal{F}\left(\frac{a}{h}, \frac{\lambda}{h}\right)$$

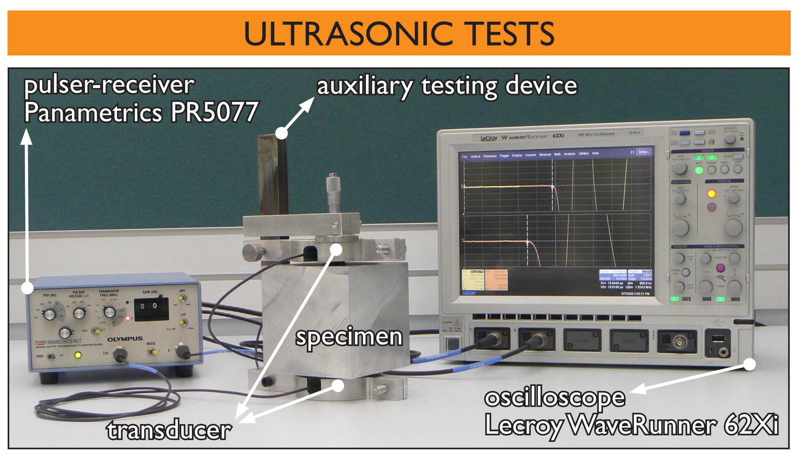
ACHIEVED REDUCTION OF NUMBER OF ARGUMENTS OF PHYSICAL PROBLEM: **5 → 2**

$v_L \ldots$ longitudinal wave velocity
$C_{1111} \ldots$ normal stiffness tensor component
$\rho \ldots$ mass density
$a \ldots$ characteristic cross-section dimension
$h \ldots$ specimen height
$\lambda \ldots$ wavelength

ELASTIC CONSTANTS

How are the elastic constants determined?

ULTRASONIC TESTS



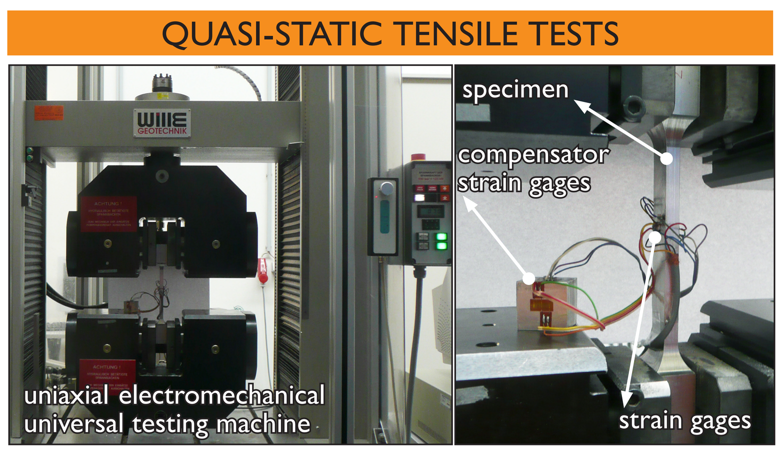
- 1 specimen: cube a=100 mm (infinite medium)
- tests at all frequencies: 0.1 - 20 MHz; L-, T-waves
- measured: density, v_L (7 values), v_T (6 values)

RELATIONSHIPS TENSOR COMP. - ENGINEERING CONSTANTS

$$\frac{C_{1111}}{E} = \frac{1-\nu}{(1+\nu)\,(1-2\,\nu)} \qquad \frac{C_{1212}}{E} = \frac{1}{2\,(1+\nu)}$$

[GPa] ([ν]=[-])	C_{1111}	C_{1212}	E	ν
mean	108.1	27.4	72.8	0.331
std. dev. [%]	0.28	0.35	-	-

QUASI-STATIC TENSILE TESTS



- 2 specimens: cross-section 30/10 mm
- load-controlled tensile tests (up to 75 MPa)
- measured: lateral and longitudinal strains

YOUNG'S MODULUS / POISSON'S RATIO

$$E = \frac{\Delta\sigma}{\Delta\varepsilon_\ell} \qquad \nu = \frac{\Delta\varepsilon_q}{\Delta\varepsilon_\ell}$$

[GPa] ([ν]=[-])	C_{1111}	C_{1212}	E	ν
specimen 1	103.2	27.6	72.7	0.318
specimen 2	102.4	27.6	72.5	0.316

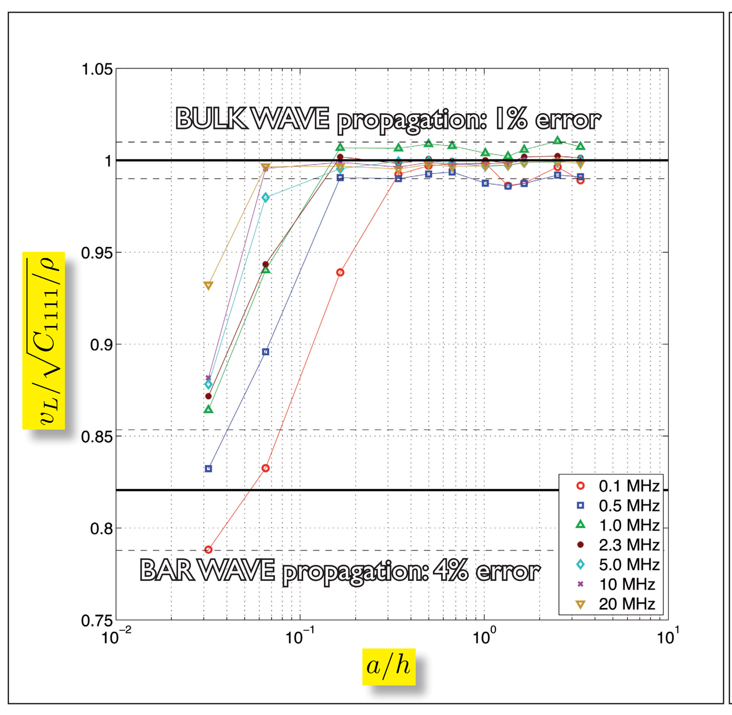
RESULTS

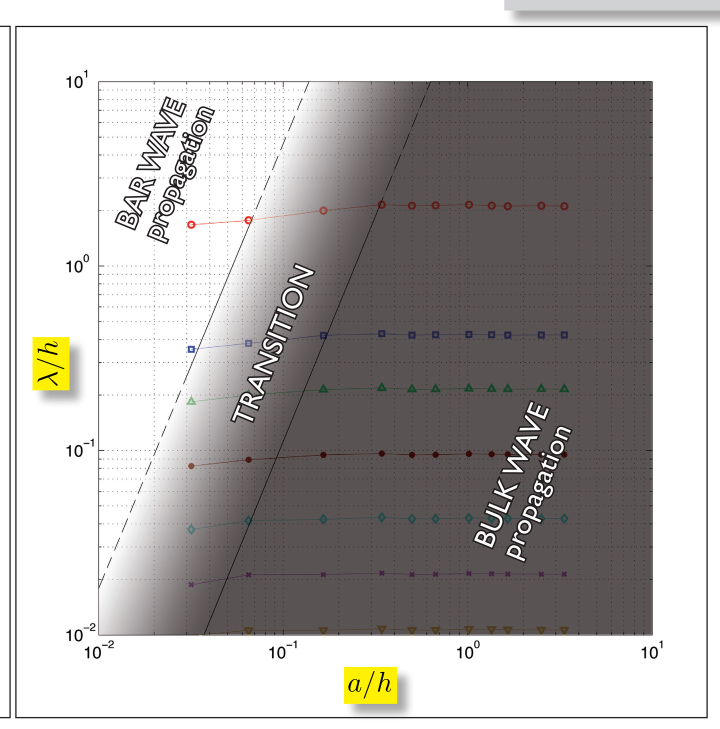
Longitudinal ultrasonic waves of 7 different frequencies (0.1-20 MHz) were sent through the aluminum specimens, leading to 77 data-points covering a range of 3 orders of magnitude of the two remaining independent variables of the dimensionless functional relation of the physical problem. In specimens with smaller cross-sectional dimension a 'Young's modulus mode' [3] occurs at lower frequencies (bar wave propagation velocity):

BAR WAVE VELOCITY

$$v_E = \sqrt{E/\rho}$$

What are the limits for bulk wave propagation?





BULK WAVE

LOW FREQUENCIES $a > h/2$

HIGH FREQUENCIES $a > h/20$

BAR WAVE

LOW FREQUENCIES $a < h/10$

HIGH FREQUENCIES $a < h/100$