Benchmark case: Cat's eye

Description

The cat's eye geometry has been analysed in a number of papers in recent years, cf. [1–7]. As shown in Figure 1, it is a sphere of radius $R = 1.0$ m with the positive octant cut out. The problem is suited for radiation and scattering. Herein, we investigate the radiation problem of the vibrating surface where it coincides with the spherical one. The plain surfaces of the missing octant are assigned a zero admittance.

The idea of investigation of a cat's eye structure with BEM is essentially based on three considerations:

- This radiator allows construction of a smooth solution that will make it easy to identify solution failures caused by the ill–conditioning of the integral operator for techniques that solve the Helmholtz equation in an integral formulation, i.e. the irregular frequencies.
- The cat's eye structure is a more complicated shape than a sphere. Hence, the solution is expected to expose more irregular frequencies in a BEM solution than the sphere. This is clearly shown in paper [3].
- For finite and boundary element solutions, sound pressure evaluation at the centre point can be challenging.

It is another advantage of this model that it easily allows to construct hierarchical meshes [4].

The sound pressure at a backside point, i.e. (x,y,z)=$(-\frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}})$ of the radiator is a smooth function asymptotically approaching the solution of a pulsating sphere [3]. This smooth function makes it easy to identify frequencies where the solution of numeral methods fail.

Since the cat's eye as a scatterer shows multiple reflections of an incoming wave [1, 4], it may be suited for methods which encounter problems in such cases. Furthermore, the sound pressure solution in the centre of the cut-out octant may require mesh refinement due to the large gradient in this region.

Mechel [5] has presented an analytical solution for the cat's eye model. It will be interesting to compare his results which are based on Fourier series and the results based on FEM, BEM and other numerical methods.

Name	Cat's eye
Field	Linear Acoustics
Code	coded UN3–1|RF or UN3–1|SF
Categories	
Bounded or Unbounded problems	Unbounded
Dimensionality of the case	3D
Scattering or Radiation problem	Scattering and Radiation
Time–domain or Frequency–domain problem	Frequency domain
Description	
PDE	Helmholtz Equation
Geometry	Sphere with radius $R = 1$ m. Octant in positive x, y, and z coordinates cut out. see Fig. 1
Propagation medium	Air ($\rho = 1.3$ kg/m^3, $c = 340$ m/s)
BCs	$Z = \infty$ at boundaries
Source	$v_n = 1$ m/s at spherical surface of the sphere.
Receiver	P_1 at $(x, y, z) = (-\frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}})$
	P_2 at $(x, y, z) = (-\frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}}, \frac{R}{\sqrt{3}})$
	P_3 at $(x, y, z) = (-\frac{R}{\sqrt{3}}, \frac{R}{\sqrt{3}}, \frac{R}{\sqrt{3}})$
	P_4 at $(x, y, z) = (\frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}})$
	P_5 at $(x, y, z) = (\frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}}, \frac{R}{\sqrt{3}})$
	P_6 at $(x, y, z) = (\frac{R}{\sqrt{3}}, \frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}})$
	P_7 at $(x, y, z) = (-\frac{R}{\sqrt{3}}, \frac{R}{\sqrt{3}}, -\frac{R}{\sqrt{3}})$
	P_8 at $(x, y, z) = (-\frac{2R}{\sqrt{3}}, -\frac{2R}{\sqrt{3}}, -\frac{2R}{\sqrt{3}})$
	P_9 at $(x, y, z) = (-\frac{10R}{\sqrt{3}}, -\frac{10R}{\sqrt{3}}, -\frac{10R}{\sqrt{3}})$
	P_{10} at $(x, y, z) = (\frac{R}{10}, \frac{R}{10}, \frac{R}{100})$
	P_{11} at $(x, y, z) = (\frac{R}{2}, \frac{R}{2}, \frac{R}{20})$
	P_{12} at $(x, y, z) = (0, 0, 0)$
Quantity to compute	Acoustic pressure
	Frequencies for computation:
	1 to 1700 Hz, with 1 Hz increment

References

[1] S. N. Makarov and M. Ochmann. An iterative solver for the Helmholtz integral equation for high frequency scattering. *Journal of the Acoustical Society of America*, 103(2):742–750, 1998.

[2] S. Marburg. The Burton and Miller method: Unlocking another mystery of its coupling parameter. *Journal of Computational Acoustics*, page 20 pages, 2015. (under review).

[3] S. Marburg and S. Amini. Cat's eye radiation with boundary elements: Comparative study on treatment of irregular frequencies. *Journal of Computational Acoustics*, 13(1):21–45, 2005.

[4] S. Marburg and S. Schneider. Performance of iterative solvers for acoustic problems. Part i: Solvers and effect of diagonal preconditioning. *Engineering Analysis with Boundary Elements*, 27(7):727–750, 2003.

[5] F. P. Mechel. The cat's eye model. *Acta Acustica united with Acustica*, 91(4):653–660, 2005.

[6] S. Schneider. Application of fast methods for acoustic scattering and radiation problems. *Journal of Computational Acoustics*, 11(3):387–401, 2003.

[7] S. Schneider and S. Marburg. Performance of iterative solvers for acoustic problems. Part ii: Acceleration by ilu–type preconditioner. *Engineering Analysis with Boundary Elements*, 27(7):751–757, 2003.

Geometrical details

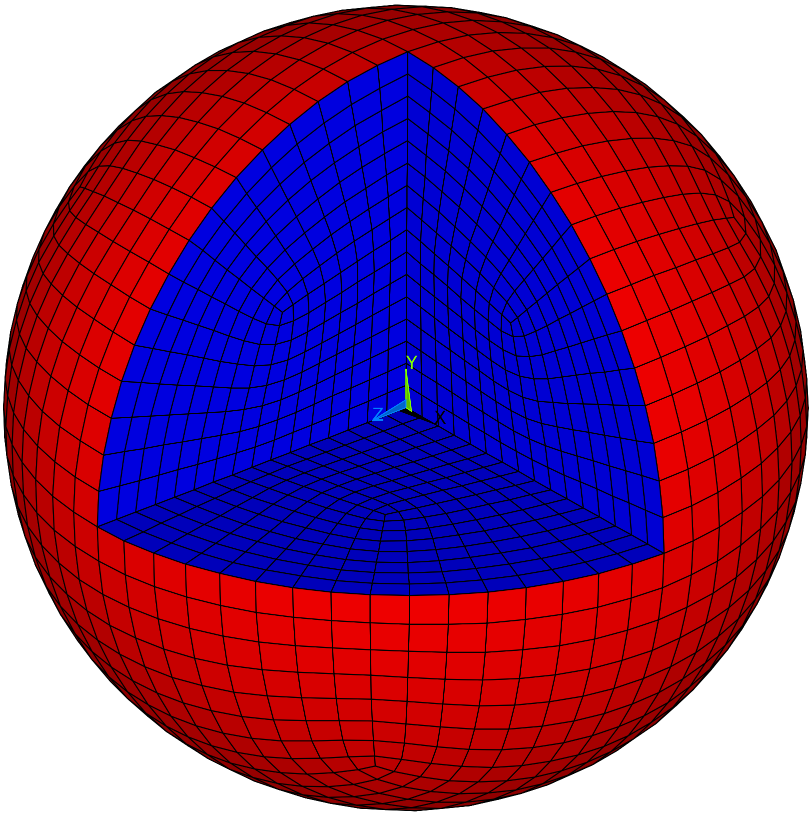

Figure 1: Benchmark case 'Cat's eye', coded UN3–1|RF or UN3–1|SF.