HYDROLOGICAL PROCESSES, VOL. 9, 313-330 (1995)

ON THE REPRESENTATIVE ELEMENTARY AREA (REA) CONCEPT AND ITS UTILITY FOR DISTRIBUTED RAINFALL-RUNOFF MODELLING

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ABSTRACT

Since the paper of Wood *et al.* (1988), the idea of a representative elementary area (REA) has captured the imagination of catchment modellers. It promises a spatial scale over which the process representations can remain simple and at which distributed catchment behaviour can be represented without the apparently undefinable complexity of local heterogeneity. This paper further investigates the REA concept and reassesses its utility for distributed parameter rainfall–runoff modelling. The analysis follows Wood *et al.* (1988) in using the same topography and the same method of generating parameter values. However, a dynamic model of catchment response is used, allowing the effects of flow routing to be investigated. Also, a 'nested catchments approach' is adopted which better enables the detection of a minimum in variability between large- and small-scale processes. This is a prerequisite of the existence of an REA.

Results indicate that, for an impervious catchment and spatially invariant precipitation, the size of the REA depends on storm duration. A 'characteristic velocity' is defined as the ratio of a characteristic length scale (the size of the REA) to a characteristic time-scale (storm duration). This 'characteristic velocity' appears to remain relatively constant for different storm durations. Spatially variable precipitation is shown to dominate when compared with the effects of infiltration and flow routing. In this instance, the size of the REA is strongly controlled by the correlation length of precipitation. For large correlation lengths of precipitation, a separation of scales in runoff is evident due to small-scale soil and topographic variability and large-scale precipitation patterns. In general, both the existence and the size of an REA will be specific to a particular catchment and a particular application. However, it is suggested that a separation of scales (and therefore the existence of an REA), while being an advantage, is not a prerequisite for obtaining simple representations of local heterogeneity.

KEY WORDS Distributed modelling Heterogeneity Spatial scale Spatial variability Continuum assumption Spectral gap Separation of scales Characteristic velocity

INTRODUCTION

Distributed parameter hydrological models are being increasingly used in investigations of spatial scale and catchment heterogeneity as well as general rainfall–runoff applications (e.g. Goodrich, 1990; Grayson *et al.*, 1993, in press; Rosso *et al.*, 1994; Sivapalan and Viney, 1994). A critical problem in the application of these models is the choice of element size, which must be able to represent the heterogeneity of the catchment

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CCC 0885-6087/95/030313-18

Received 20 December 1993
Accepted 22 September 1994

response, be compatible with the algorithms used in the model, and must enable definition of the model parameters. The choice of element size relates to the representation of spatial heterogeneity of catchment response in general, and of the model parameters in particular. The representative elementary area (REA) work of Wood *et al.* (1988) was an initial attempt to determine at what scale, if any, the description of catchment properties becomes simple. This relates directly to an 'ideal' or 'preferred' element size for distributed parameter catchment modelling and will depend on the effect on catchment response of the heterogeneity of soils, topography and precipitation at various spatial scales.

Since the initial work of Wood *et al.* in 1988, the idea of an REA has captured the imagination of catchment modellers. It promises a spatial scale over which the process representations can remain simple and at which distributed catchment behaviour can be represented without the apparently undefinable complexity of local heterogeneity.

Wood *et al.*'s original work was based on a modified version of TOPMODEL (Beven and Kirkby, 1979) and the topography of the Coweeta catchment in North Carolina. Wood *et al.* (1988) used TOPMODEL to compute the total runoff volume for a one hour period of rainfall, but did not include dynamic effects such as the routing of 'runoff waves'. Spatially correlated stationary random fields of soil hydraulic conductivity and precipitation were used to investigate the spatial characteristics of runoff. For each realization of soil and rainfall characteristics, Wood *et al.* (1988) determined the runoff volume from 148 subcatchments. These runoff volumes were ranked on the basis of subcatchment size, irrespective of their relative position in the catchment. The average of a 15 element filter, moving in steps of five, was plotted versus area (expressed as number of pixels). These plots were then used to determine the REA, defined as the area where the curve joining points of increasing subcatchment area flattened out. This was shown by Wood *et al.* (1988) to be approximately 1 km^2. The choice of different rainfall correlation lengths varying from 125 to 1250 m or spatially invariant precipitation did not significantly change this result. They concluded that the REA was strongly influenced by topography and that the length scales of soils and rainfall characteristics played only a secondary part.

In a more recent discussion of the REA concept, Beven (1991) highlighted a number of aspects of the original work that require further investigation. These included the influence of larger correlation lengths for rainfall, the effects of flow routing on the volume and timing of runoff, and the influence of geology and large-scale synoptic effects.

The objective of this paper is to investigate further the utility of the REA concept. Specifically, it goes beyond the original work of Wood *et al.* (1988) in the following ways:

1. Sets of nested subcatchments are used here, whereas Wood *et al.* (1988) used averaging over non-nested subcatchments.
2. A dynamic model of catchment response is used, allowing the effects of flow routing to be investigated.
3. Rainfall correlation length is explored over a wider range and rainfall duration is varied.
4. The utility of the REA concept for distributed parameter rainfall–runoff modelling is reassessed.

The reasons for using nested catchments are discussed in detail later but, briefly, it is because it enables the detection of a separation of scales between large- and small-scale variability.

The paper is separated into nine sections. The first is a discussion of the theoretical considerations that underlie the concept of the REA. The second and third present the modelling assumptions used in the subsequent simulations and the methods used to test for the existence of an REA. The following sections present the results of the simulations starting from the simplest case of spatially uniform parameters, then introducing spatially variable parameters in steps. The last two sections are a discussion and conclusion, respectively, including a review of the utility of the REA concept.

CONTINUUM ASSUMPTIONS, REV AND REA

Heterogeneity in natural catchments is of stunning complexity and so far has virtually defied detailed description and/or measurement. Many branches of science which deal with similar heterogeneity adopt a 'continuum approach' and ignore the local (microscale) heterogeneity for modelling purposes, replacing the real system by a fictitious continuous medium at the macroscale. For example, in groundwater hydrology, the detailed patterns of pore structure at the microscopic level are replaced by a continuous field of porosity at the macroscopic level. Porosity is a coefficient which describes the lumped effect of the microscopic pattern (Hubbert, 1956; Wheatcraft and Cushman, 1991). Bear and Bachmat (1990) suggest the following advantages of the continuum approach: (a) no knowledge of the microscale patterns is required; (b) the continuous medium is differentiable; and (c) the continuum (potentially) represents measurable quantities.

The continuum can be arrived at either by ensemble averaging or by space averaging. For the ensemble average, the medium is assumed to be a realization of a random process. A property such as porosity can then be defined, at a given point in space, as the average over all possible realizations of the same process (deMarsily, 1986; Cushman, 1987). The space average can be a volume or area average and defines a property, at a given point in space, as the average property of a certain volume (or area) of material surrounding it. Two area averaging methods have been suggested in the context of continuum assumptions in catchment hydrology: Gupta *et al.* (1986) proposed elevation bands in high relief terrain, whereas Wood *et al.* (1988; 1990) suggested subcatchments. Here we will follow Wood *et al.* (1988).

In principle, an arbitrary elementary area (AEA) of *any* size can be selected as an averaging area for passing from the microscopic to the macroscopic level (Bear and Bachmat, 1990: 16; Stull, 1988: 420). It is likely, however, that different AEAs will yield different averaged values for each quantity of interest, which makes model parameterization difficult. The question then is whether we can determine an AEA where the averaged variables become insensitive to variations in the size of the AEA. Hubbert (1956; 227) addressed this very problem in the context of defining macroscopic porosity at a particular point in a porous medium:

> About this point we take a finite volume element ΔV, which is large as compared with the grain or pore size of the rock. Within this volume element the average porosity is defined to be $\bar{f} = \Delta V_f/\Delta V$ where ΔV_f is the pore volume within ΔV. We then allow ΔV to contract about the point P and note the value of $\bar{f}$ as ΔV diminishes. If we plot $\bar{f}$ as a function of ΔV (Hubbert's Fig. 5), it will approach smoothly a limiting value as ΔV diminishes until ΔV approaches the grain or pore size of the solid. At this stage $\bar{f}$ will begin to vary erratically and will ultimately attain the value of either 1 or 0, depending upon whether P falls within the void or the solid space.

It is important to note that Hubbert (and also Bear, 1972: 16, who based his ideas on those of Hubbert), considered one fixed reference point with a volume of changing size around it. Bear (1972) denoted, as the representative elementary volume (REV), the order of magnitude where 'f (*porosity*) approaches smoothly a limiting value' (i.e. varies only smoothly with changing volume). In direct analogy, Wood *et al.* (1988), coined the term Representative Elementary Area (REA) in catchment hydrology.

The existence of a scale where macroscopic properties vary smoothly is tantamount to the existence of a 'separation of scales' (Gelhar, 1986). 'Separation of scales' refers to a process consisting of a small-scale (or fast) component superimposed on a much larger (or slow) component. The scales are called 'separated' if there is a minimum in the power spectrum. In meteorology, for example, microscale turbulence and synoptic scale processes are often separated by a 'spectral gap' which shows up as a minimum in the power spectrum of wind speed. Typically, this spectral gap is at a frequency of 1/hour; both higher and lower frequencies are much more common (Stull, 1988). Although Wood *et al.* (1988) did not use this terminology, an REA would exist if there was a 'spectral gap' (or equivalently a separation of scales) in catchment variability.

The application of the continuum approach to distributed hydrological modelling requires the deterministic representation of large-scale variability (as different values in different model elements) and the parameterization of the small-scale processes within an element (see, e.g. Kirnbauer *et al.*, 1994). Variability within an element is often referred to as 'sub-grid' variability. Wood *et al.* (1988) suggested the use of distribution functions for parameterizing the sub-grid variability of infiltration. Provided a separation of scales exists, the same parameterization of sub-grid processes can be used for a range of element sizes. This becomes possible when the element size falls into the spectral gap or, in other words, when the element size is of the order of magnitude of the size of the REA. A small change in element size does not then significantly change the relative contributions of sub-grid and large-scale variability.

It should be emphasized that the existence of a spectral gap (i.e. the existence of an REA) is not a prerequisite for the parameterization of processes within an element. Small-scale processes can be parameterized for any element size. The advantage of the existence of a spectral gap is that the same parameterization can be used for a range of element sizes and so would be more generally applicable. If such a gap existed and was 'universal', then there would be a basis for developing generalized macroscale modelling approaches.

METHODS

The simulation exercise in this paper uses the distributed parameter model known as THALES (Grayson *et al.*, 1992), which is based on the terrain analysis software TAPES-C (Moore and Grayson, 1991). The general framework of the model is described in the following, as are differences from the model as presented in Grayson *et al.* (1992; in press).

Following the work of Wood *et al.* (1988), a 30 m Digital Elevation Model (DEM) of the 17 km^2 Coweeta catchment in North Carolina is used. Elevations range from 680 m to 1600 m. From the DEM, contours with variable spacing from 5 to 40 m are derived and used to determine the flow paths. Adjacent flow paths and contours define elements. The resulting element network contains 6206 elements with an average element size of 50 by 50 m (Figure 1). Model parameters are assumed to be constant within each element. Runoff is produced via infiltration excess or saturation excess mechanisms and flows downslope. The infiltration equation of Philip (1957) is used, based on the parameter description of Wood *et al.* (1988). Surface runoff is described as overland sheet flow and is routed using the kinematic wave assumptions with a Manning's n of 0·4. The infiltration of surface runoff in downslope elements (runon) is allowed in these simulations and the channel network is defined by a threshold upslope contributing area of 10 ha. Elements with areas greater than this threshold are assumed to contain channels with a width of 5 m and a Manning's n of 0·1. Although in the original version of THALES, a soil depth above an impermeable boundary is defined, for the simulations

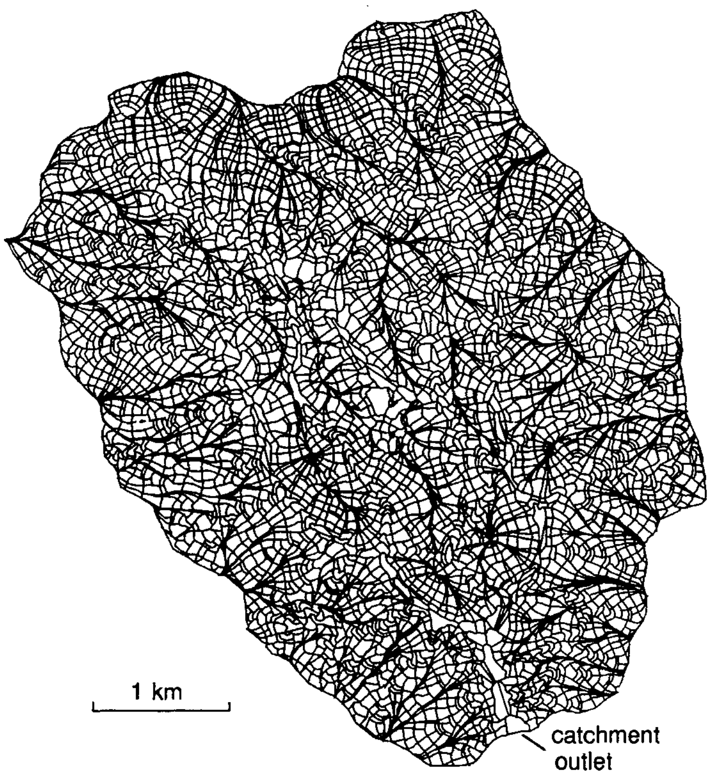


Figure 1. Element network for the Coweeta topography, North Carolina, USA. The elements are defined by adjacent flow paths and contours

Table I. Parameter sets used in the simulations

	Manning's *n* overland flow	Manning's *n* channel flow	(Average) hydraulic conductivity (mm/h)	Correlation length of conductivity (m)	Distribution of initial sat. deficit	Initial saturated area (%)	Storm duration	(Average) intensity of precip. (mm/h)	Correlation length of precipitation (m)
Figure 2abc	0·4	0·1	5	100	Topogr.	30	1 h	5	Invar.
Figure 4	0·4	0·1	5	100	Topogr.	30	1 h	5	Invar.
Figure 5	0·4	no chann.	Impervious		Impervious		1 h	5	Invar.
Figure 6a	0·4	no chann.	Impervious		Impervious		12 min	5	Invar.
Figure 6b	0·4	no chann.	Impervious		Impervious		2 h	5	Invar.
Figure 7	0·04	no chann.	Impervious		Impervious		12 min	5	Invar.
Figure 8a	0·4	no chann.	5	100	Topogr.	30	12 min	5	Invar.
Figure 8b	0·4	no chann.	5	100	Topogr.	70	12 min	5	Invar.
Figure 9a	0·4	0·1	5	100	Topogr.	30	12 min	5	Invar.
Figure 9b	0·4	0·1	5	100	Topogr.	30	1 h	5	Invar.
Figure 10a	0·4	0·1	5	100	Topogr.	30	1 h	5	125
Figure 10b	0·4	0·1	5	100	Topogr.	30	1 h	5	2500
Figure 10c	0·4	0·1	5	100	Topogr.	30	1 h	5	625
Figure 11a	0·4	0·1	5	100	Topogr.	30	1 h	5	125
Figure 11b	0·4	0·1	5	100	Topogr.	30	1 h	5	2500
Figure 12 I	0·4	0·1	Impervious		Impervious		1 h	5	2500
Figure 12 V	0·4	0·1	5	100	Topogr.	30	1 h	5	2500
Figure 12 T	0·4	0·1	5	Invar.	Topogr.	30	1 h	5	2500
Figure 12 R	0·4	0·1	5	Invar.	Rand.field	30	1 h	5	2500

Abbreviations: sat. deficit = saturation deficit; precip. = precipitation; no chann. = no channels; topogr. = based on topographic wetness index; invar. = spatially invariant; and rand. field = random field.

in this paper, the 'saturation deficit' concept of Beven and Kirkby (1979) is used to ensure consistency with the work of Wood *et al.* (1988). A simulation time step of two minutes is used.

The use of THALES as a basis for the simulations in this paper introduces some differences from the work presented by Wood *et al.* (1988). The model is dynamic and both channel and overland flow are actually routed though the catchment so runoff peaks and storm volumes can be defined. This introduces the ability to represent the influence of channelized versus overland flow.

Spatially, model and input parameters are assumed to be either constant, or stationary correlated random fields. Specifically, in the case of precipitation, the correlation length refers to a double exponential correlation function while in the case of the hydraulic conductivity it refers to a Bessel type correlation function. The exception is the initial saturation deficit which is either represented by a random field of exponential correlation structure or by the topographic wetness index of Beven and Kirkby (1979). The methods and parameters used strictly follow the work of Wood *et al.* (1988, Appendix A and B) unless where otherwise indicated. Precipitation is assumed to be a rain block of 12 min, 1 hour or 2 hour duration. Table 1 gives the parameter sets used in the simulations.

EFFECT OF AVERAGING

The analysis of Wood *et al.* (1988) involved the ranking of runoff volumes according to subcatchment size and subsequent filtering. Our concern is that this method of averaging might influence the definition and size of the REA. This section presents an analysis similar to that of Wood *et al.* (1988) and explores the effect of different averaging methods.

Simulations were performed for uniform precipitation, spatially variable hydraulic conductivity and topographically defined initial saturation deficit based on the $\ln(a/\tan\beta)$ index of Beven and Kirkby (1979) where a is the upslope contributing area per unit contour length and β is the local slope. Subcatchments were chosen using the following procedure: ten ridge points were chosen at random from the element network and the flow paths from each of these points to the catchment outlet were determined. The upslope contributing area for each element on the flow path defined a subcatchment. Figure 3 shows some of the subcatchments defined by this procedure for one flow path. This resulted in 161 subcatchments, which is close to the 148 used by Wood *et al.* (1988).

The flow volumes for each subcatchment were ranked according to subcatchment size in an identical manner to that of Wood *et al.* (1988). The same filtering procedure based on a window of 15 subcatchments, moving in steps of five, was used resulting in 30 classes for the 161 subcatchments. The resulting average flow volumes are plotted versus subcatchment area (Figure 2a). Figure 2a is very similar to Figure 4a in Wood *et al.* (1988), with the response flattening out above approximately 1 km^2.

Figure 2b shows the same simulation results, but with 1394 subcatchments selected for averaging using a larger number of flow paths. The same number of filtering classes as in Figure 2a were used (30 in each instance). For the 1394 subcatchments, this corresponds to a window of 135 subcatchments moving in steps of 45. Clearly, the window to window variations are smoothed out when a sufficient number of subcatchments per window are selected. The shape of the curve, with lower volumes in small catchments, reflects the influence of the topographic index on saturation deficit with large deficits for the small subcatchments near the ridges (i.e. lower $\ln(a \tan\beta)$).

Given the smoothing effect of the number of subcatchments in a window on average volumes, it may be more consistent with the requirement for a 'minimum of variance', to examine the variability within each window. Figure 2c shows the variance in the windows used to construct Figure 2a, which indeed is much lower above a threshold area of 1 km^2 than for smaller subcatchments. However, the meaning of such a quantity in the context of distributed rainfall–runoff modelling is unclear, as it compares catchments of similar size, irrespective of their relative position or distance apart. In fact, any approach that ignores the relative position of subcatchments will be unable to detect an increase in variance at large scales. This is illustrated by the following example. Two small adjacent catchments differ only due to small-scale variability. Two small catchments, separated by a large distance, differ due to both the small-scale variability as well as any large-scale variability present in the landscape. If, on the other hand, we compare two large catchments, small-scale

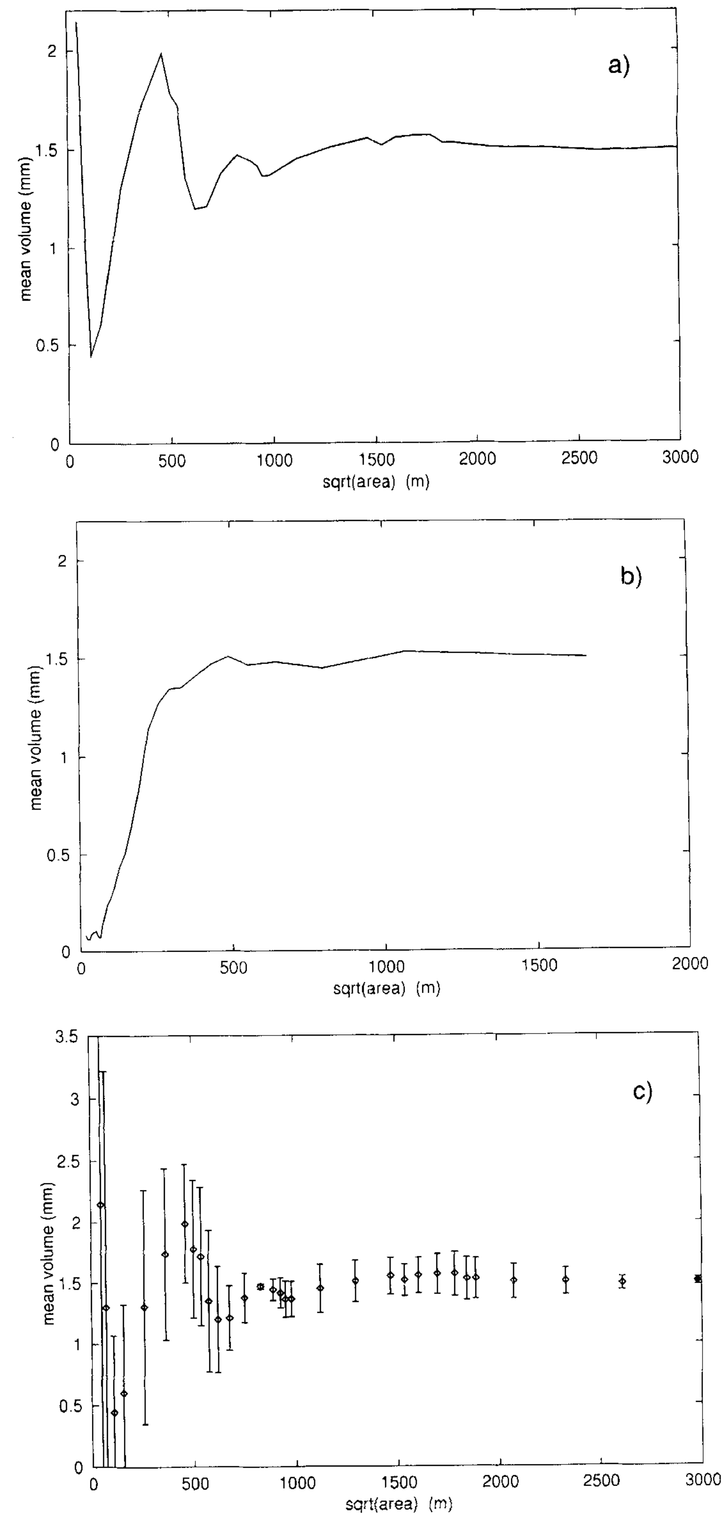


Figure 2. Mean runoff volume versus catchment size for spatially variable soils and constant precipitation. (a) Filtering based on a window of 15 subcatchments moving in steps of five, resulting in 30 classes as in Figure 4a of Wood *et al.* (1988). (b) Filtering based on a window of 135 subcatchments moving in steps of 45, resulting in 30 classes. (c) Standard deviations within windows for (a)

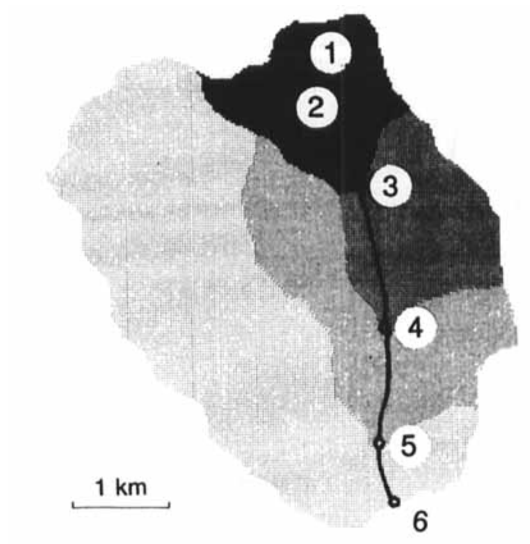


Figure 3. Example of a set of nested catchments. Numbers indicate some of the subcatchment outlets and refer to those in Figure 4.

variability is averaged out within each catchment and we have variability due only to large-scale effects. It is therefore apparent that large catchments will *never* appear more variable than widely spaced small catchments. To show true changes in variability and detect a spectral gap, adjacent areas must be compared.

The averaging method should therefore consider one fixed reference point and a domain of variable size around it. In catchment hydrology, this is a set of nested catchments, as illustrated in Figure 3. In this example, the fixed reference point is a ridge point at the top of Figure 3 and the 'domain of variable size' is defined by subcatchments of increasing size. This approach is analogous to that of Hubbert (1956) and Bear (1972), used in their definition of the REV. The difference from their analysis is that the nested catchments are grouped asymmetrically around the reference point, rather than symmetrically as in the work of Hubbert (1956) and Bear (1972). This is because water flows downhill. The outlets of the subcatchments lie on a flow path from the ridge point to the catchment outlet.

The 'averaged properties' are peaks and volumes of the hydrographs at the respective subcatchment outlets. These are plotted versus the square root of area for the same conditions as Figure 2a in Figure 4. The

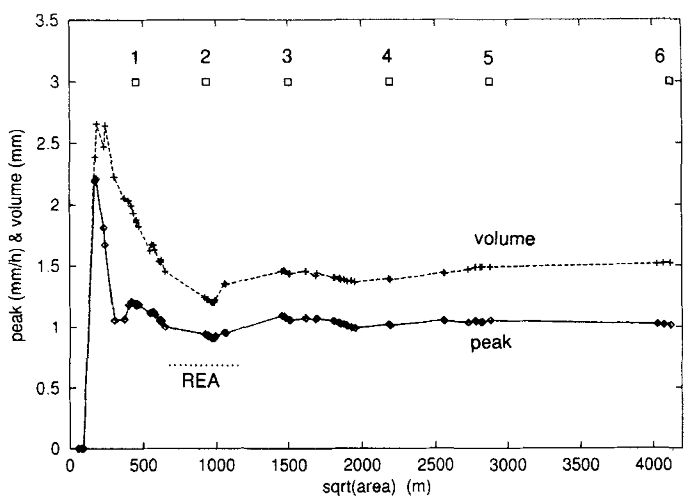


Figure 4. Runoff volume and peak versus subcatchment size for one set of nested catchments as shown in Figure 3. Spatially variable soils and constant precipitation

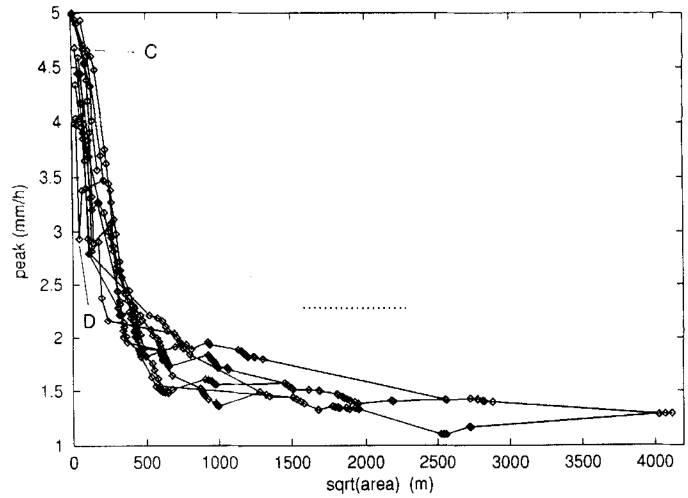


Figure 5. Peak flow versus subcatchment area for 10 sets of nested catchments. Impervious conditions, no channels, Manning's $n = 0{\cdot}4$, one hour rainblock. Precipitation is spatially invariant. The dotted line suggests the approximate size of the REA. C refers to a convergent element and D refers to a divergent element

use of length units rather than area for the x-axis allows closer examination of small scales and is more in keeping with the notion of 'length scales'. Following the approach of Hubbert (1956) and Bear (1972), the REA is described as the scale where $|dq/da|$ is small, with q being the hydrograph peak or volume and a the subcatchment area. For Figure 4 this occurs around 600 to 1200 m as indicated by the dotted line.

It is important to note that Figure 4 shows the results of one set of nested catchments without any filtering. This set of nested catchments can be interpreted as a realization of topography. In the following sections, 10 such sets (realizations) are presented in each graph. Runoff peaks will be used rather than runoff volume because they better capture the dynamics of runoff processes and may be more relevant to practical applications.

EFFECT OF TOPOGRAPHY ON ROUTING AND THE 'CHARACTERISTIC VELOCITY'

This section examines the effect of topography for impervious conditions and spatially invariant precipitation of variable duration.

Figure 5 shows the results for a one hour rainblock of 5 mm/h intensity. As in Figure 4, peak flow at one subcatchment outlet plots as a marker in Figure 5 and nested subcatchments are joined by lines. Peaks are dampened significantly with increasing flow length (subcatchment area). There is substantial scatter between the individual sets of nested catchments, which is largely a consequence of convergent and divergent flow across the hillslopes. Examples of convergent and divergent elements (i.e. subcatchment outlets) are marked in Figure 5 by the letters C and D, respectively. The convergent element is 50 m long and its width decreases from 119 to 56 m downhill. Consequently, the peak flow is significantly higher (4·7 mm/h) than the average of other elements with the same upslope contributing area (3·6 mm/h). The width of the divergent element increases from 7 to 28 m downhill and gives a peak of 2·9 mm/h as opposed to the average of 4·4 mm/h. These peak flows are also affected by the convergence/divergence of the upslope neighbouring elements, which have similar shapes to the elements examined. The curves flatten out at around 2000 m, indicating the length scale of the REA.

Figure 6 illustrates the effect of storm duration using the same rainfall intensity of 5 mm/h. A shorter storm duration of 12 min decreases the length scale of the REA significantly (to 300 m; Figure 6a), whereas an increase in storm duration increases the length scale of the REA to 4000 m or larger (Figure 6b). This indicates some relationship between length scale and storm duration. If we choose the square root of the REA as a characteristic length scale and the storm duration as a characteristic time-scale, a 'characteristic velocity' can

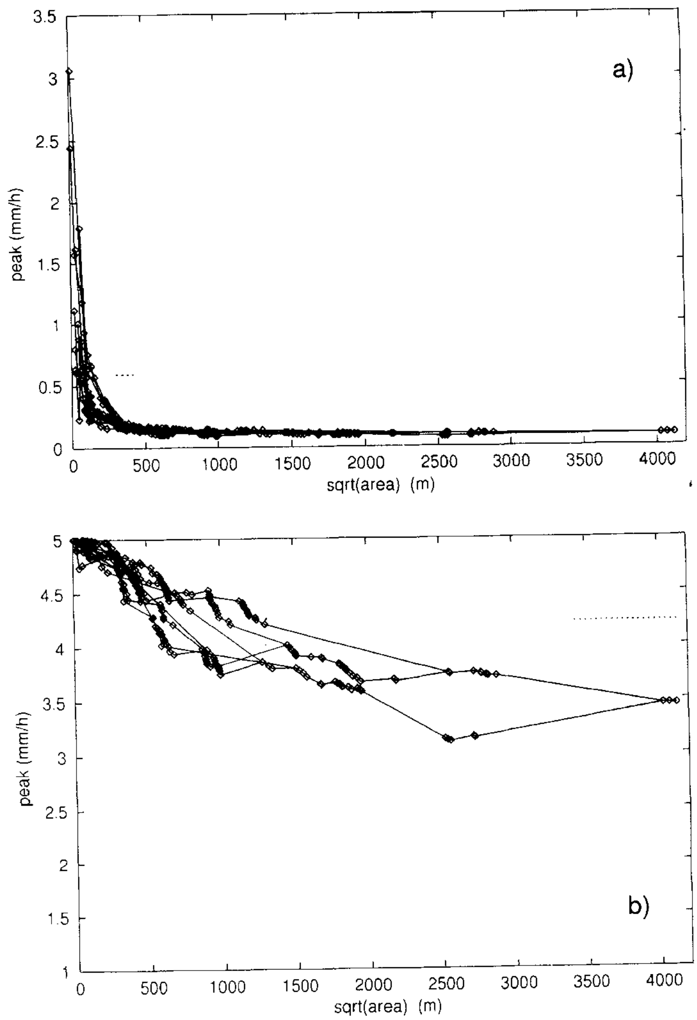


Figure 6. Peak flow versus subcatchment area for 10 sets of nested catchments. Impervious conditions, no channels, Manning's $n=0{\cdot}4$. (a) 12 minute rainblock; and (b) two hour rainblock. Precipitation is spatially invariant

be defined. In the case of the simulation in Figure 6, this is about 2 km/h This characteristic velocity is expected to be either related to wave celerity or particle speed, so a decreased roughness should increase the characteristic velocity V and consequently the characteristic length scale L (i.e. $\sqrt{\text{REA}}$) for a given characteristic time-scale T according to:

$$L = V * T$$

Figure 7 shows this effect. Using a 12 minute storm (as in Figure 6a), but smoother surface conditions (Manning's $n=0{\cdot}04$ rather than $n=0{\cdot}4$) gives a characteristic length scale of about 700 to 1000 m, which corresponds to a characteristic velocity of 3·5 to 5 km/h. This is more than double the value of the rougher case *in Figure 6a* ($n=0{\cdot}4$).

EFFECT OF INFILTRATION

In this section, runoff can be produced by saturation excess and infiltration excess mechanisms. Hydraulic conductivity is spatially variable (with an average of 5 mm/h and a correlation length of 100 m as in Wood

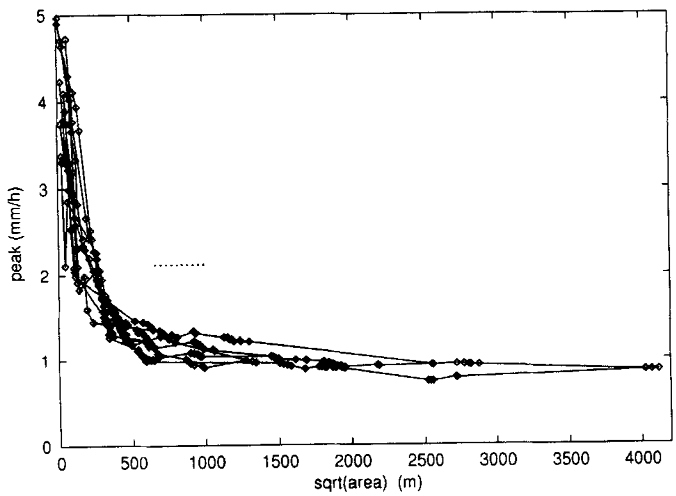


Figure 7. Peak flow versus subcatchment area for 10 sets of nested catchments. Impervious conditions, no channels, 12 minutes rainblock, Manning's $n = 0{\cdot}04$. Precipitation is spatially invariant (same as Figure 6a, but lower roughness)

et al., 1988), but precipitation is spatially invariant. A 12 minute rainblock of 5 mm/h is used and the initial saturation deficit is based on the topographic wetness index.

Figure 8a presents the results for initial conditions as used in Wood *et al.* (1988), where 30% of the area is saturated at the beginning of the storm ($m = 30$ mm and $\bar{S} = 15{\cdot}3$ mm in Wood *et al.*, Appendix A1). Allowing spatially variable infiltration introduces significant scatter between the sets of nested catchments (as compared with Figure 6a) and increases the length scale of the REA to about 800–1200 m. The saturated part of the catchment behaves as an 'impervious surface' for runoff generation, so a larger saturated area is expected to reduce the length scale of the REA, closer to that of the impervious case in Figure 6a. This is borne out by the simulations shown in Figure 8b, which have a 70% saturated area at the beginning of the storm ($m = 30$ mm and $\bar{S} = -34{\cdot}7$ in Wood *et al.*, Appendix A1). Figure 8b indicates a length scale of the REA of about 500–600 m.

EFFECT OF CHANNELS

This section examines the effect of assuming channelized flow in elements which have an upslope contributing area larger than 10 ha, superimposed on the effects of topography and infiltration.

Figure 9a presents the results for a 12 minute rainblock of 5 mm/h intensity. As compared with Figure 8a (sheet flow only), the length scale of the REA is increased. This might be related to the higher conveyance of the channels (due to the geometry and the smoother surface condition), which should increase the characteristic velocity. However, using a one hour rainblock does not increase the length scale of the REA to the same extent (Figure 9b).

EFFECT OF SPATIALLY VARIABLE PRECIPITATION

For ease of comparison with the original work of Wood *et al.* (1988), a one hour rainblock is used in the following analyses. Spatially variable precipitation fields with correlation lengths of 125, 625 m and 2500 m are used. Spatially variable conductivity, infiltration and channel flow are represented as in Figure 9b.

Figure 10a shows results for 10 realizations of precipitation with correlation lengths of 125 m for one set of nested catchments. As compared with Figure 9b (spatially invariant precipitation), peaks are much higher for small catchments. The curves flatten out at about 1000 m. Using a correlation length of 2500 m changes the results dramatically (Figure 10b). There is no single scale where the curves flatten out. Indeed, Figure 10b

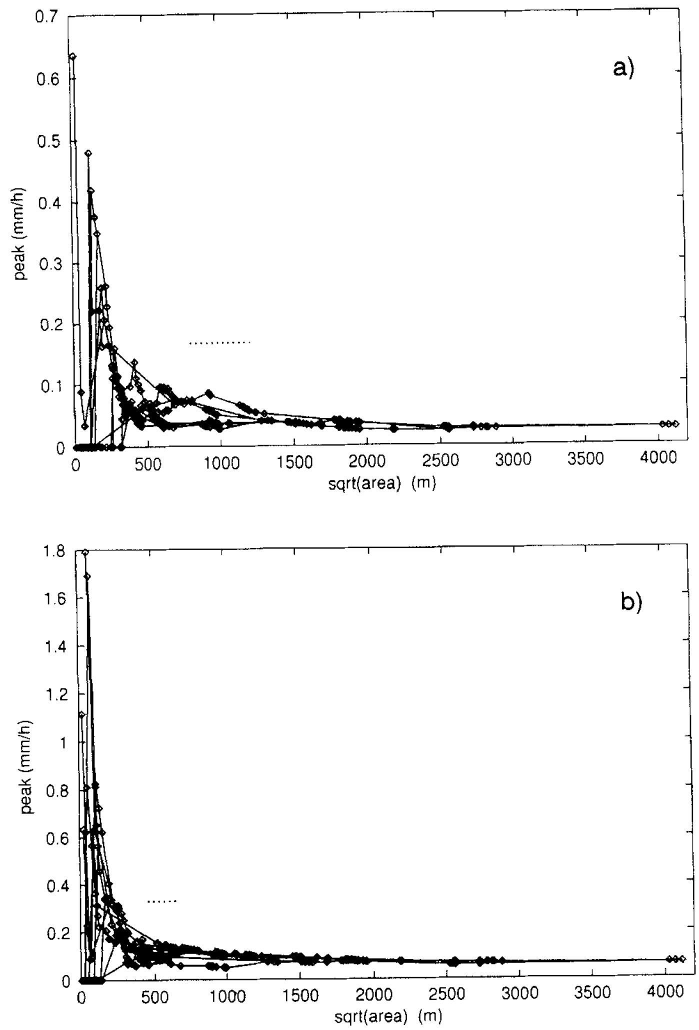
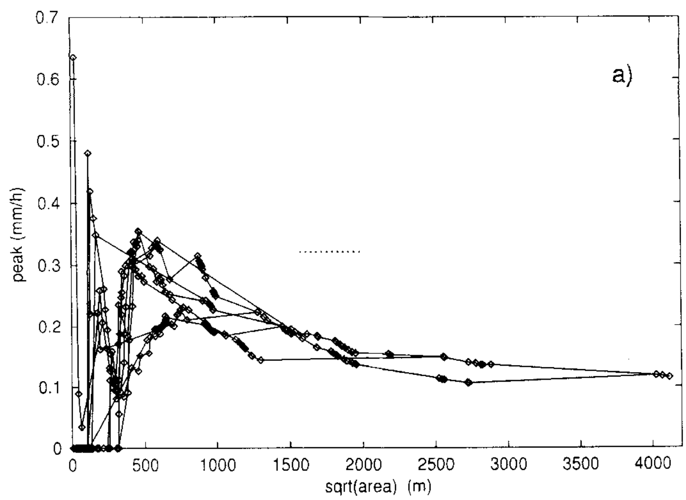


Figure 8. Peak flow versus subcatchment area for 10 sets of nested catchments. Infiltration allowed, no channels, 12 minute rainblock, Manning's $n = 0{\cdot}4$. (a) Initial saturated area is 30%. (b) Initial saturated area is 70%. Precipitation is spatially invariant and soils are spatially variable

suggests that there is both variability at the small scale (< 500 m) and at the large scale (> 2500 m) with little variability in between. It is important to note that each of the curves in Figure 10b should be inspected individually and the minimum in variability appears where $|dq/da|$ is small (dq/da is the change in peak flow with change in subcatchment area for a set of nested subcatchments). Using a correlation length which is between the two extremes (Figure 10c) gives length scales between those of Figure 10a and 10b as would be expected. Similar results are found for other sets of nested catchments (Figure 11).

To illustrate the combined effect on the REA of different assumptions of infiltration and precipitation, two realizations of precipitation are investigated in more detail for one set of nested catchments. Figure 12 shows results of using a correlation length of 2500 m for precipitation and either:

- impervious surface conditions (marked I in Figure 12),
- spatially variable conductivity and topographically defined initial saturation deficit (marked V in Figure 12),
- spatially invariant conductivity and topographically defined initial saturation deficit (marked T in Figure 12),
- spatially invariant conductivity and random initial saturation deficit (marked R in Figure 12).



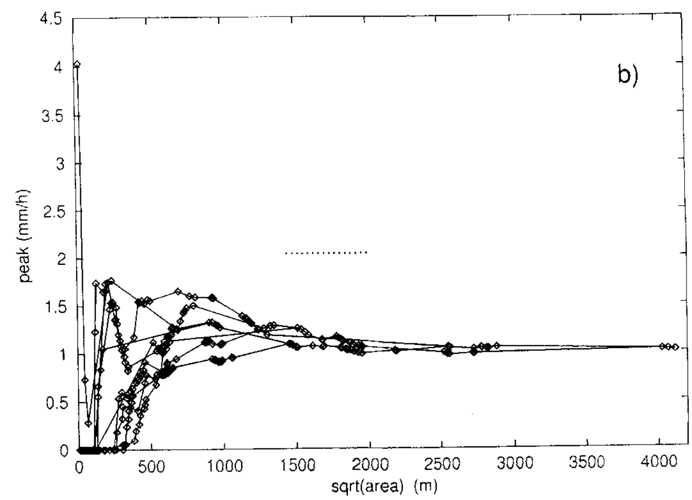


Figure 9. Peak flow versus subcatchment area for 10 sets of nested catchments. Infiltration allowed, 30% initial saturated area, Manning's $n = 0{\cdot}4$, channelized flow for upslope contributing areas > 10 ha. (a) 12 minute rainblock; and (b) one hour rainblock. Precipitation is spatially invariant and soils are spatially variable

The 'random initial saturation deficit' was generated as a stationary random field with the same distribution function and covariance function as the topographically defined initial saturation deficit. Figure 12 indicates that the different assumptions on infiltration markedly affect the magnitude of the runoff peaks. However, the change of flow peaks with scale (and consequently the size of the REA) is dominated by the precipitation pattern. Again, Figure 12 shows small-scale variability below 500 m and large-scale variability above 2000 m.

DISCUSSION

Peak flows in nested catchments are used to examine the effects of various assumptions on the REA.

For the impervious case, storm duration is shown to be dominant. A characteristic velocity is defined which relates the length scale of the REA to storm duration. It is about 2 km/h and suggests a larger REA for longer storm durations. In the context of distributed parameter rainfall–runoff modelling, this finding is not surprising. Essentially, it means that coarser temporal resolutions in precipitation are consistent with larger element sizes. In fact, many rainfall–runoff modelling applications use, say, time-scales of minutes for

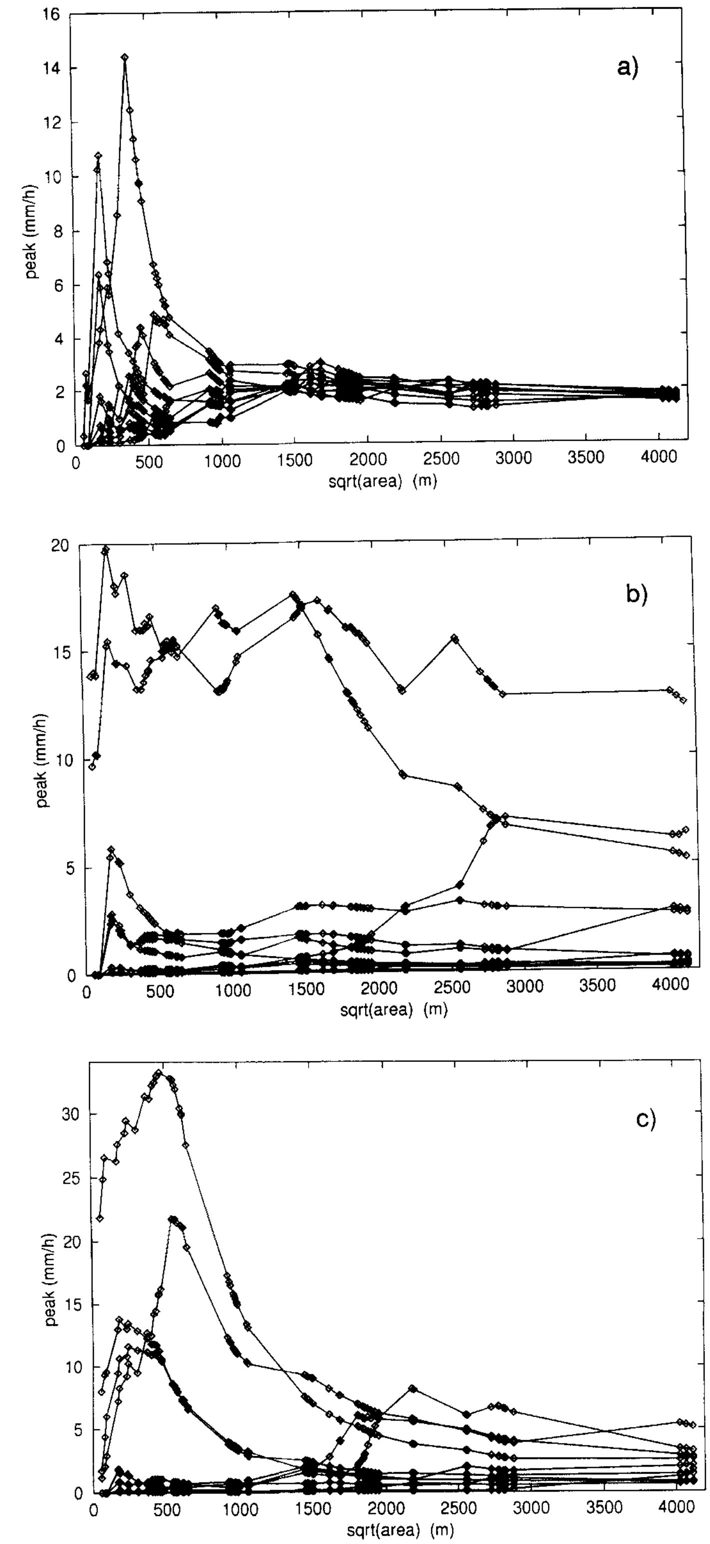


Figure 10. Peak flow versus subcatchment area for the set of nested catchments shown in Figure 3. Infiltration characteristics and channels as in Figure 9. Ten realizations of spatially variable precipitation of one hour duration with correlation lengths of (a) 125 m; (b) 2500 m; and (c) 625 m

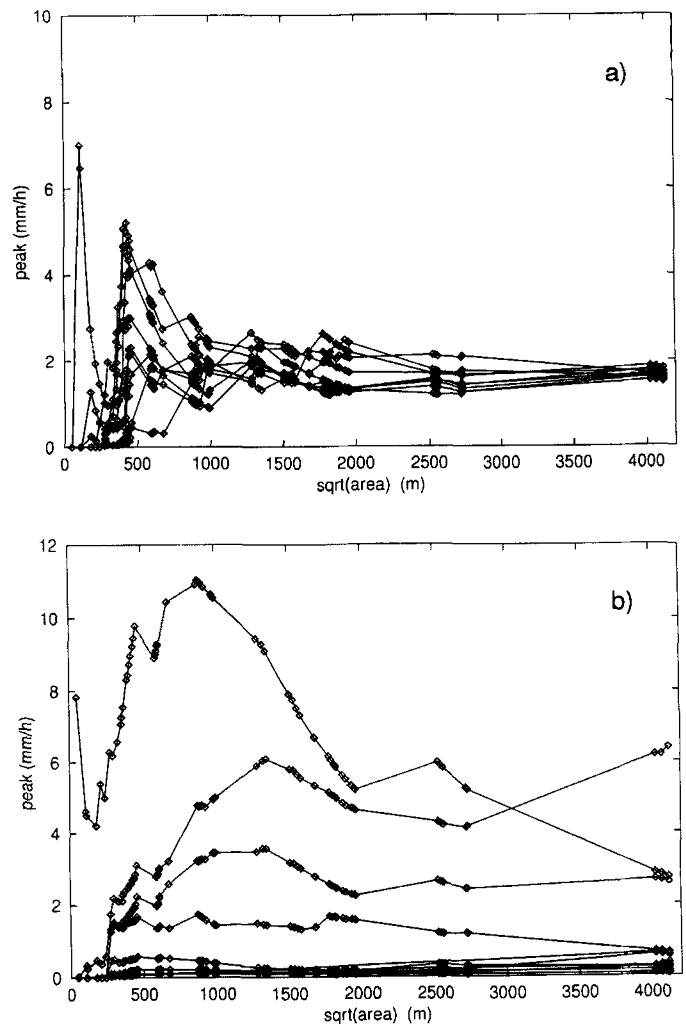


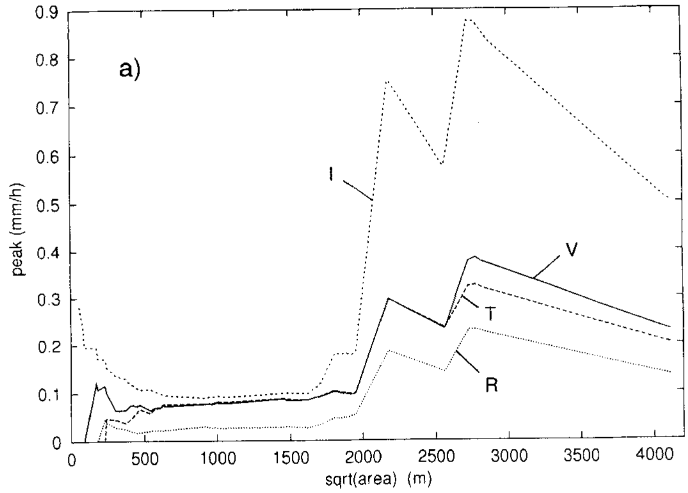
Figure 11. As Figure 10 for another set of nested catchments. Correlation length of precipitation is (a) 125 m; and (b) 2500 m. The highest intensity realization is not shown in (b)

catchments of several hectares, whereas time steps of hours or days are typically used for catchments of tens or hundreds of square kilometres.

Also, the notion of a characteristic velocity is known from meteorology. For many atmospheric processes, the characteristic velocity, relating time and length scales of the processes, is about 10 m/s. Physically, this may correspond to particle velocities (such as the typical vertical velocities in convective cells) or wave celerities (such as the celerities of large scale frontal systems) (Haltiner and Williams, 1980; Fortak, 1982; Stull, 1985). The much smaller characteristic velocities found here (2 km/h = 0·6 m/s) are clearly consistent with the perception of overland flow in catchments.

It is interesting to note that the effect of the characteristic velocity seems not to be related to one particular catchment size. Simulation runs (not shown here) using the Coweeta topography scaled up by a factor of 10 (i.e. area = 1700 km^2), without changing the slopes, gave very similar characteristic velocities to the real topography for the same value of surface roughness.

Another way of thinking about this is to consider the 'area of influence' upstream of any point in the catchment, i.e. the actual contributing area of any point at peak discharge. The results in Figure 6 indicate that, for a given roughness (or wave speed) this area depends on the rainfall duration. That is to be expected as a longer storm will increase the contributing area at peak discharge. If the roughness is reduced, a similar



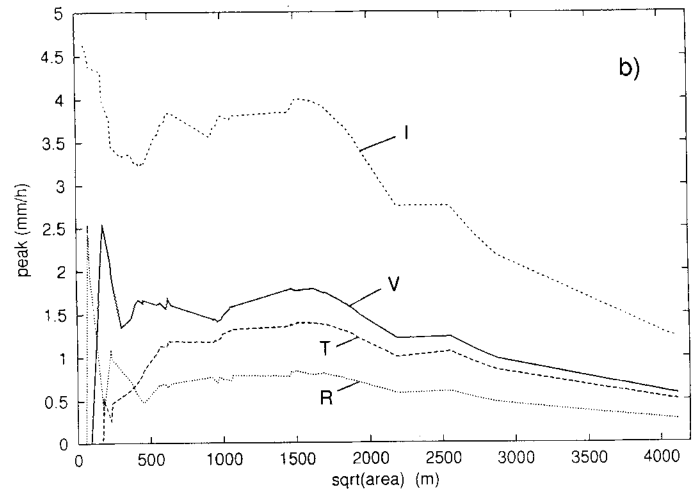


Figure 12. As Figure 10, but two realizations of spatially variable precipitation only [shown in (a) and (b)]; Correlation length of precipitation is 2500 m in both instances. The different graphs relate to different assumptions on infiltration (see text)

effect to that of increasing storm duration is expected as the wave speed increases and therefore the contributing area at peak discharge increases. Figure 7 illustrates this point.

When allowing for infiltration and channel flow, the effect of storm duration shows up less clearly and seems to be largely dominated by the former. Specifically, for the parameter sets used here, the results suggest that the individual effects of infiltration and channel flow are of the same order of magnitude as those of routing a one hour rainblock on an impervious catchment. Therefore, superposition of these processes gives a combined length scale of the REA, which is similar to that of the individual processes. This is about 1000–2000 m.

Introducing spatially variable precipitation clearly dominates over routing and infiltration. It also dominates over storm duration, as results not shown here indicate. The size of the REA seems to be directly related to the correlation length of precipitation. For very large correlation lengths, however, the results indicate both large-scale and small-scale variability, with little variability in between. Figure 12 shows that different assumptions about infiltration do not affect the large-scale variation, but they significantly affect the small-scale variability. Clearly, the small-scale variability is related to the effect of variable soil parameters and topography, whereas the large-scale variability is due to precipitation patterns. A minimum in variance in between indicates that a 'separation of scales' in runoff is present.

The following example is used to show the implications of a separation of scales for distributed catchment modelling. Assume that a distributed rainfall–runoff model for the catchment and the conditions used here is to be set up. It is then prudent to choose the element size of the model somewhere between 500 and 2000 m. The large-scale variability due to precipitation would be represented explicitly as element to element variations. The small-scale variability due to soils and topography would be parameterized as subgrid variability. For example, the parameterization may be performed by a distribution function approach (e.g. Beven, 1991). The advantage of the existence of a separation of scales now is that the same distribution function for parameterizing the small-scale variability can be used for any element size between 500 and 2000 m. If no separation of scales existed in the above example, the catchment might still be modelled with an element size of, say, 2000 m and the small-scale variability could still be parameterized by a distribution function. However, when the element size was then changed to, say, 500 m, the distribution function representing small-scale variability would also have to change.

Following Wood *et al.* (1988), stationary random fields have been used in this study for simulating the spatial variation of rainfall and soil properties. The stationarity assumption (i.e. no large-scale variability) is the reason for most graphs shown in this paper levelling out at a certain scale. The same applies to the case of spatially constant soil properties and precipitation as these also assume no large-scale variability. Most of this paper was therefore concerned with the *size* of the REA and its controls rather than with the *existence* of an REA. However, hydrological processes are rarely stationary. It is now being argued that precipitation may be a multiscale phenomenon (or be fractal) and therefore have no preferred scale (e.g. Lovejoy and Schertzer, 1985; Kumar and Foufoula-Georgiou, 1993). There is also an increasing awareness that geological controls are operative at a multitude of scales (Gelhar, 1986; Wheatcraft and Cushman, 1991). Given the dominant controls of the correlation lengths of soils and precipitation on both the existence and the size of the REA shown in this paper, it may be that, generally, an REA will not exist. Although a spectral gap in runoff (and hence an REA) may exist for a particular catchment it seems that the size of the REA will be highly catchment-specific. Conceptually then, one universal size of an REA or one universal 'optimum element size' seems unlikely, although research will continue to determine if the REA concept is valuable at a regional level.

The existence of a spectral gap in runoff (and hence of an REA) is *not* a requirement for parameterizing subgrid processes. For distributed rainfall–runoff modelling in the future, the question of how to parameterize may become more important than the question of an optimum scale at which to parameterize.

CONCLUSIONS

1. It is suggested that nested catchments be used for testing the existence and size of an REA. The length scale of the REA is defined as the scale where, with increasing area, changes of peak flows (or flow volumes) become small for one set of nested catchments. The existence of an REA is equivalent to the presence of separation of scales in runoff. The approach is consistent with the original definition of the REV of Hubbert (1956) and Bear (1972). It is shown to be capable of detecting a minimum in variability between large- and small-scale processes.
2. For impervious catchments of a given roughness, and spatially invariant precipitation, the size of the REA is shown to be controlled by a characteristic velocity. The characteristic velocity is defined as the ratio of a characteristic length scale (the size of the REA) and a characteristic time-scale (storm duration). The characteristic velocity is about 2 km/h for the conditions examined here.
3. Infiltration and channel flow exhibit less obvious controls on the REA. When allowing for these processes, length scales of the REA are of the order of 1000 m for the Coweeta catchment topography and conditions simulated in this study.
4. Spatially variable precipitation is shown to dominate over other processes considered. The size of the REA is strongly controlled by the correlation length of precipitation. The results indicate a separation of scales in runoff for large correlation lengths of precipitation and the conditions considered here. In this instance, large-scale variability is due to precipitation, whereas small-scale variability is related to soil characteristics and topography.
5. The simulations show that the size of the REA depends on many factors, including storm duration and variability, flow routing and infiltration characteristics. It is therefore apparent that the size of the REA

will be specific to a particular catchment and a particular application. There is no evidence for one universal size of an REA or one universal 'optimum element size' in the context of distributed rainfall–runoff modelling.

6. Following Wood *et al.* (1988), this study has used stationary random fields for simulating the spatial variation of rainfall and soil properties. Multiscale input fields might be more realistic. Given the dominant effect of the correlation lengths on the REA, it seems unlikely that for multiscale controls, a spectral gap (and hence an REA) should exist. This requires further study.
7. The existence of a spectral gap in runoff (and hence of an REA) is not a requirement for parameterizing sub-grid processes. It simply makes any parameterization more generally applicable. There remains a major challenge in deriving parameterizations for sub-grid variability and for distributed rainfall–runoff modelling in the future, the question of how to parameterize may be more important than the question of an optimum scale at which to parameterize.

ACKNOWLEDGEMENTS

The authors thank the Fonds zur Förderung der Wissenschaftlichen Forschung, Vienna, project no. J0699-PHY for financial support. The authors also thank Keith Beven and Ross Woods for their useful comments on the manuscript.

REFERENCES

Bear, J. 1972. *Dynamics of Fluids in Porous Media.* American Elsevier, New York. 764pp.

Bear, J., and Bachmat, Y. 1990. *Introduction to Modeling of Transport Phenomena in Porous Media.* Kluwer, Dordrecht. 533pp.

Beven, K. 1991. 'Scale considerations' in Bowles, D.S. and O'Connell, P.E. (Eds), *Recent Advances in the Modeling of Hydrologic Systems.* Kluwer, Dordrecht. pp.357–371.

Beven, K., and Kirkby, M. 1979. 'A physically based variable contributing area model of basin hydrology', *Hydrol. Sci. Bull.*, **24**, 43–69.

Cushman, J. H. 1987. 'More on stochastic models', *Wat. Resour. Res.*, **23**, 750–752.

deMarsily, G. 1986. *Quantitative Hydrogeology.* Academic Press, London. 440pp.

Fortak, H. 1982. *Meteorologie.* Dietrich Reimer, Berlin. 293pp.

Gelhar, L. W. 1986. 'Stochastic subsurface hydrology from theory to applications', *Wat. Resour. Res.*, **22**, 135S–145S.

Goodrich, D. C. 1990. 'Geometric simplification of a distributed rainfall-runoff model over a range of basin scales', *PhDThesis*, The University of Arizona. 361pp.

Grayson, R. B., Moore, I. D., and McMahon, T. A. 1992. 'Physically based hydrologic modeling, 1. A terrain-based model for investigative purposes', *Wat. Resour. Res.*, **28**, 2639–2658.

Grayson, R. B., Blöschl, G., Barling, R. D., and Moore, I. D. 1993. 'Process, scale and constraints to hydrologic modelling in GIS' in *HydroGIS93, Proc. Vienna Symp., April 1993.* Kovar, K. and Nachtnebel, H. P. (Eds) *IAHS Publ.*, **211**, 83–92.

Grayson, R. B., Blöschl, G., and Moore, I. D. Distributed parameter hydrologic modelling using vector elevation data: THALES and TAPES-C. in Singh, V. P. (Ed), *Computer Models of Watershed Hydrology.* CRC Press, Boca Raton, in press.

Gupta, V. K., Waymire, E., and Rodríguez-Iturbe, I. 1986. 'On scales, gravity and network structure in basin runoff' in Gupta, V.K., Rodríguez-Iturbe, I., and Wood, E.F. (Eds), *Scale Problems in Hydrology*, Reidel, Dordrecht. pp.159–184.

Haltiner, G. J., and Williams, R. T. 1980. *Numerical Prediction and Dynamic Meteorology.* Wiley, Chichester 477pp.

Hubbert, M. K. 1956. 'Darcy's law and the field equations of the flow of underground fluids', *Trans. Am. Inst. Min. Met. Engln.*, **207**, 222–239.

Kirnbauer, R., Blöschl, G., and Gutknecht, D. 1994. 'Entering the era of distributed snow models', *Nordic Hydrol.*, **25**, 1–24.

Kumar, P. and Foufoula-Georgiou, E. 1993. 'A multicomponent decomposition of spatial rainfall fields, 2. Self-similarity in fluctuations', *Wat. Resour. Res.*, **29**, 2533–2544.

Lovejoy, S., and Schertzer, D. 1985. Generalized scale invariance and fractal models of rain. *Wat. Resour. Res.*, **21**, 1233–1250.

Moore, I. D. and Grayson, R. B. 1991. 'Terrain-based catchment partitioning and runoff prediction using vector elevation data', *Wat. Resour. Res.*, **27**, 1177–1191.

Philip, J. R. 1957. 'The theory of infiltration, 1–7', *Soil Sci.*, **83**, **84**, and **85**.

Rosso, R., Peano, A., Becchi, I., and Bemporad, G. A. 1994. 'Advances in distributed hydrology' in *Proceedings of a Workshop held in Bergamo, Italy, 25–26 June 1992.* Water Resources Publications, Littleton, CO, 416 pp.

Sivapalan, M. and Viney, N. R. 1994. 'Large scale catchment modelling to predict the effects of land use and climate', *Water, J. Austr. Water and Wastewater Assoc.*, **21**, 33–37.

Stull, R. B. 1985. 'Predictability and scales of motion', *Bull. Am. Meteorol. Soc.*, **66**, 432–436.

Stull, R. B. 1988. *An Introduction to Boundary Layer Meteorology.* Kluwer, Dordrecht. 666pp.

Wheatcraft, S. W. and Cushman, J. H. 1991. 'Hierarchical approaches to transport in heterogeneous porous media', *Rev. Geophys., Suppl.*, 263–269.

Wood, E. F., Sivapalan, M., Beven, K., and Band, L. 1988. 'Effects of spatial variability and scale with implications to hydrologic modeling', *J. Hydrol.*, **102**, 29–47.

Wood, E. F., Sivapalan, M., and Beven, K. 1990. 'Similarity and scale in catchment storm response', *Rev. Geophys.*, **28**, 1–18.