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Probabilistic envelope curves for extreme rainfall events

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SUMMARY

This study extends the concept of the regional envelope curve (REC) of flood flows to extreme rainstorm events by introducing the Depth-Duration Envelope Curves (DDEC). DDEC are defined as regional upper bounds on observed rainfall maxima for several rainfall durations. The study adapts the probabilistic interpretation recently proposed for REC, which enables one to estimate the recurrence interval T of the curve, to DDEC. The study also assesses the suitability of DDEC for estimating the T-year rainfall event associated with a given duration and large T values. We illustrate an application of DDEC to annual maximum series of rainfall depth with duration spanning from 15 min to 24 h collected at 208 raingauges located in northern-central Italy. The accuracy of rainfall quantiles retrieved for ungauged sites from DDEC is assessed through a comparison with a Regional Depth-Duration-Frequency Equation that was recently proposed for the same study area.

Introduction

Producing a reliable estimate of the design storm, herein defined as the point rainfall depth for a given storm duration and probability of occurrence (or recurrence interval), is an essential task in many problems related to the definition of urban and rural planning strategies and water resources management. Also, an estimation of the frequency regime of rainfall extremes is often needed when evaluating peak river flows by using conceptual rainfall–runoff models or when deriving the flood frequency curve from the rainfall frequency curve through simplified methods (Brath and Rosso, 1993; Sivapalan et al., 2005; Merz et al., 2008). This approach is frequently used to support the design of river engineering works when considering ungauged river basins. Indirect estimations of the design flood are also frequently used when the selected recurrence interval is large or very large, as in the case of the design of major flood protection works.

The problem of estimating the design storm at ungauged locations, or at gauged sites for which the available rainfall record is significantly shorter than the recurrence interval of interest, is frequently addressed by means of regional frequency analyses of rainfall extremes by pooling together the rainfall information collected at several raingauges that are climatically similar (see e.g., Schaefer, 1990; Buishand, 1991; Faulkner, 1999; Brath et al., 2003). An alternative approach to estimating an extreme design storm for ungauged sites is to refer to the Probable Maximum Precipitation (PMP), defined as "theoretically the greatest depth of precipitation for a given duration that is physically possible over a given size storm area at a particular geographical location at a certain time of the year" (World Meteorological Organization, WMO, 1986). PMP has been extensively employed for estimating the Possible Maximum Flood (PMF), the largest flood that may occur in a given basin, to be used for designing major flood protection works. Although broadly accepted, the concept of PMP is still highly criticized (see e.g., Benson, 1973; Dooge, 1986; Dingman, 1994; Koutsoyiannis, 1999). For instance, Koutsoyiannis (1999) shows that Hershfield's statistical method for evaluating PMP (Hershfield, 1961) is based upon rainfall records that actually suggest to reject the hypothesis of existence of a physical upper limit, therefore contradicting the theoretical definition of PMP itself.

This study reconsiders the concepts of regionalization of rainstorms and definition of a statistical upper bound on the observed rainfall extremes. The main aim is to develop a graphical tool for the estimation of design storms associated with high and very high-recurrence intervals for a broad range of timescales (conventionally referred to as durations) and for gauged and ungauged locations.

In particular, the study adapts the idea of envelope curves of flood flows to extreme rainfall depths. Regional Envelope Curves (REC) summarize the current bound on our experience of extreme floods in a region. REC have continued to be constructed for many areas in the world (see e.g., conterminous Unites States - Jarvis,

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1925; Italy - Marchetti, 1955; Western Greece - Mimikou, 1984; Japan - Kadoya, 1992) and are viewed mainly as summary accounts of record floods. Castellarin et al. (2005) and Castellarin (2007) presented a probabilistic interpretation of REC which enables one to associate to the curve an estimate of the non-exceedance probability (i.e., recurrence interval). The probabilistic interpretation of the curve offers opportunities for several engineering applications which seek to exploit regional flood information to augment the effective record length associated with design flood estimates.

First we extend the probabilistic interpretation of regional envelope curves to extremes rainfall events by introducing the Depth-Duration Envelope Curve (DDEC). DDEC is defined as the regional upper bound on all observed maximum rainfall depths for a given duration. Second we adapt to DDEC the probabilistic interpretation originally proposed for RECs and assess the suitability of DDEC for estimating the design storm in ungauged sites. The paper is structured as follows: first the theoretical background of the probabilistic interpretation of RECs is briefly recalled from Castellarin et al. (2005) and Castellarin (2007); second, a definition of DDEC as a function of the local value of the Mean Annual Precipitation (MAP) is proposed and discussed in the light of the indications on regional frequency analysis of rainfall extremes reported in the literature; then, the probabilistic interpretation of RECs is extended to DDEC; finally, DDEC are constructed for a large geographical Italian region for which a rather dense network of raingauges is available and the estimates of the design storm that can be retrieved from the curves are validated against a regional model proposed by the scientific literature.

Probabilistic envelope curves for floods

Several studies (e.g., Jarvis, 1926; Marchetti, 1955; Castellarin et al., 2005) define a REC using,

$$\ln\frac{Q}{A} = a + b\cdot\ln(A) \tag{1}$$

where Q is the envelope flood for a given basin, A is drainage area (i.e., Q/A is the unit envelope flood), a and b are two regional coefficients. Castellarin et al. (2005) proposed a probabilistic interpretation of RECs constructed from groups of Annual Maximum Series (AMS) of flood flows. The interpretation adopts two fundamental assumptions: (i) the groups of AMS (sites) is homogeneous in the sense of the index-flood hypothesis (see e.g., Dalrymple, 1960); and (ii) the relationship between the index-flood μ_X (e.g., mean annual flood) and A is of the form,

$$\mu_X = C\cdot A^{b+1} \tag{2}$$

where b and C are constants and b is the same as in (1). Under these assumptions the authors developed an estimator of the exceedance probability p_{EE} of the REC and showed that under the adopted hypotheses the problem of estimating p_{EE} reduces to estimating the exceedance probability of the largest value in a regional sample of standardised annual maximum peak flows (i.e., observed peak flows divided by the mean annual flood). The primary challenge of their work involved estimation of the regional information content of cross-correlated flood series. Castellarin et al. (2005) used results introduced by Matalas and Langbein (1962) and Stedinger (1983) to quantify the regional information content using the concept of the equivalent number of independent annual maxima. The authors developed an empirical estimator of the equivalent number of independent sequences for a group of cross-correlated and concurrent annual sequences of equal length. The authors generated the sequences according to the underlying hypotheses through Monte Carlo experiments.

Castellarin (2007) relaxed the need for concurrent series, proposing an estimator of the equivalent number of independent annual observations for real-world regional datasets. For M individual AMS that globally span n years, the actual distribution of the flood series in time (e.g., missing data, different installation years for different gauges, etc.) can be taken into account as follows. First, one identifies the number of years, n_1, for which the original dataset includes only one observation of the annual maximum discharge, that is $M-1$ observations are missing (for example, some gages may not be operational, or may not be installed yet). These $n1$ observations are effective (independent) by definition. Second, the dataset containing the n-$n1$ remaining years is subdivided into $N_s \leqslant (n-n_1)$ subsets; each one of them (say subset s) is selected in such a way that all its $L_s \leqslant M$ sequences are concurrent and of equal length l_s and therefore suitable for the application of the estimator proposed by Castellarin et al. (2005). Using this splitting criterion, the effective number of observations n_{eff} can be estimated as the summation of the effective sample years of data estimated for all N_s subsets,

$$\hat{n}_{eff} = n_1 + \sum_{s=1}^{N_s}\hat{n}_{eff,s} = n_1 + \sum_{s=1}^{N_s}\frac{L_s\cdot l_s}{1+[\bar{\rho}^{\beta}]_{L_s}\cdot(L_s-1)},$$
$$\text{with } \beta = 1.4\frac{(L_s\cdot l_s)^{0.176}}{[(1-\bar{\rho})^{0.376}]_{L_s}}, \tag{3}$$

where overlines indicate average values of the corresponding functions of the correlation coefficient (i.e., $[\bar{\rho}^{\beta}]_{L_s}$ is the average of the $L_s(L_s-1)/2$ values of $\rho_{k,j}^{\beta}$, where $\rho_{k,j} = \rho_{j,k}$ is the correlation coefficient between annual maximum floods at sites k and j, with $1 \leqslant k < j \leqslant L_s$).

The application of (3) requires the selections of a suitable cross-correlation model for representing intersite correlation. Castellarin (2007) showed that the selection of the cross-correlation model has limited impact on (3) and suggested to use the model introduced by Tasker and Stedinger (1989) to approximate the true annual peak cross-correlation function $\rho_{i,j}$ as a function of the distance $d_{i,j}$ among sites i and j,

$$\rho_{i,j} = \exp\frac{-\lambda_1\cdot d_{i,j}}{1+\lambda_2\cdot d_{i,j}} \tag{4}$$

where $\lambda_1 > 0$ and $\lambda_2 \geqslant 0$ are regional parameters that may be estimated by ordinary or weighted least squares procedures.

Once n_{eff} has been estimated, a suitable plotting position needs to be selected for evaluating p_{EE}. The general plotting position reads (Cunnane, 1978)

$$p_{\hat{E}E} = 1 - \frac{\hat{n}_{eff}-\eta}{\hat{n}_{eff}+1-2\eta} \tag{5}$$

where η is the plotting position parameter and $\hat{n}_{eff}$ is the empirical estimate of n_{eff} given in (3). Among several possible options for selecting the η value, a quantile unbiased-plotting position should be used for estimating p_{EE}. A traditional choice is the Hazen plotting position ($\eta = 0.5$), which Castellarin (2007) showed to be particularly suitable for use when the annual maxima follow a Generalized Extreme Value (GEV) distribution (Jenkinson, 1955). GEV distribution has been shown to satisfactorily reproduce the sample frequency distribution of hydrological extremes around the world (see e.g., Stedinger et al., 1993; Vogel and Wilson, 1996; Robson and Reed, 1999; Castellarin et al., 2001; Di Baldassarre et al., 2006).

The algorithm proposed by Castellarin (2007) was developed considering annual sequences of flood flows, but its applicability is not confined to floods. The algorithm can be used to estimate the effective number of observations for groups of cross-correlated sequences of annual maxima in general. This study presents the first application to sequences of AMS of rainfall depth for given durations.

Regional envelope curve for rainfall extremes

The first main goal of this study is to provide a graphical representation of the maximum observed point rainfall depth (record rainfall depth) over a region for a given duration. The second main aim is to quantify the exceedance probability of the rainfall depth of record to be used for design purposes.

The graphical representation of the envelope of maximum rainfall depths observed at various sites in a region can be based upon the findings of several studies on regional frequency analysis of rainstorms (Schaefer, 1990; Alila, 1999; Brath et al., 2003; Di Baldassarre et al., 2006). These studies show that the statistics of rainfall extremes vary systematically with location, expressed in terms of mean annual precipitation (MAP). In particular, the studies illustrate for different regions of the world how the coefficients of variation and skewness (or *L*-variation –*L*–*Cv*– and *L*-skewness –*L*–*Cs*–, see e.g., Hosking (1990) for a definition of *L*-moments) of rainfall extremes tend to decrease as the local value of MAP increases. Di Baldassarre et al. (2006) formalised for a wide geographical region of northern-central Italy the relationship between *L*-statistics of rainfall extremes and MAP through a Horton-type curve (Horton, 1939),

$$L-Cx(\mathrm{MAP}) = a + (b-a)\cdot\exp(-c\cdot\mathrm{MAP}) \quad (6)$$

where *L*–*Cx* represents *L*–*Cv* or *L*–*Cs* relative to the annual maximum series (*AMS*) of rainfall depth with storm duration *t*, while *a*, *b*, *c*, with $0 \leqslant a \leqslant b$ and $c \geqslant 0$, are the parameters of the empirical model and depend on *t*. Fig. 1 reports an example for $t = 24$ h of the relationships between *L*–*Cv* and *L*–*Cs* and MAP for the study region analysed by Di Baldassarre et al. (2006).

The rainfall depth associated with duration *t* and a given exceedance probability, expressed in terms of recurrence interval *T*, $h_{t,T}$ can be represented as a function of MAP through a suitable probabilistic model by adopting the relationships reported in the literature between the statistics of rainfall extremes and MAP and by assuming that a non-decreasing relationship holds between the mean annual maximum rainfall depth for duration *t*, m_t, and MAP. Fig. 2 reports the empirical values of $m_{t,}$ with $t = 0.25$, 1, 6 and 24 h against the corresponding MAP values for the AMS of rainfall depths observed in the study area considered by Di Baldassarre et al. (2006). A strong positive relationship exists between m_t and MAP for long durations ($t \geqslant 12$ h), whereas the relationship gets weaker as the duration decreases; m_t tends to become independent of MAP for very short durations (hourly and sub-hourly durations). The behaviour illustrated in Fig. 2 for sub-hourly durations holds for various regions of the world and is well documented in the literature (e.g., Hershfield, 1961; Bell, 1969; Ferreri and Ferro, 1990; Alila, 1999).

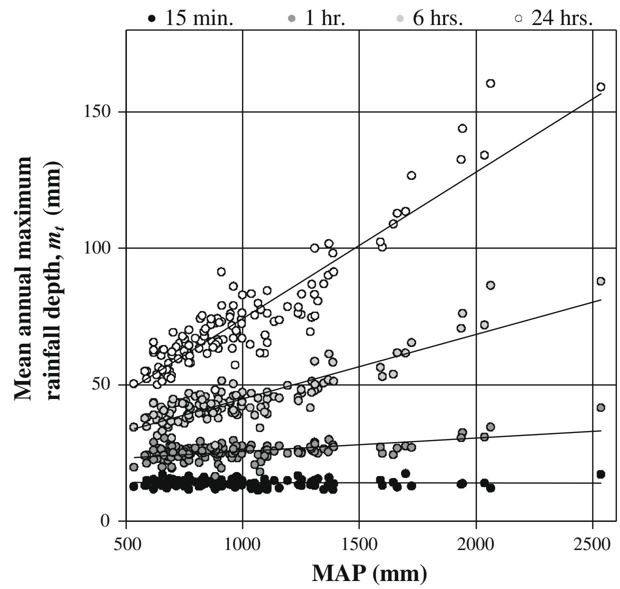

Fig. 2. Empirical values of the mean annual maximum rainfall depth for duration *t* vs. MAP for $t = 0.25$, 1, 6 and 24 h and the study area considered by Di Baldassarre et al. (2006).

Fig. 3 illustrates the dependence on MAP of the ratio between $h_{t,T}$ and MAP (hereafter also referred to as $\eta_{t,T}$) for $t = 1$ and 24 h and $T = 10$, 100 and 1000 years. The curves illustrated in Fig. 3 apply (6) with coefficients *a*, *b*, and *c* reported in Di Baldassarre et al. (2006, Table 2) to express the *L*-statistics of rainfall extremes as a function of MAP. Also, the curves adopt the linear relationships depicted in Fig. 2 to express the link between m_t and MAP, and utilise the EV1 (see Gumbel, 1958) or GEV (see Jenkinson, 1955) distributions as parent distributions.

The curves in Fig. 3 can be well approximated by a linear relationship in the log–log scale independently of the considered recurrence interval and parent distribution for $t = 15$ min. For $t = 24$ h the approximation is acceptable for the EV1 parent distribution, while it gets less satisfactory for the GEV parent when low values of MAP are considered.

Given the relationship between $\eta_{t,T}$ and MAP illustrated in Fig. 3, it seemed reasonable to represent the regional upper bound of observed maximum point rainfall depths for duration *t* through the following mathematical log-linear expression,

$$\ln\frac{h_{t,MAX}}{\mathrm{MAP}} = A(t) + B(t)\cdot\ln(\mathrm{MAP}) \quad (7)$$

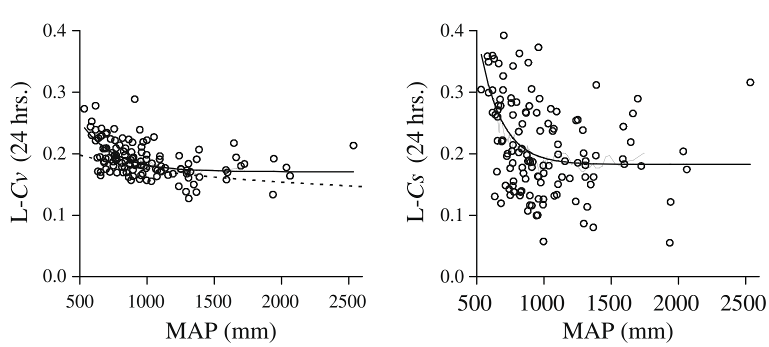

Fig. 1. AMS of 24 h rainfall for a large region in northern-central Italy: sample *L* moments vs. MAP (circles); weighted moving average curves (gray thick line); relationships between *L*–*Cv* and *L*–*Cs* and MAP identified by Di Baldassarre et al. (2006) (solid black line) and between *L*–*Cv* and MAP identified for Canada by Alila (1999, Table 3, p. 650) (dashed black line).

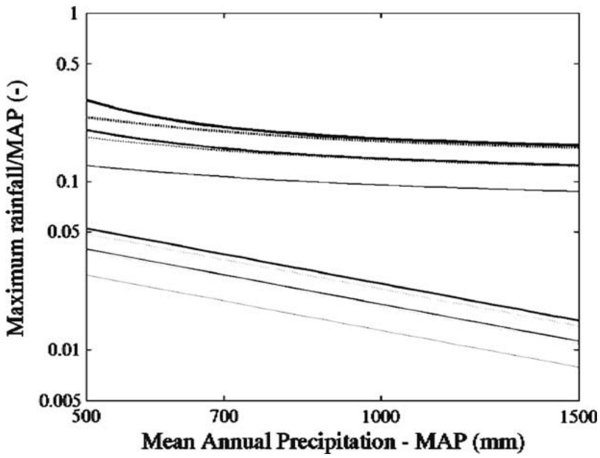

Fig. 3. Example of relationship between MAP and $\eta_{t,T}$ for t = 15 min (grey lines) and 24 h (black lines), T = 10, 100 and 1000 years (thin, medium and thick lines, respectively), and EV1 (dashed lines) and GEV (solid lines) parent distributions.

where $h_{t,MAX}$ represents the envelope rainfall depth for duration t, whereas A and B are regional coefficients that depends on duration t.

It can be observed that there exists a formal analogy between (1) that describes the regional envelope curve for flood flows and (7), which we term Depth-Duration Envelope Curve (DDEC). The suitability of DDEC expressed as in (7) for describing the upper bound of observed point rainfall and the applicability of the probabilistic interpretation of REC proposed by Castellarin et al. (2005) and Castellarin (2007) to DDEC are discussed in the next sections.

Study area

The study area (≈35,800 km^2) includes the Italian administrative regions of Emilia-Romagna and Marche. The area is bounded by the Po River to the north, the Adriatic Sea to the east, and the Apenninic divide to the southwest (see Fig. 4). The north-eastern portion of the study region is mainly flat, while the south-western and coastal parts are predominantly hilly and mountainous.

The database of extreme rainfall consists of the annual series of precipitation maxima with duration t equal to 15, 30 and 45 min and 1, 3, 6, 12 and 24 h that were obtained for a dense network of rain gauges from the National Hydrographical Service of Italy (SIMN) in the period 1935–1989. The available rainfall data are summarised in Table 1 in terms of number of gauges and overall sample-years of data for all durations of interest.

A regional frequency analysis of the dates of occurrence of short-duration rainfall extremes (i.e. annual maximum rainfall depths with duration less than 3 h) points out a very limited variance of the dates around the mean value, which varies for the whole study region between the end of July and the beginning of August. The short-duration rainfall extremes are almost invariantly summer showers generated by local convective cells. A regional frequency analysis of the dates of occurrence of long-duration extremes (i.e., duration from 12 to 24 h) shows a larger variability of the dates around the mean value, which ranges from the beginning of September to the beginning of November for all of the considered raingauges (Castellarin and Brath, 2002).

The mean annual precipitation (MAP) varies on the study region from about 600–2500 mm. Altitude is the factor that most affects MAP, which exceeds 1500 mm starting from altitudes higher than 400 m a.s.l. and exhibits the highest values along the Apenninic divide. Regional frequency analysis of rainfall annual maxima indicated that the GEV distribution is a suitable parent distribution for all durations of interest (Brath et al. 2003; Di Baldassarre et al., 2006).

A reliable and accurate representation of the true cross-correlation structure of the observations is critical to the estimation of the exceedance probability of an envelope curve of hydrological extremes constructed on the basis of annual maximum sequences (Castellarin et al., 2005; Castellarin, 2007). The sample correlation-coefficients for the considered annual sequences were

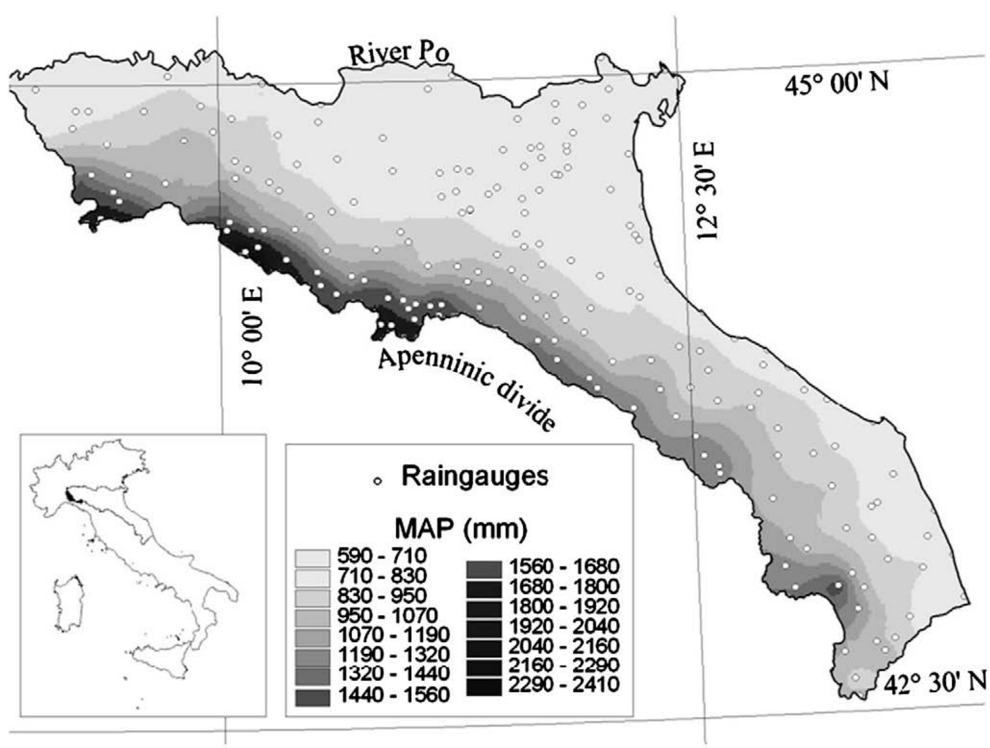

Fig. 4. Study area, location of raingauges and isoline representation of MAP (mm).

Table 1
Characteristics of the AMS of rainfall depth; calibrated coefficients of the cross-correlation formula (4); empirical DDEC intercept, slope and estimated recurrence interval.

Duration t (h)	24	12	6	3	1	0.75	0.50	0.25
Duration t (min)	1440	720	360	180	60	45	30	15
No. of sites	208	208	208	208	208	174	207	205
No. of observations (obs.)	7619	7625	6349	6740	7615	796	3492	2033
No. of single obs. n_1	1	1	7	8	1	5	0	0
No. of effective obs. n_{eff}	3060.3	4479.9	4983.4	6005.0	7103.7	729.7	3340.9	1909.4
λ_1 (km^{-1})	0.04085	0.06634	0.11216	0.18430	0.22210	0.19600	0.23550	0.21200
λ_2 (km^{-1})	0.01285	0.02226	0.03828	0.06408	0.07108	0.06220	0.06904	0.06620
DDEC slope, B	−0.4726	−0.534	−0.7557	−0.8091	−0.8793	−0.8691	−0.9378	−1.0159
DDEC intercept, A	2.2096	2.5013	3.7282	4.0156	3.8177	3.5835	3.7414	4.0386
Recurrence interval (years)	6121	8960	9967	12010	14207	1459	6682	3819

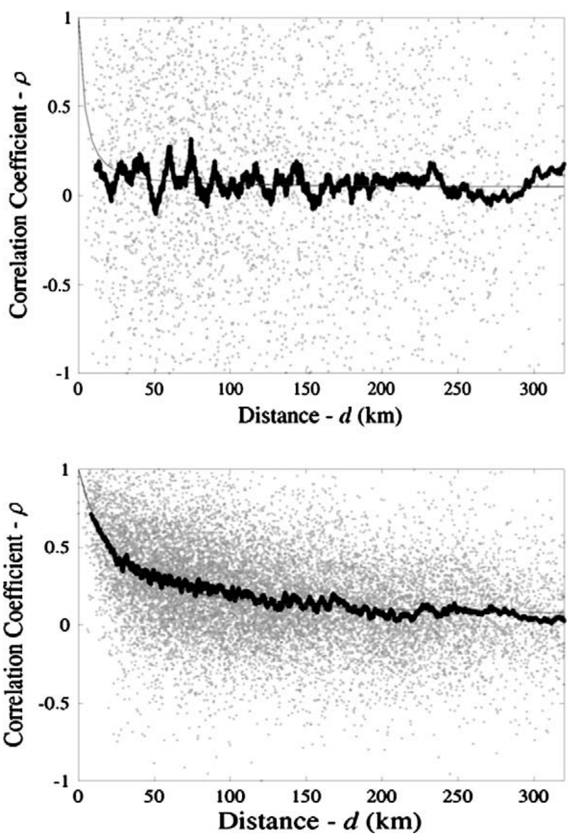

Fig. 5. Sample cross-correlation coefficients (gray dots); weighted moving average curve (thick line); correlation formula (4) calibrated for the whole study area (thin line) for duration t = 15 min. (top) and 24 h (bottom).

computed using sample estimators proposed in the scientific literature (see e.g., Stedinger, 1981) and the true annual peak cross-correlation function $\rho_{i,j}$ was modelled through (4) as a function of the distance $d_{i,j}$ among sites i and j (Tasker and Stedinger, 1989; Castellarin, 2007). Fig. 5 reports for t equal to 15 min and 24 h: the sample cross-correlation coefficients; a weighted moving average curve, which weights each sample coefficient proportionally to the record length; and the correlation formula (4) calibrated through a weighted least squares optimization procedure.

As it was expected, the cross-correlation between annual sequences becomes stronger as duration increases (see Fig. 5). Consequently, the calibrated values of the coefficients λ_1 and λ_2 show a strong relationship with the considered storm duration t (Fig. 6). The interpolation of the empirical values reported on Fig. 6 enabled us to describe the true intersite correlation for all of the durations and distances of interest in the study (Fig. 7).

Envelope curves and exceedance probability

Construction of the DDEC

Fig. 8 illustrates the DDEC curves obtained for the study area and all durations of interest from t = 45 min to 24 h. Figures report point rainfall depths standardised by the local value of MAP and they illustrate the observed maximum rainfall depths and envelope curves. The slope $B(t)$ of each curve was estimated by regressing the standardised rainfall maxima against the local value of MAP. Then, the intercept $A(t)$ of the curve was identified by enveloping all rainfall maxima observed in the study region through the following equation:

$$A(t) = \max_{j=1,2,\ldots,M}\left\{\ln\frac{h_{t,MAX,j}}{MAP_j} - \hat{B}(t)\cdot\ln(\mathrm{MAP}_j)\right\} \quad (8)$$

where is the estimated slope, $h_{t,MAX,j}$ denotes the maximum rainfall depth observed for duration t at site $j = 1,2,\ldots,M$ and M is the

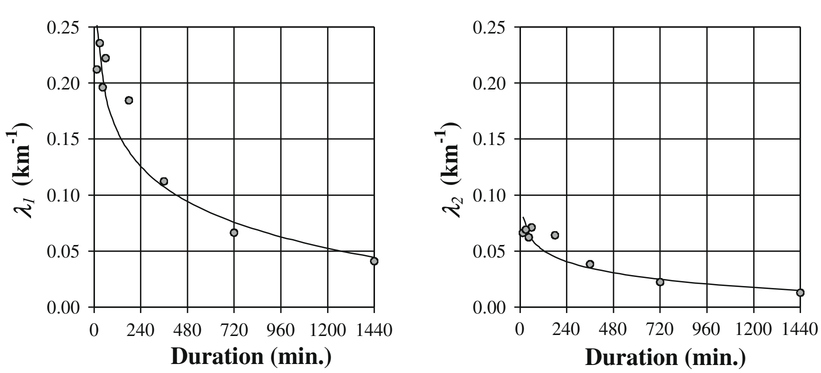

Fig. 6. Relationship between the calibrated values of the coefficients λ_1 and λ_2 of correlation formula (4) and duration.

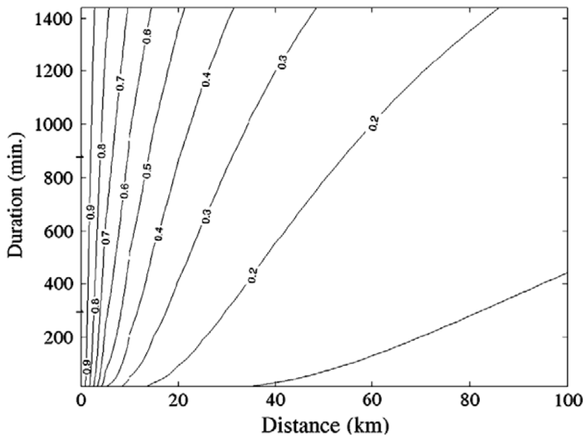

Fig. 7. Cross-correlation between AMS of rainfall depth as a function of duration t and intersite distance.

number of sites in the region, while MAP_j is the local value of the mean annual precipitation. Slopes and intercepts for all DDEC are listed in Table 1.

The schematisation of the cross-correlation structure illustrated in Figs. 6 and 7 and the application of the algorithm (3) described in 'Probabilistic envelope curves for floods' to the available rainfall data enabled us to estimate the exceedance probability of the envelope curves. The overall sample-years of annual maximum rainfall depths, the single observations n_1, the estimated equivalent number of independent observation n_{eff}, and the estimated value of the recurrence interval obtained by applying the Hazen plotting position to n_{eff} are listed in Table 1 for all durations of interest.

The envelope curves illustrated in Fig. 8 and the corresponding estimates of the recurrence interval listed in Table 1 represent an easy-to-use graphical tool to (i) identify a plausible value of the extreme point rainfall depth at any location of the considered study area as a function of the local value of MAP (see Fig. 4) for durations ranging from 15 min to 24 h and to (ii) attach to this rainfall depth an estimate of the exceedance probability, expressed in terms of recurrence interval.

Depth–duration envelope curves and regional depth–duration–frequency equations

The assessment of the accuracy of rainfall depth quantiles that can be retrieved from the DDEC is not an easy task due to the high values of the recurrence intervals associated with the curves (see Table 1). We obtained an indication on the accuracy of the DDEC quantiles by performing a comparison with the rainfall quantiles computed by applying a regional model proposed in the literature for the study area.

We performed such a comparison for t equal to 1 and 24 h, selected to represent convective and frontal rainstorm. The DDEC quantiles for the comparison were computed through a "leave-one-out" procedure (see e.g., Castellarin, 2007). For each of the 208 available gauges (see Table 1) we carried out the following steps: (1) we neglected the available hourly annual maximum data, therefore assuming to have information about MAP only; (2) we constructed the DDEC for the durations of 1 and 24 h on the basis of the information collected at the 207 remaining gauges (DDEC*); (3) we applied the algorithm presented in 'Probabilistic envelope curves for floods' and estimated the recurrence interval T^* for each of the two DDEC*; (4) we retrieved the envelope rainstorms for t equal to 1 and 24 h for the raingauge of interest from the two DDEC* as a function of the local value of MAP and we associated each envelope rainstorm with the T^* of the corresponding DDEC*. We repeated these four steps for all 208 available gauges.

Finally, we compared the rainfall depths we obtained through the procedure outlined above, which we termed $h_{T,t}^{DDEC}$, with the rainfall quantiles obtained for each site and the same T^* and t values by applying the Regional Depth-Duration Frequency Equation (RDDFE) proposed by Brath et al. (2003). We termed these reference rainfall quantiles as $h_{T,t}^{RDDFE}$. The equation has the following expression:

$$h_{T,t} = 0.138 \cdot t^{0.624} \cdot h_{10y,24h} \cdot \left[f \cdot \ln \frac{T}{10} + 1 \right] + (24 - t)^{0.770} \cdot [0.474 \cdot \ln(T) + 0.951] \quad (9a)$$

where $h_{T,t}$ ($h_{10y,24h}$) is the point rainfall depth with recurrence interval T (10 years) and duration t (24 h) and is expressed in mm, f is expressed as,

$$f = 0.602 - 0.055 \cdot \ln(\mathrm{MAP}) \quad (9b)$$

with MAP in mm. Eq. (9) can be used for estimating $h_{T,t}$ in any location of the study region for $1\ \mathrm{h} \leqslant t \leqslant 24\ \mathrm{h}$, provided the local value of MAP and an estimate of $h_{10y,24h}$. Brath et al. (2003) identified the equation by referring to T values that are significantly lower than the ones estimated for the DDEC of the whole study area. Nevertheless, as the results of the comparison show, the extrapolation of the RDDFE is functional to the discussion of the accuracy of DDEC rainfall quantiles and enables us to draw some concluding remarks.

The comparison produced limited values of the relative residuals between the two sets of rainfall quantiles for $t = 1$ h. For this duration 90% of residuals falls within the interval $-11\% \leqslant \varepsilon \leqslant 17\%$, whereas 50% of the residuals are between $-5\% \leqslant \varepsilon \leqslant 4\%$. Larger absolute values were found for the duration $t = 24$ h. In this case 90% of the residuals falls between $-56\% \leqslant \varepsilon \leqslant 12\%$, 50% between $-34\% \leqslant \varepsilon \leqslant 12\%$, and $h_{T,t}^{DDEC}$ values generally overestimate $h_{T,t}^{RDDFE}$ values. It is worth remarking here that DDEC quantiles are computed on the basis of the local value of MAP alone (see 'Regional envelope curve for rainfall extremes' and 'Construction of the DDEC').

The scatter-plots of Fig. 9 illustrate the results of the comparison. The scatter-plot relative to $t = 24$ h shows that the DDEC rainfall quantiles are significantly underestimated by RDDFE equations. The differences between the two sets of quantiles tend to be smaller for higher values of the rainfall depth, for which the two approaches show a better agreement. This result may in part be a consequence of the extrapolation of RDDFE equations, which are used in this context for values of T definitely larger than the ones adopted for the identification of the equation. Nevertheless, a generalized overestimation could also be associated with the selected log-linear shape of the envelope. We show in 'Regional envelope curve for rainfall extremes' that for long (e.g., daily) storm durations and GEV parents the envelope presents a concavity in the log–log space (see Fig. 3) .

The scatter-plot relative to $t = 1$ h shows large absolute values of the residuals also for high rainfall depths. In this case, though, differently from what observed for $t = 24$ h or for $t = 1$ h and smaller rainfall quantiles, DDEC quantiles significantly underestimate RDDFE quantiles. The significant differences between DDEC and RDDFE rainfall quantiles for $t = 1$ h are probably due to the extrapolation of RDDFE for large T, and should not be ascribed to a limited accuracy of DDEC rainfall quantiles. This consideration can be explained as follows. First, the relationship between the envelope and MAP for short storm durations is expected to be approximately linear in the log–log space independently of the selected recurrence interval T and parent distribution (see Fig. 3). Second, it has to be noted that the 24 h 10 year rainfall quantile is one of the parameters of RDDFE (see Eq. (9a)). Therefore, RDDFE is expected to perform better for $t = 24$ h than for $t = 1$ h (Brath et al., 2003).

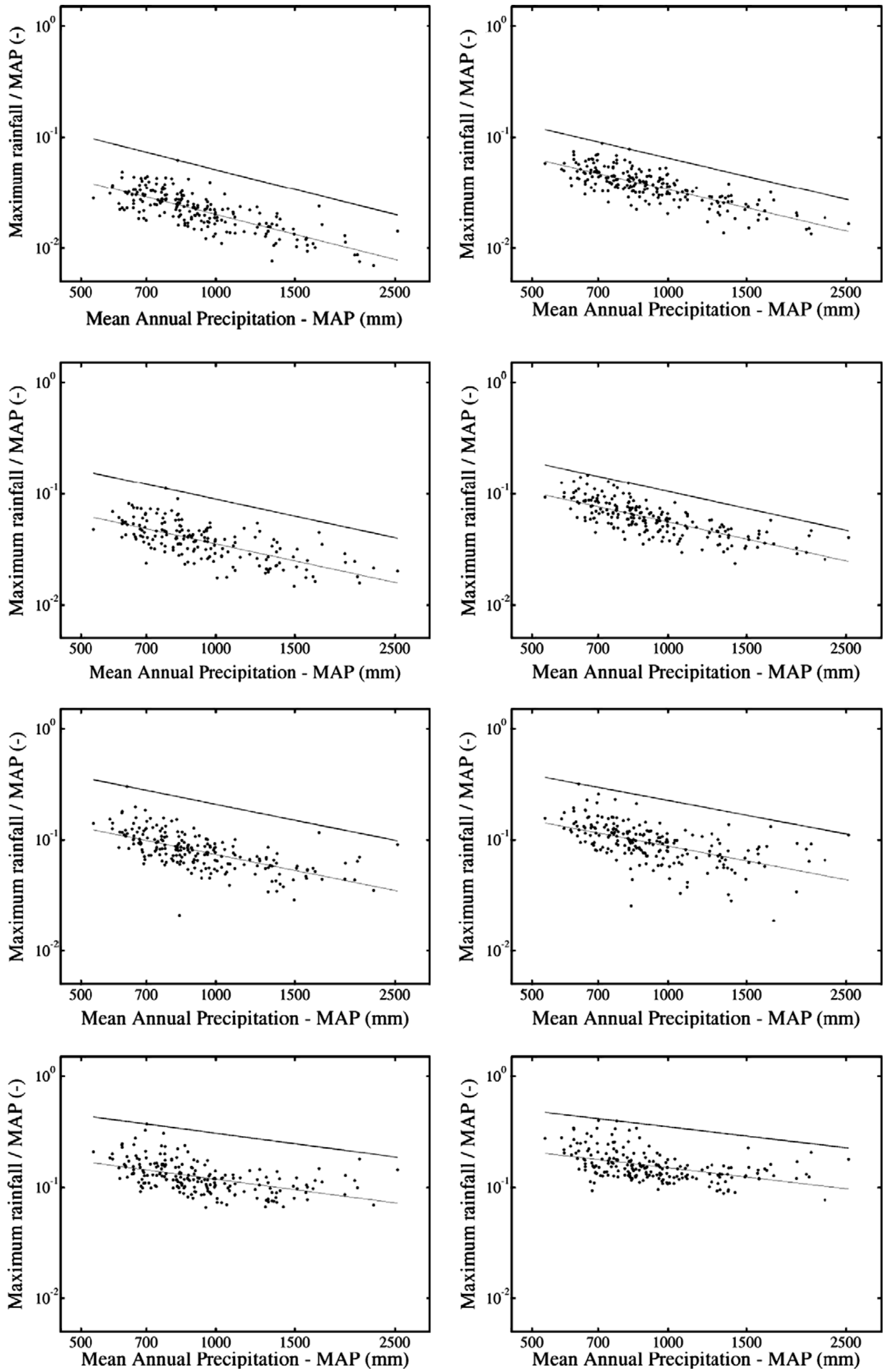

Fig. 8. DDEC (thick lines) constructed for the study area for duration (from top to bottom and from left to right): 15, 30 and 45 min and 1, 3, 6, 12 and 24 h (right), each diagram reports also the observed rainfall maxima (dots) and the regression line between maxima and MAP (thin line).

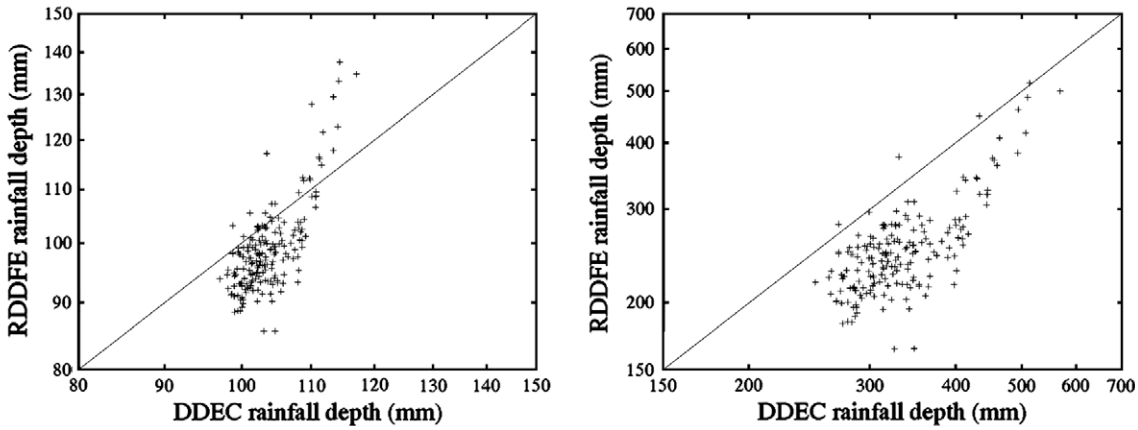

Fig. 9. DDEC rainfall quantiles vs. RDDFE rainfall quantiles for duration t of 1 h (left) and 24 h (right).

Conclusions and final recommendations

The main goal of this study is the representation of the upper bound on our experience of extreme rainstorms in a region. To this aim, we reconsider the recent advances in the field of regional frequency analysis of rainstorms and we introduce a simple mathematical formulation of the upper bound on observed rainfall maxima. The formulation adopts Mean Annual Precipitation (MAP) as a surrogate of location. The result is a graphical tool, which we call Depth-Duration Envelope Curve (DDEC), that can be used for determining plausible extreme rainfall events at gauged and ungauged sites as a function of local climatic condition, described by MAP.

We propose a procedure for estimating the exceedance probability of DDEC, which is based on an adaptation of the algorithm for the evaluation of the exceedance probability of Regional Envelope Curve (REC) of flood flows reported in the literature. The estimation of the exceedance probability of a DDEC makes it a design tool that is suitable for addressing engineering problems such as the definition of urban and rural planning strategies and the design of river engineering works or major flood protection works.

The concept of DDEC is applied and assessed in this study for a wide geographical region located in northern-central Italy. For this region annual maximum series (AMS) of rainfall depth with duration spanning from 15 min to 24 h are available for a rather dense gauging network. DDEC were constructed for all durations considered in the regional dataset.

An accurate quantification of intersite correlation among AMS recorded at different raingauges is fundamental to the evaluation of the exceedance probability of the DDEC. We propose a cross-correlation model that expresses the correlation degree between the annual sequences of the study region as a function of the intersite distance and storm duration. The cross-correlation model is then applied to estimate the effective regional sample-year of data (equivalent overall number of independent annual maxima) used to construct the DDEC for the various storm durations of interest. The estimation of the effective sample-years of data is the core of the algorithm for the quantification of the exceedance probability of the envelope curve. Once we constructed the envelope curves and estimated their exceedance probability, we compared the rainfall quantiles (rainfall depths associated with a given duration and recurrence interval) retrieved from DDEC with the corresponding quantiles computed through a regional depth-duration frequency equation proposed by the scientific literature for the region of interest. The results of the analysis indicate that the proposed DDEC can be effectively employed to determine plausible extreme values of rainfall depth for different storm-duration at gauged and ungauged sites (deterministic interpretation of the envelope curve) and may also be used to provide a realistic estimate of the recurrence intervals associated with such rainfall events (probabilistic interpretation of the envelope curve). Our results are still preliminary, nevertheless this study represents an initial effort at the representation of a probabilistic upper bound on observed rainfall maxima. Further analysis should: (i) investigate alternative mathematical formulations of the upper bound, the log-linear envelope curve considered in present study may not provide an accurate representation of the envelope for long durations (e.g., 12–24 h); (ii) compare the suggested DDEC with other methods proposed in the literature for predicting high-recurrence interval rainstorms (e.g., Probable Maximum Precipitation); (iii) consider different regions of the world; (iv) address the problem of areal rainfall estimation, which is more important than point rainfall in many engineering applications.

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