Quantum gravitational states of ultracold neutrons as a tool for probing of beyond-Riemann gravity

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A B S T R A C T

We analyze a possibility to probe beyond-Riemann gravity (BRG) contributions, introduced by Kostelecký and Li (see Kostelecký and Li (2021) [6]) on the basis of the Effective Field Theory (EFT) by Kostelecký (2004) [4]. We carry out such an analysis by calculating the BRG contributions to the transition frequencies of the quantum gravitational states of ultracold neutrons (UCNs). These states are being used for a test of interactions beyond the Standard Model (SM) and General Relativity (GR) in the *q*BOUNCE experiments. We improve by order of magnitude some constraints obtained by Kostelecký and Li (2021) [6].

1. Introduction

Nowadays the effective field theories (EFT) are a powerful tool for the analysis of the Nature [1–3]. The general EFT by Kostelecký [4], based on the General Gravity (GR) coupled to the Standard Model (SM), has been extended by Kostelecký and Li [5,6] by the contributions of interactions, caused by beyond-Riemann gravity (BRG). These contributions are closely overlapped with the contributions of interactions violating Lorentz-invariance [7–13]. In [6] Kostelecký and Li have proposed to investigate the BRG as well as Lorentz-invariance violation (LV) contributions to the energy spectrum and transition frequencies of the quantum gravitational states of ultracold neutrons (UCNs).

This paper is addressed to the analysis of the BRG and LV contributions to the energy spectrum and transition frequencies of the quantum gravitational states of ultracold neutrons (UCNs). For the analysis of these problems we follow [14]. In [14] we have calculated the LV contributions to the energy spectrum and transition frequencies of the quantum gravitational states of UCNs, caused by the effective low-energy potential, derived by Kostelecký and Lane [15] in the frame work of the Standard Model Extension (SME) [7,8] (see also [4]) by using the Foldy-Wouthuysen transformations [16,17].

The paper is organized as follows. In section 2 we discuss the effective low-energy potential, derived by Kostelecký and Li [6] for the analysis of BRG interactions in the terrestrial laboratories. We define such a potential in the standard coordinate frame related to the laboratory at the Institut Laue Langevin (ILL) in Grenoble. We specify the BRG and LV contributions to the phenomenological coupling constants of this potential. We adduce the wave functions of the quantum gravitational states of polarized and unpolarized UCNs. In section 3 we calculate the BRG and LV contributions to the energy spectrum and transition frequencies of the quantum gravitational states of polarized and unpolarized UCNs. Using the current experimental sensitivity of the *q*BOUNCE experiments we give some estimates of the phenomenological constants of the BRG and LV interactions. In section 4 we discuss the obtained results and perspectives of further investigations of the BRG and LV interactions by using the quantum gravitational states of UCNs.

2. Effective non–relativistic potential of beyond-Riemann gravity interactions

For the experimental analysis of the BRG and LV interactions in the terrestrial laboratories by using the quantum gravitational states of UCNs Kostelecký and Li propose to use the following Hamilton operator [6]

$$H = H_0 + \Phi_{RG} + \Phi_{BRG} = \frac{\vec{p}^{\,2}}{2m} - m\vec{g}\cdot\vec{z} + \Phi_{nRG} + \Phi_{nBRG}, \tag{1}$$

where the first two terms are the operators of the UCN energy and the Newtonian gravitational potential of the gravitational field of the Earth, respectively, with the gravitational acceleration $\vec{g}$ such as $\vec{g}\cdot\vec{z} = -gz$ [6]. Then, Φ_{nRG} is the effective low-energy potential of the neutron-gravity interaction, calculated to next-to-leading order in the large neutron mass m expansion and related to the contribution of Riemann gravity. It is equal to [6]

$$\Phi_{nRG} = \frac{3}{4m}(\vec{\sigma}\times\vec{p})\cdot\vec{g} - \frac{3}{4m}(\vec{p}^{\,2}\vec{g}\cdot\vec{z} + \vec{g}\cdot\vec{z}\,\vec{p}^{\,2}). \tag{2}$$

In turn, the potential Φ_{nBRG} describes the BRG and LV contributions to neutron-gravity interactions

$$\Phi_{nBRG} = H_\phi + H_{\sigma\phi} + H_g + H_{\sigma g}, \tag{3}$$

where the operators H_j for $j = \phi, \sigma\phi, g$ and σg are equal to [6]

$$H_\phi = (k_\phi^{NR})_n \vec{g}\cdot\vec{z} + (k_{\phi p}^{NR})_n^j \frac{1}{2}\Big(p^j(\vec{g}\cdot\vec{z}) + (\vec{g}\cdot\vec{z})p^j\Big) + (k_{\phi pp}^{NR})_n^{jk}\frac{1}{2}\Big(p^j p^k(\vec{g}\cdot\vec{z}) + (\vec{g}\cdot\vec{z})p^j p^k\Big),$$

$$H_{\sigma\phi} = (k_{\sigma\phi}^{NR})_n^j \sigma^j(\vec{g}\cdot\vec{z}) + (k_{\sigma\phi p}^{(NR)})_n^{jk}\frac{1}{2}\sigma^j\Big(p^k(\vec{g}\cdot\vec{z}) + (\vec{g}\cdot\vec{z})p^k\Big) + (k_{\sigma\phi pp}^{NR})_n^{jk\ell}\frac{1}{2}\sigma^j\Big(p^k p^\ell(\vec{g}\cdot\vec{z}) + (\vec{g}\cdot\vec{z})p^k p^\ell\Big),$$

$$H_g = (k_g^{(NR)})_n^j g^j + (k_{gp}^{(NR)})_n^{jk} p^j g^k + (k_{gpp}^{(NR)})_n^{jk\ell} p^j p^k g^\ell,$$

$$H_{\sigma g} = (k_{\sigma g}^{NR})_n^{jk}\sigma^j g^k + (k_{\sigma gp}^{NR})_n^{jk\ell}\sigma^j p^k g^\ell + (k_{\sigma gpp}^{NR})_n^{jk\ell m}\sigma^j p^k p^\ell g^m. \tag{4}$$

The non-relativistic Hamilton operator Eq. (1) is written in the coordinate system shown in Fig. 1, where m is the neutron mass, $\vec{z}$ is a radius-vector of a position of an UCN on the z-axis, $\vec{p} = -i\nabla$ is a 3-momentum of an UCN and $\vec{\sigma}$ is the Pauli 2×2 matrix of the UCN spin [17]. The coefficients $(k_\phi^{NR})_n$, $(k_{...}^{NR})_n^j$, $(k_{...}^{NR})_n^{jk}$, $(k_{...}^{NR})_n^{jk\ell}$, and $(k_{...}^{NR})_n^{jk\ell m}$ define the BRG and LV contributions, which can be tested in experiments with neutrons [6] in the following way.

The system of a Schrödinger quantum particle with mass m bouncing in a linear gravitational field is known as the quantum bouncer [19–22]. Above a horizontal mirror, the linear gravity potential leads to discrete energy eigenstates of a bouncing quantum particle. An UCN, bound on a reflecting mirror in the gravity potential of the earth, can be found in a superposition of quantum gravitational energy eigen-states. The quantum gravitational states of UCNs have been verified and investigated [28–31] at the UCN beamline PF2 at the Institute Laue-Langevin (ILL), where the highest UCN flux is available worldwide. The qBOUNCE collaboration develops a gravitational resonant spectroscopy (GRS) method [40], which allows to measure the energy difference between quantum gravitational states with increasing accuracy. Recent activities [34], and a summary can be found in [33]. The energy difference can be related to the frequency of a mechanical modulator, in analogy to the Nuclear Magnetic Resonance technique, where the Zeeman energy splitting of a magnetic moment in an outer magnetic field is connected to the frequency of a radio-frequency field. The frequency range in GRS used so far is in the acoustic frequency range between 100 and 1000 Hz. The quantum gravitational states of UCNs have peV energy, on a much lower energy scale compared to other bound quantum systems. Any gravity-like potential or a deviation from Riemann gravity would shift these energy levels [35,41,44,50] and an observation would point to new physical understanding.

Our choice of the laboratory frame is related to the following. Indeed, the qBOUNCE experiments are being performed in the laboratory at Institut Laue Langevin (ILL) in Grenoble. The ILL laboratory is fixed to the surface of the Earth in the northern hemisphere. Following [9–13] (see also [6,14]) we choose the ILL laboratory or the standard laboratory frame with coordinates (t, x, y, z), where the x, y and z axes point south, east and vertically upwards, respectively, with northern and southern poles on the axis of the Earth's rotation with the Earth's sidereal frequency $\Omega_\oplus = 2\pi/(23\,\text{hr}\,56\,\text{min}\,4.09\,\text{s} = 7.2921159\times 10^{-5}\,\text{rad/s}$. The position of the ILL laboratory on the surface of the Earth is determined by the angles χ and ϕ, where $\chi = 44.83333^0$ N is the colatitude of the laboratory and ϕ is the longitude of the laboratory measured to east with the value $\phi = 5.71667^0$ E [18]. The beam of UCNs moves from south to north antiparallel to the x–direction and with energies of UCNs quantized in the z–direction. The gravitational acceleration in Grenoble is $g = 9.80507\,\text{m/s}^2$ [14,18]. Following [14] we may neglect the Earth's rotation assuming that the ILL laboratory frame is an inertial one.

2.1. The analysis of contributions to the effective low-energy potential Φ_{nBRG} in Eq. (3) violating of Lorentz-invariance

Before we proceed to calculating the contributions of the effective potential Eq. (3) to the energy spectrum and transition frequencies of the quantum gravitational states of UCNs we would like to compare the potential Φ_{nBRG} with the effective low-energy potential Φ_{nLV} of the LV interactions (see Eq. (4) in [14]), calculated in [15]. The effective low-energy potential Φ_{nLV} is equal to

$$\Phi_{nLV} = \Big(-b_\ell^n + md_{\ell 0}^n - \frac{1}{2}m\varepsilon_{\ell kj}g_{kj0}^n + \frac{1}{2}\varepsilon_{\ell kj}H_{kj}^n\Big)\sigma_\ell + \frac{1}{m}\Big(-a_j^n + m(c_{0j}^n + c_{j0}^n) + me_j^n\Big)p_j$$

$$+ \frac{1}{m}\Big(b_0^n\delta_{j\ell} - m(d_{\ell j}^n + d_{00}^n\delta_{\ell j}) - \frac{1}{2}m\varepsilon_{\ell km}(g_{mkj}^n + 2g_{m00}^n\delta_{jk}) - \varepsilon_{j\ell k}H_{k0}^n\Big)p_j\sigma_\ell - \frac{1}{2m}\Big(2c_{jk}^n + c_{00}^n\delta_{jk}\Big)p_j p_k$$

$$+ \Big[\frac{1}{4m}\Big((4d_{0j}^n + 2d_{j0}^n - \varepsilon_{jmn}g_{mn0}^n)\delta_{k\ell} + \varepsilon_{\ell mn}g_{mn0}^n\delta_{jk} - 2\varepsilon_{j\ell m}(g_{m0k}^n + g_{mk0}^n)\Big)$$

$$+ \frac{1}{2m^2}\Big((-b_j - \frac{1}{2}\varepsilon_{jmn}H_{mn})\delta_{k\ell} + b_\ell\delta_{jk}\Big)\Big]p_j p_k\sigma_\ell. \tag{5}$$

The LV contributions to the energy spectrum and transition frequencies of the quantum gravitational states of UCNs, induced by the effective low-energy potential Eq. (5), have been calculated in [14].
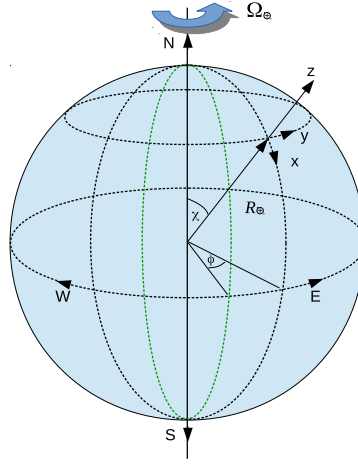
Fig. 1. The position of the ILL laboratory of the *q*BOUNCE experiments on the surface of the Earth.

From Eq. (4) one may see that the effective low-energy interactions H_ϕ, $H_{\sigma\phi}$ and $(k_g^{(NR)})_n^j g^j$ in H_g are new in comparison with Eq. (5). So this means that the coefficients or the phenomenological coupling constants in these interactions are induced by the BRG interactions. Of course, these terms are able to contain the LV contributions (see Table III of Ref. [6]) but such contributions should not dominate in them.

In turn, the effective low-energy neutron-gravity interactions, defined by H_g and $H_{\sigma g}$, have the structure of the effective low-energy potential Φ_{nLV} in Eq. (5). From the comparison we may write the following relations

$$
(k_{\sigma g}^{NR})_n^{jk} g^k = -b_j^n + m d_{j0}^n - \frac{1}{2} m \, \varepsilon_{jk\ell} g_{k\ell 0}^n + \frac{1}{2} \, \varepsilon_{jk\ell} H_{k\ell}^n + \dots,
$$

$$
(k_{gp}^{NR})_n^{jk} g^k = \frac{1}{m} \left(-a_j^n + m(c_{0j}^n + c_{j0}^n) + m e_j^n \right) + \dots,
$$

$$
(k_{\sigma gp}^{NR})_n^{jk\ell} g^\ell = \frac{1}{m} \left(b_0^n \delta_{jk} - m(d_{kj}^n + d_{00}^n \delta_{kj}) - \frac{1}{2} m \, \varepsilon_{jtm} \left(g_{mtk}^n + 2 g_{m00}^n \delta_{jk} \right) - \varepsilon_{jkm} H_{m0}^n \right) + \dots,
$$

$$
(k_{gpp}^{NR})_n^{jk\ell} g^\ell = -\frac{1}{2m} \left(2 c_{jk}^n + c_{00}^n \delta_{jk} \right) + \dots,
$$

$$
(k_{\sigma gpp}^{NR})_n^{jk\ell m} g^m = \frac{1}{4m} \left(\left(4 d_{0j}^n + 2 d_{j0}^n - \varepsilon_{jmn} g_{mn0}^n \right) \delta_{k\ell} + \varepsilon_{\ell mn} g_{mn0}^n \delta_{jk} - 2 \, \varepsilon_{j\ell m} \left(g_{m0k}^n + g_{mk0}^n \right) \right)
$$
$$
+ \frac{1}{2m^2} \left(\left(-b_j - \frac{1}{2} \, \varepsilon_{jmn} H_{mn} \right) \delta_{k\ell} + b_\ell \delta_{jk} \right) + \dots,
$$

$$(6)$$

where ellipses denote the BRG contributions of neutron-gravity interactions (see Table III in Ref. [6]).

2.2. The rotation-invariant effective low-energy potential $\Phi_{nBRG}^{(RI)}$ for the qBOUNCE experiments

For the experimental analysis of the BRG as well as LV interactions by the quantum gravitational states of UCNs Kostelecký and Li proposed to use the following rotation-invariant (RI) effective low-energy potential [6]

$$
\Phi_{BRG}^{(RI)} = (k_\phi^{NR})_n \, \vec{g} \cdot \vec{z} + (k_{\sigma g}^{NR})_n' \, \vec{\sigma} \cdot \vec{g} + (k_{\sigma g\phi}^{NR})_n' \, (\vec{\sigma} \times \vec{p}) \cdot \vec{g} + \frac{1}{2} (k_{\sigma \phi p}^{NR})_n' \left((\vec{\sigma} \cdot \vec{p})(\vec{g} \cdot \vec{z}) + (\vec{g} \cdot \vec{z})(\vec{\sigma} \cdot \vec{p}) \right)
$$
$$
+ (k_{\sigma gpp}^{NR})_n' \, \vec{p}^{\,2} \vec{\sigma} \cdot \vec{g} + (k_{\sigma gpp}^{NR})_n'' (\vec{\sigma} \cdot \vec{p})(\vec{g} \cdot \vec{p}).
$$

$$(7)$$

In this expression the coefficients with primes denote suitably normalized irreducible representations of the rotation group obtained from the nonrelativistic coefficients in Eq. (4) (see [6]). Then, according to Kostelecky and Li [6], the effective low-energy potential Eq. (4) is of interest for certain experimental applications, in part because the rotation invariance ensures that all terms take the same form at leading order when expressed either in the laboratory frame or the Sun-centered frame. The latter implies, for example, no leading-order dependence on the local sidereal time or laboratory colatitude in experimental signals for these terms [6].

Since the effective neutron-gravity interactions, proportional to $(k_{\sigma g\phi}^{(NR)})_n'$ and $(k_{\sigma \phi p}^{(NR)})_n'$, do not contribute to the energy spectrum of the quantum gravitational states of UCNs, the possible contributions should be proportional to the coefficients $(k_\phi^{NR})_n$, $(k_{\sigma g}^{NR})_n'$, $(k_{\sigma gpp}^{NR})_n'$ and $(k_{\sigma gpp}^{NR})_n''$, respectively. According to our discussion above, the coefficient $(k_\phi^{NR})_n$ is caused by the BRG interactions, whereas the coefficients $(k_{\sigma g}^{NR})_n'$, $(k_{\sigma gpp}^{NR})_n'$ and $(k_{\sigma gpp}^{NR})_n''$ should be saturated by the LV ones [14].

2.3. Wave functions and energy spectrum of quantum gravitational states of UCNs

The non-perturbed quantum gravitational states of UCNs obey the Schrödinger-Pauli equation [19]

$$i \frac{\partial \Psi^{(0)}_{\vec{p}_\perp k\sigma}(t, \vec{r})}{\partial t} = \left(\frac{\vec{p}^2}{2m} + mgz \right) \Psi^{(0)}_{\vec{p}_\perp k\sigma}(t, \vec{r}) \tag{8}$$

where $\vec{r} = \vec{z} + \vec{r}_\perp$ is a radius-vector of a position of an UCN with $\vec{r}_\perp = (x, y)$, the wave function $\Psi_{\vec{p}_\perp k\sigma}(t, \vec{r})$ is equal to $\Psi_{\vec{p}_\perp k\sigma}(t, \vec{r}) = \Psi^{(0)}_{k\sigma}(z) e^{i\vec{p}_\perp \cdot \vec{r}_\perp - i(E_{\vec{p}} + E_k^{(0)})t}/2\pi = |\vec{p}_\perp k\sigma\rangle$, $\vec{p}_\perp$ and $E_{\vec{p}_\perp} = \vec{p}_\perp^2/2m \sim 10^{-7}$ eV (for $|\vec{p}_\perp| \sim 24$ eV or $v_\perp \sim 7$ m/s) are the momentum and kinetic energy of UCNs. Below all BRG and LV contributions will be calculated at $\vec{p}_\perp = 0$ [14], and $k = 1, 2, \ldots$ is the principal quantum number [19]. The wave function $\Psi_{\vec{p}_\perp k\sigma}(t, \vec{r})$ is normalized by $\langle \sigma' k' \vec{p}_\perp' | \vec{p}_\perp k\sigma \rangle = \delta^{(2)}(\vec{p}_\perp' - \vec{p}_\perp) \delta_{k'k} \delta_{\sigma'\sigma}$ [23,24]. Then, $\Psi^{(0)}_{k\sigma}(z) = \psi^{(0)}_k(z) \chi_\sigma = |k\sigma\rangle$ is a two–component spinorial wave function of UCNs in the k–gravitational state with the binding energy $E_k^{(0)}$, and in a spin eigenstate χ_σ with $\sigma = \uparrow$ or $\downarrow$. They are normalized by $\langle \sigma' k' | k\sigma \rangle = \delta_{k'k} \delta_{\sigma'\sigma}$. The wave functions $\psi^{(0)}_k(z)$ are given by [19]

$$\psi^{(0)}_k(z) = \frac{\mathrm{Ai}(\xi - \xi_k)}{\sqrt{\ell}\, |\mathrm{Ai}'(-\xi_k)|} e^{i\alpha} \quad , \quad \int_0^\infty dz\, \psi^{(0)*}_{k'}(z) \psi^{(0)}_k(z) = \delta_{k'k}, \tag{9}$$

where $\xi = z/\ell$, $\mathrm{Ai}(\xi - \xi_k)$ is the Airy-function and $\mathrm{Ai}'(-\xi_k)$ its derivative at $z = 0$ [19,25–27], $e^{i\alpha}$ is a constant complex factor, $\ell = (2m^2 g)^{-1/3} = 5.88$ μm is the scale of the quantum gravitational states of UCNs and ξ_k is the root of the equation $\mathrm{Ai}(-\xi_k) = 0$, cased by the boundary condition $\psi^{(0)}_k(0) = 0$ [19]. The latter defines the energy spectrum of the quantum gravitational states of UCNs $E_k^{(0)} = E_0 \xi_k$ for $k = 1, 2, \ldots$ with $E_0 = mg\ell = \sqrt[3]{mg^2/2} = 0.6016$ peV [14]. Experimentally the quantum gravitational states of UCNs have been investigated in [28–30].

The wave functions $\Psi^{(0)}_{k\sigma}(z) = \psi^{(0)}_k(z) \chi_\sigma$ describe the quantum gravitational states of polarized UCNs, whereas for the quantum gravitational states of unpolarized UCNs the wave functions are given by [14]

$$\Psi^{(0)}_k(z) = \psi^{(0)}_k(z) c_\uparrow \chi_\uparrow + \psi^{(0)}_k(z) c_\downarrow \chi_\downarrow, \tag{10}$$

where the coefficients $c_\uparrow$ and $c_\downarrow$ are normalized by $|c_\uparrow|^2 + |c_\downarrow|^2 = 1$ and determine the probabilities to find an UCN in the k–quantum gravitational state with spin *up* and *down*, respectively. The quantum gravitational states of UCNs with the wave function Eq. (6) are 2–fold degenerate [23,24].

3. The BRG and LV contributions to the energy spectrum and transition frequencies of quantum gravitational states of UCNs

The energy spectrum of the quantum gravitational states of polarized UCNs with the RG, BRG and LV corrections are defined by the integrals

$$E_{k\sigma} = \langle \sigma k | \mathrm{H} | k\sigma \rangle = \int_0^\infty dz\, \Psi^{(0)\dagger}_{k\sigma}(z) \mathrm{H} \Psi^{(0)}_{k\sigma}(z) = E_k^{(0)} + \langle \sigma k | \Phi_{\mathrm{nRG}} | k\sigma \rangle + \langle \sigma k | \Phi^{(\mathrm{RI})}_{\mathrm{nBRG}} | k\sigma \rangle. \tag{11}$$

Using the table of integrals in [26,27] we obtain the RG, BRG and LV contributions to the energy spectrum of the quantum gravitation states of unpolarized UCNs. We get

$$\langle \sigma k | \Phi_{\mathrm{nRG}} | k\sigma \rangle = \int_0^\infty dz\, \Psi^{(0)\dagger}_{k\sigma}(z) \Phi_{\mathrm{nRG}} \Psi^{(0)}_{k\sigma}(z) = \frac{2}{5} \frac{(E_k^{(0)})^2}{m},$$

$$\langle \uparrow k | \Phi^{(\mathrm{RI})}_{\mathrm{nBRG}} | k \uparrow \rangle = \int_0^\infty dz\, \Psi^{(0)\dagger}_{k\uparrow}(z) \Phi^{(\mathrm{RI})}_{\mathrm{nBRG}} \Psi^{(0)}_{k\uparrow}(z) = -\frac{2}{3} (k^{(\mathrm{NR})}_\phi)_n \frac{E_k^{(0)}}{m} - g (k^{(\mathrm{NR})}_{\sigma g})'_n - \frac{2}{3} mg E_k^{(0)} \left((k^{(\mathrm{NR})}_{\sigma gpp})'_n + (k^{(\mathrm{NR})}_{\sigma gpp})''_n \right),$$

$$\langle \downarrow k | \Phi^{(\mathrm{RI})}_{\mathrm{nBRG}} | k \downarrow \rangle = \int_0^\infty dz\, \Psi^{(0)\dagger}_{k\uparrow}(z) \Phi^{(\mathrm{RI})}_{\mathrm{nBRG}} \Psi^{(0)}_{k\uparrow}(z) = -\frac{2}{3} (k^{(\mathrm{NR})}_\phi)_n \frac{E_k^{(0)}}{m} + g (k^{(\mathrm{NR})}_{\sigma g})'_n + \frac{2}{3} mg E_k^{(0)} \left((k^{(\mathrm{NR})}_{\sigma gpp})'_n + (k^{(\mathrm{NR})}_{\sigma gpp})''_n \right). \tag{12}$$

Since the binding energies of the quantum gravitational states of UCNs are of a few parts of 10^{-12} eV, the RG contribution is of order of a few parts of 10^{-33} eV and can be neglected. This concerns also the contributions proportional to $\frac{2}{3} mg E_k^{(0)} \leq 10^{-25}$ eV^3 for $k \leq 10$ [33,34].

As a result, the energy spectrum of the quantum gravitational states of UCNs together with the BRG and LV contributions is equal to

$$E_{k\uparrow/k\downarrow} = E_k^{(0)} - \frac{2}{3} (k^{(\mathrm{NR})}_\phi)_n \frac{E_k^{(0)}}{m} \mp g (k^{(\mathrm{NR})}_{\sigma g})'_n, \tag{13}$$

where $g = 2.15 \times 10^{-23}$ eV [14]. The LV contribution, proportional to $(k^{(\mathrm{NR})}_{\sigma g})'_n$, is the same for all energy level. It depends only on the neutron spin-polarization.

According to the energy spectrum Eq. (13), for the non-spin-flip $|k\sigma\rangle \to |k'\sigma\rangle$ and spin-flip $|k\sigma\rangle \to |k'\sigma'\rangle$ transitions we get [14]

$$\delta\nu_{k'\sigma k\sigma} = -\left(k_\phi^{\mathrm{NR}}\right)_n \frac{E_{k'}^{(0)} - E_k^{(0)}}{3\pi m},$$

$$\delta\nu_{k'\sigma' k\sigma} = \pm\frac{g}{\pi}\left(k_{\sigma g}^{(\mathrm{NR})}\right)_n' - \left(k_\phi^{\mathrm{NR}}\right)_n \frac{E_{k'}^{(0)} - E_k^{(0)}}{3\pi m}, \tag{14}$$

for $(\sigma = \uparrow, \sigma' = \downarrow)$ or $(\sigma = \downarrow, \sigma' = \uparrow)$, respectively.

For current sensitivity $\Delta E = 2 \times 10^{-15}$ eV [33] (see also [14]) and for the $|1\rangle \to |4\rangle$ transition [14] we are able to obtain the upper bound on the BRG contribution $\left|(k_\phi^{(\mathrm{NR})})_n\right|$ and an estimate for $(k_{\sigma g}^{(\mathrm{NR})})_n'$, i.e.,

$$\left|(k_\phi^{(\mathrm{NR})})_n\right| < 10^{-3} \text{ GeV} \quad , \quad \left(k_{\sigma g}^{(\mathrm{NR})}\right)_n' = 0. \tag{15}$$

The upper bound $\left|(k_\phi^{(\mathrm{NR})})_n\right| < 10^{-3}$ GeV is one order of magnitude better in comparison with the result $\left|(k_\phi^{(\mathrm{NR})})_n\right| < 1.3 \times 10^{-2}$ GeV, obtained in [6]. Then, our result $(k_{\sigma g}^{(\mathrm{NR})})_n' = 0$ agrees well with that by Kostelecký and Li [6]. The spin-flip transitions may also admit an upper bound $\left|(k_{\sigma g}^{(\mathrm{NR})})_n'\right| < 10^8$. However, it seems unrealistic, since the main contribution to $(k_{\sigma g}^{(\mathrm{NR})})_n'$ is caused by LV interactions [32].

It is important to emphasize that in the coefficient $(k_\phi^{\mathrm{NR}})_n$ the dominate role belongs to the BRG interactions. According to [6], the coefficient $(k_\phi^{\mathrm{NR}})_n$ has the following structure (see Table III in Ref. [6]):

$$\left(k_\phi^{\mathrm{NR}}\right)_n = 2\,(m'^{\mathrm{L}})_n^{ss} - 2\,(a^{\mathrm{L}})_n^{tss} + 2m\,(e_h^{\mathrm{L}})_n^{tss} - 2m\,(c_h^{\mathrm{L}})_n^{tss} + 2m^2\,(m_h^{(5)\mathrm{L}})_n^{ttss} - 2m^2\,(a_h^{(5)\mathrm{L}})_n^{ttss}, \tag{16}$$

where the phenomenological coupling constants in the right-hand-side of Eq. (16) are fully induced by the BRG interactions (see Table I in Ref. [6]).

The energy spectrum of the quantum gravitational states of unpolarized UCNs, calculated by taking into account the 2-fold degeneracy of the energy levels [14] (see also [23,24]), is equal to

$$E_k^{(\pm)} = E_k^{(0)} - \frac{2}{3}\left(k_\phi^{(\mathrm{NR})}\right)_n \frac{E_k^{(0)}}{m} \pm \left|g\left(k_{\sigma g}^{(\mathrm{NR})}\right)_n'\right|. \tag{17}$$

Using Eq. (15) in Ref. [14] we define the contributions to the transition frequencies of the quantum gravitational states of UCNs

$$\delta\nu_{k'k}^{(\pm\pm)} = -\left(k_\phi^{(\mathrm{NR})}\right)_n \frac{E_{k'}^{(0)} - E_k^{(0)}}{3\pi m}, \tag{18}$$

and

$$\delta\nu_{k'k}^{(\pm\mp)} = -\left(k_\phi^{(\mathrm{NR})}\right)_n \frac{E_{k'}^{(0)} - E_k^{(0)}}{3\pi m} \pm \left|\frac{g}{\pi}\left(k_{\sigma g}^{(\mathrm{NR})}\right)_n'\right|. \tag{19}$$

One may see that the experimental analysis of the transition frequencies between the quantum gravitational states of unpolarized UCNs should lead to the estimates Eq. (15).

4. Discussion

We have analyzed a possibility to test contributions of interactions, induced by non-Riemann geometry beyond the standard Riemann General Relativity and the Lorentz-invariance violation (LV), by Kostelecký and Li [6]. Using the effective low-energy potential, derived in [6], we have calculated the contributions of beyond-Riemann gravity or the BRG contributions and the LV contributions to the energy spectrum and transition frequencies of the quantum gravitational states of polarized and unpolarized UCNs. Such UCNs are used as test-particles for probes of contributions of interactions beyond the Standard Model (SM) and Einstein's gravity [33,34,36–51]. We have got the following constraints $\left|(k_\phi^{(\mathrm{NR})})_n\right| < 10^{-3}$ GeV and $(k_{\sigma g}^{(\mathrm{NR})})_n' = 0$. The upper bound $\left|(k_\phi^{(\mathrm{NR})})_n\right| < 10^{-3}$ GeV is one order of magnitude better in comparison with the constraint obtained in [6]. Then, from our analysis of the transition frequencies of the quantum gravitational states of UCNs follows that $(k_{\sigma g}^{(\mathrm{NR})})_n' = 0$, whereas in [6] such a value $(k_{\sigma g}^{(\mathrm{NR})})_n' = 0$ has been imposed by assumption. It is important to emphasize that for the experimental sensitivity $\Delta E = 2 \times 10^{-17}$ eV, which should be reached in the qBOUNCE experiments in a nearest future [40], we should expect the upper bound $\left|(k_\phi^{(\mathrm{NR})})_n\right| < 10^{-5}$ GeV.

As has been pointed out by Kostelecký and Li [6], the coefficient $(k_\phi^{\mathrm{NR}})_n$ should appear in the nonrelativistic Hamilton operator in Minkowski spacetime [52] but it produces no measurable effects in that context because it amounts to an unobservable redefinition of the zero of energy or, equivalently, because it can be removed from the theory via field redefinitions [7,8]. The observability of $(k_\phi^{\mathrm{NR}})_n$ is thus confirmed to be a consequence of the coupling to the gravitational potential, the presence of which restricts the applicability of field redefinitions [5].

In the perspective of the further analysis of BRG and LV interactions by the quantum gravitation states of UCNs we see in the use of i) the total effective low-energy potential Eq. (3) for the calculation of the BRG and LV contributions to the energy spectrum and the transition frequencies of the quantum gravitational states of UCNs, and of ii) the quantum bouncing ball experiments with a free fall of UCNs in the gravitational field of the Earth [42,43].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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