Hadronic EDMs from Dyson Schwinger

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Introduction

- Introduction
- Part A: Overview of Techniques
- Part B: The Theoretical Framework
- Part C: *The ρ Meson*
- Part D: The Nucleon

Introduction: The Energy Scale



Introduction: The Energy Scale & Effective EDM Operators for dim \geq 4 at scale \sim 1 GeV

Calculation of hadronic EDMs naturally splits into 2 parts

- Calculation of Wilson coefficients by integrating out short distances
- Switching from perturbative quark-gluon description to non-perturbative treatment
 - (much harder and larger uncertainties)

Effective EDM Operators for dim \geq 4 at scale \sim 1 GeV

$$\mathcal{L}_{M}^{1GeV} = \frac{g_{s}^{2}}{32\pi^{2}} \Theta G_{\mu\nu}^{a} \tilde{G}^{a,\mu\nu}$$

$$-\frac{i}{2} \sum_{i=e,u,d,s} d_{i} \bar{\psi}_{i} (F \cdot \sigma) \gamma^{5} \psi_{i} - \frac{i}{2} g_{s} \sum_{i=u,d,s} \tilde{d}_{i} \bar{\psi}_{i} (G \cdot \sigma) \gamma^{5} \psi_{i}$$

$$+ \sum_{i,j=e,u,d,s} C_{ij} (\bar{\psi}_{i} \psi_{i}) (\bar{\psi}_{j} i \gamma^{5} \psi_{j}) + \cdots$$

Part A: Overview of Techniques

Non-perturbative techniques:

- Lattice
- QCD sum rules
- Wave functions (including light-cone input)
- 1/N_c approximation
- Relativistic Quark-Parton Model
- Chiral Quark Model
- χPT
- DSE

Non-relativistic SU(6) Quark Model

- most widely used for quark EDMs to nEDM
- **disadvantage:** cannot be used for a wider class of *CP*-odd sources relevant for d_n and $\bar{g}_{\pi NN}$
- result: $d_n(d_q) = \frac{1}{3}(4d_d d_u)$

Naive dimensional analysis

- conceptually simplest approach/ QCD power counting
- **disadvantage:** does not allow in general to combine different contributions
- results: $d_n(d_q,\mu) \sim d_q(\mu)$

$$d_n(\Theta_q,\mu) \sim e \,\Theta_q(\mu) rac{m_q(\mu)}{\Lambda_{had}^2} \ d_n(\tilde{d}_q,\mu) \sim rac{e}{4\pi} \tilde{d}_q(\mu)$$

Chiral techniques

- *CP* odd $\bar{g}_{\pi NN}$ contributes to nEDM via π^- loop
- **disadvantage:** *limited applicability*, chiral enhancement is no numerical enhancement in the physical regime $(m_{\pi} \neq 0)$

• result:
$$d_n^{\chi \log} = rac{e}{4 \pi^2 M_n} \, g_{\pi NN} \, ar{g}_{\pi NN}^{(0)} \ln rac{\Lambda}{m_\pi}$$

QCD sum-rules techniques

- start at high energies, use operator product expansion, construct QCD sum rules
- **disadvantage:** dependence on partly know spectral function

• results:
$$d_n^{est} = \frac{4}{3}d_d - \frac{1}{3}d_u - \frac{2m_\pi^2 e}{m_n(m_u+m_d)} \left(\frac{2}{3}\tilde{d}_d + \frac{1}{3}\tilde{d}_u\right)$$
 (PQ)

Lattice

- assumedly ultimate choice for calculating hadronic matrix elements but *difficult*
- disadvantage: modest finite volume, quark masses higher than real physical values => extrapolation to real physical conditions by other techniques (e.g. chiral techniques)

Conclusion

- Many different techniques available
- Each with varying advantages and disadvantages
- Have been used complimentary
- MUCH room for improvement

Part B: The Theoretical Framework

1. Dyson-Schwinger Equations





Dyson-Schwinger Equation

- Non-perturbative continuum approach to any QFT
- A shift in the integration variable (φ(x) → φ(x) + λ(x)), does not change the path integral for suitable b.c., i.e.

$$\int D[\varphi] \, \frac{\delta}{\delta \varphi} f[\varphi] = 0$$

• Application to the *generating functional* Z[J] yields

$$\int D[\varphi] \left[-\frac{\delta S}{\delta \varphi} + J \right] e^{-S + \int d^4 x J \varphi} = 0$$

with the action $S = \int d^4x \mathcal{L}$. This can be rewritten as

$$\left[-\frac{\delta S}{\delta \varphi}\left(\frac{\delta}{\delta J}\right) + J\right] Z[J] = 0$$

Dyson-Schwinger Equation

In QCD the fermion propagator is obtained by derivation of

$$\left[-\frac{\delta S}{\delta\bar{\psi}(x)}\left(\frac{\delta}{\delta\bar{\eta}},-\frac{\delta}{\delta\eta},\frac{\delta}{\delta J_{\mu}}\right)+\eta(x)\right]Z[\eta,\bar{\eta},J]=0$$

with respect to η leading after several formal manipulations to the

Gap Equation for the *quark propagator*

$$S_{F}(p)^{-1} = ip Z_{2} + m_{q}(\mu) Z_{4}$$

+ $Z_{1} \int \frac{d^{4}q}{(2\pi)^{4}} g^{2} D_{\mu\nu}(k-p) \gamma_{\mu} \frac{\lambda^{i}}{2} S_{F}(k) \Gamma_{\nu}(k,p)$

Dyson-Schwinger Equation

- **Gap equation** contains the *full vertex* Γ_{μ} and *full gluon propagator* $D_{\mu\nu}(k-p)$, each satisfies it's own DSE
- **2** DSE for the *full vertex* Γ_{μ} contains the *four-point vertex*, which has it's own DSE...
- ⇒ DSE is an infinite tower of equations relating all correlation functions

- DSE are exact relations and are the quantum Euler-Lagrange equations for any QFT
- Perturbative Expansion yields standard perturbative QFT

DSE provides *Dynamical Chiral Symmetry Breaking*, i.e *dynamically induced mass* even in the *chiral limit* ("mass from nothing")

"Dressed" mass in Weak coupling expansion

$$M(p^2) \simeq m \left(1 - \frac{3\alpha_s}{4\pi} \ln[p^2/m^2] + \ldots\right)$$

 \implies *chiral limit* ($m \rightarrow 0$) yields *vanishing mass correction* (i.e. $m_N, \ldots \rightarrow 0$)

One has to apply *non-perturbative* methods, i.e. one has to sum an *infinite number of diagrams* for *DCSB*

2. Bethe-Salpeter Equations



Bethe-Salpeter Equations

Bethe-Salpeter equation is the DSE describing a *bound 2 body system*

Obtained by *four derivatives of the generating functional* and *several formal manipulations*

$$\Gamma(k;P) = \int \frac{d^4q}{(2\pi)^4} K(q,k;P) S_F\left(q + \frac{P}{2}\right) \Gamma(q;P) S_F\left(q - \frac{P}{2}\right)$$

Solutions for discrete set P² yield mass spectra



3. Ward-Takahashi Identities & Approximations





Ward-Takahashi Identities & Truncation

Observing that "a gauge transformation does not change the path integral" \implies

- Vector WTI ensures EM current conservation
- Axial-vector WTI gives large mass splittings between chiral partners & ensures massless ground state PS mesons in chiral limit despite strong enhancement of quark mass function in the IR

Calculations in *non-perturbative regime* demand a *symmetry-preserving truncation* of the *infinite set of DSEs* which has to respect *relevant (global) symmetries of QCD* such as

- Chiral symmetry
- Lorentz invariance and
- Renormalisation group invariance
- EM current conservation

Rainbow-Ladder Truncation

A viable truncation is the *rainbow-ladder truncation* in combination with the *impulse approximation*

1. In BSE kernel

$$K(p,p';k,k') \to -\mathcal{G}\ell(q^2)D^{\mathsf{free}}_{\mu\nu}(q)rac{\lambda^a}{2}\gamma_\mu\otimesrac{\lambda^a}{2}\gamma_
u$$

2. In gap equation

$$Z_1 g^2 D_{\mu\nu}(q) \Gamma^a_{\nu}(k,p) \to \mathcal{G}\ell(q^2) D^{\mathsf{free}}_{\mu\nu}(q) \frac{\lambda^a}{2} \gamma_{\nu}$$

- R-LT is first term in systematic expansion of qq̄ scattering kernel K(p, p'; k, k')
- Reduces to leading-order perturbation theory asymptotically

Gluon Propagator

• DSE and unquenched QCD lattice studies show that the

full gluon propagator

$$D^{ab}_{\mu
u}(p) = \delta^{ab} rac{\mathcal{G}\ell(p^2)}{p^2} \left(\delta_{\mu
u} - rac{p_{\mu}p_{
u}}{p^2}
ight)$$

is IR finite, i.e.

$$\lim_{p^2 \to 0} D^{ab}_{\mu\nu}(p) = finite$$

- the gluon has dynamically generated mass in the IR

 EM Observables in the static limit (q_µ → 0) probe gluon propagator for small transversed momenta ⇒

point-like vector \times vector contact interaction

$$g^2 D^{ab}_{\mu\nu}(p) = \delta^{ab} \delta_{\mu\nu} \frac{1}{m_G^2}$$

Contact Interaction Model

This implies

- Non-renormalizable theory
- Introduce proper-time regularization
- $\Lambda_{uv} = 1/\tau_{uv}$ cannot be removed but plays a dynamical role and sets the scale of all dimensioned quantities
- $\Lambda_{ir} = 1/\tau_{ir}$ implements *confinement* by ensuring the *absence of quark production tresholds*
 - Scale *m_G*, is set in agreement with *observables*
 - In the static limit $q^2 \rightarrow 0$ results "indistinguishable" from any other however sophisticated DSE approach
 - For q² ≥ M²_{dressed} deviations are expected from other experimental values

Part C: The ρ Meson

The ρ Meson

- "Per se" from an experimental point of view uninteresting
- Short lifetime (~ 10⁻²⁴ s) makes EDM measurements hard (or rather impossible)
- Simplest system possibly providing EDM and hence perfect prototype particle
- Results available in QCD sum rules and other techniques

Profile

$$I^{G}(J^{PC}) = 1^{+}(1^{--})$$

2
$$m = 775.49 \pm 0.34$$
 MeV,

 $\Gamma = 149.1 \pm 0.8 \text{ MeV}$

3 Primary decay mode (~ 100%): $\rho \rightarrow \pi \pi$

The ρ -Meson in Impulse Approximation



EDM sources induce CP violating corrections to the

- $q\gamma q$ vertex
- 2 Bethe-Salpeter amplitude
- Propagator

The $\rho\gamma\rho$ vertex

$$\begin{aligned} d_{\alpha\nu}(p)\Gamma_{\nu\mu\sigma}d_{\sigma\beta}(p') &= \\ d_{\alpha\nu}(p)\Big\{(p+p')_{\mu}[-\delta_{\nu\sigma}\,\mathcal{E}(q^2) + q_{\nu}q_{\sigma}\,\mathcal{Q}(q^2)] \\ &+ (\delta_{\mu\nu}q_{\sigma} - \delta_{\mu\sigma}q_{\nu})\,\mathcal{M}(q^2) - i\varepsilon_{\nu\sigma\mu\rho}q_{\rho}\,\mathcal{D}(q^2)\Big\}d_{\sigma\beta}(p') \end{aligned}$$

with CP conserving form factors

- for *charge* $\mathcal{E}(0) = 1$
- magnetic dipole moment $\mathcal{M}(0)$ in units $e/(2m_{\rho})$

• *quadrupole moment* Q(0) with form factor $Q(0) = (2/m^2)(Q + \mu - 1)$

and CP violating term

• *electric dipole moment* $\mathcal{D}(0)$ in units $e/(2m_{\rho})$

Form factors will be *projected out* by appropriate *projection operators*

The Magnetic Moment

Results for $\mathcal{M}(0)$ in units $e/(2m_{\rho})$

DSE - CIM	2.11
DSE - RL RGI-improved	2.01
DSE - EF parametrisation	2.69
LF - <i>CQM</i>	2.14
LF - <i>CQM</i>	1.92
QCD sum rules	1.8 ± 0.3
point particle	2

$$\mathcal{L}_{\mathsf{eff}} = -i ar{\Theta} rac{g_s^2}{32\pi^2} \, G^a_{\mu
u} ilde{G}^a_{\mu
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- No suppression by heavy scale (strong CP problem)
- *U*(1)_A anomaly allows to rotate it into complex mass for evaluation (*effective propagator correction*)



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The Quark-EDM

• The intrinsic EDM of a quark itself

$$\mathcal{L}_{\mathsf{eff}} = -\frac{i}{2} \sum_{q=u,d} d_q \, \bar{q} \sigma_{\mu\nu} \gamma^5 q \, F_{\mu\nu}$$

• Effective qyq vertex correction



DSE - CIM	$0.79\left(d_u-d_d\right)$
DSE	$0.72\left(d_u-d_d\right)$
Bag Model	$0.83\left(d_u-d_d\right)$
QCD sum rules	$0.51\left(d_u-d_d\right)$
Non-relativistic quark model	$1.00\left(d_u-d_d\right)$

• The Intrinsic Chromo-EDM of a quark itself

$$\mathcal{L}_{\mathsf{eff}} = -\frac{i}{2} g_s \sum_{q=u,d} \tilde{d}_q \bar{q} \frac{1}{2} \lambda^a \sigma_{\mu\nu} \gamma^5 q G^a_{\mu\nu}$$

• Effective $q\gamma q$ vertex correction



DSE - $q\gamma q$	$-0.07 \widetilde{ed}_{-} - 0.20 \widetilde{ed}_{+}$
DSE - BSA	$-0.12 \tilde{ed}_{-} + 0.11 \tilde{ed}_{+}$
DSE - Propagator	$1.35\tilde{ed}_{-} - 0.60\tilde{ed}_{+}$
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- No results obtained in other methods yet
- Effective qyq vertex correction



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DSE	$-1.79 imes10^{-5}\mathcal{K}e u_H/\Lambda^2$

$$\mathcal{L} = i \frac{\mathcal{K}}{\Lambda^2} \varepsilon_{ij} (\bar{Q}_i d) (\bar{Q}_j \gamma^5 u) \quad \text{with} \quad \bar{Q}_i = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

- No results obtained in other methods yet
- Effective Bethe-Salpeter amplitude correction



$$\mathcal{L} = i \frac{\mathcal{K}}{\Lambda^2} \varepsilon_{ij} (\bar{Q}_i d) (\bar{Q}_j \gamma^5 u) \quad \text{with} \quad \bar{Q}_i = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

- No results obtained in other methods yet
- Effective propagator correction



DSE - $q\gamma q$	$-1.00 imes 10^{-5} \mathcal{K}e u_H / \Lambda^2$
DSE - BSA	$-9.11 imes10^{-7}\mathcal{K}e u_H/\Lambda^2$
DSE - Propagator	$-6.91 imes10^{-6}\mathcal{K}e u_{H}/\Lambda^{2}$
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Conclusion I

- Chromo-EDM contribution (despite loop suppression) is of comparable size with quark-EDM contribution (corroborates QCD SR result by Ritz & Pospelov)
- Some models suggest $d_f = D_q \frac{e \nu_H}{\Lambda^2} \sim \frac{m_f}{\nu_H} \frac{e \nu_H}{\Lambda^2} \sim 10^{-5} \frac{e \nu_H}{\Lambda^2}$ in which case EDM sources considered here (except θ) give comparable contributions
- DSE CIM yields reliable results for a whole class of observables in the static limit, which makes it important for EDM calculations

Part D: The Nucleon

- DSE neutron calculation not yet available
- Nucleon *scalar and tensor form factor* calculations are currently done
- Tensor form factor of the neutron ~ qEDM



$$\begin{split} \Lambda^{+}(p)\mathcal{S}(-p) &\int \frac{d^{4}\ell}{(2\pi)^{4}} \, S^{(u)}(\ell+p) \sigma_{\mu\nu} S^{(u)}(\ell+p) \Delta^{0^{+}}(-\ell) \mathcal{S}(p) \Lambda^{+}(p) \\ &= \mathcal{N} \, \delta d \, \Lambda^{+}(p) \sigma_{\mu\nu} \Lambda^{+}(p) \end{split}$$

$$\mathcal{S}(p) = s(p) \mathbf{1}_D \qquad (s(p) = 0.8810)$$
$$\Lambda^+(p) = \frac{1}{2m_N} (-i\gamma \cdot p + m_N)$$



$$\begin{split} \Lambda^{+}(p)\mathcal{A}^{i}_{\alpha}(-p) &\int \frac{d^{4}\ell}{(2\pi)^{4}} S^{(q)}(\ell+p)\sigma_{\mu\nu}S^{(q)}(\ell+p)\Delta^{1+}_{\alpha\beta}(-\ell)\mathcal{A}^{i}_{\beta}(p)\Lambda^{+}(p) \\ &= \mathcal{N}\,\delta q\,\Lambda^{+}(p)\sigma_{\mu\nu}\Lambda^{+}(p) \end{split}$$

$$\mathcal{A}^{i}_{\mu}(p) = a^{i}_{1}(p) \gamma_{5} \gamma_{\mu} + a^{i}_{2}(p) \gamma_{5} \hat{p}_{\mu} \qquad \left(\hat{p}^{2} = -1, i = +, 0\right)$$
$$a^{+}_{1} = -0.380, a^{+}_{2} = -0.065, a^{0}_{1} = 0.270, a^{0}_{2} = 0.046$$



$$\begin{split} \Lambda^{+} \mathcal{A}^{i}_{\alpha}(-p) \int \frac{d^{4}\ell}{(2\pi)^{4}} \,\Delta^{1+}_{\alpha\alpha'}(\ell+p) \Lambda_{\alpha'\mu\nu\beta'} \Delta^{1+}_{\beta'\beta}(\ell+p) S^{(q)}(-\ell) \mathcal{A}^{i}_{\beta}(p) \Lambda^{+} \\ &= \mathcal{N} \,\delta q \,\Lambda^{+}(p) \sigma_{\mu\nu} \Lambda^{+}(p) \end{split}$$

$$\begin{aligned} \mathcal{A}^{i}_{\mu}(p) &= a^{i}_{1}(p) \,\gamma_{5}\gamma_{\mu} + a^{i}_{2}(p) \,\gamma_{5}\hat{p}_{\mu} & \left(\hat{p}^{2} = -1, i = +, 0\right) \\ a^{+}_{1} &= -0.380, \, a^{+}_{2} = -0.065, \, a^{0}_{1} = 0.270, \, a^{0}_{2} = 0.046 \end{aligned}$$



$$\Lambda^{+}(p)\mathcal{S}(-p)\int \frac{d^{4}\ell}{(2\pi)^{4}}\,\Delta^{0^{+}}_{\alpha\alpha'}(\ell+p)\Lambda_{\mu\nu}\Delta^{0^{+}}_{\beta'\beta}(\ell+p)S^{(q)}(-\ell)\mathcal{S}(p)\Lambda^{+}(p)$$
$$= 0$$

"A spinless particle cannot have a vectorial/tensorial structure of any kind!"



$$\begin{split} \Lambda^{+}(p)\mathcal{S}(-p) \int \frac{d^{4}\ell}{(2\pi)^{4}} \,\Delta^{0^{+}}(\ell+p)\Lambda_{\mu\nu\alpha}\Delta^{1^{+}}_{\alpha\beta}(\ell+p)S^{(u)}(-\ell)\mathcal{A}^{0}_{\beta}(p)\Lambda^{+}(p) \\ &= \mathcal{N}\,\delta d\,\Lambda^{+}(p)\sigma_{\mu\nu}\Lambda^{+}(p) \end{split}$$

$$S(p) = s(p) \mathbf{1}_D \qquad (s(p) = 0.8810)$$

$$\mathcal{A}^0_\mu(p) = a^0_1(p) \gamma_5 \gamma_\mu + a^0_2(p) \gamma_5 \hat{p}_\mu \qquad (a^0_1 = 0.270, a^0_2 = 0.046)$$



$$\Lambda^{+}(p)\mathcal{A}^{0}_{\alpha}(-p)\int \frac{d^{4}\ell}{(2\pi)^{4}} \Delta^{1+}_{\alpha\beta}(\ell+p)\Lambda_{\beta\mu\nu}\Delta^{0+}(\ell+p)S^{(u)}(-\ell)\mathcal{S}(p)\Lambda^{+}(p)$$
$$= \mathcal{N}\,\delta d\,\Lambda^{+}(p)\sigma_{\mu\nu}\Lambda^{+}(p)$$

$$S(p) = s(p) \mathbf{1}_D \qquad (s(p) = 0.8810)$$

$$\mathcal{A}^0_\mu(p) = a^0_1(p) \gamma_5 \gamma_\mu + a^0_2(p) \gamma_5 \hat{p}_\mu \qquad (a^0_1 = 0.270, a^0_2 = 0.046)$$

Conclusion II

Continuum approach to any QFT

- Originates at the QCD current quark/gluon level, i.e. all operators are "implemented" at that level
- *Truncations* have to be performed (RL: dressed quark-gluon vertex, dressed gluon propagator)
- Works good for 3 external particles (PS mesons, vector particles, N, Δ) but bad for ≥ 4 e.p. (ππ scattering...)
- "rigid structure" few model parameters
- has shown to work well in the CP conserving sector
- still a lot to do: error bars, PQ, ...

The results, expounded in this talk, were obtained in Collaboration with

- Craig D. Roberts ANL
- Michael J. Ramsey-Musolf UMass Amherst
- Chien-Yeah Seng UMass Amherst

Thank You For Your Attention!