

Sensitivity of Neutron Beta Decay Observables to BSM Interactions

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Introduction

Part A: Neutron β -Decay

- Introduction
- Lifetime, Correlation Coefficients, Experimental Asymmetries
- SM next-to-leading order

Part B: Matrix Element V_{ud}

- Extraction from Neutron β -Decay
- Current Results

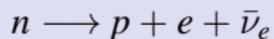
Part C: BSM

- Phenomenological Coupling Constants
- Access via Correlation Coefficients & CKM Unitarity

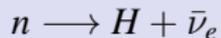
"Part A: Neutron β -Decay"

Different Kinds of Neutron Decays

Continuum State β^- - decay



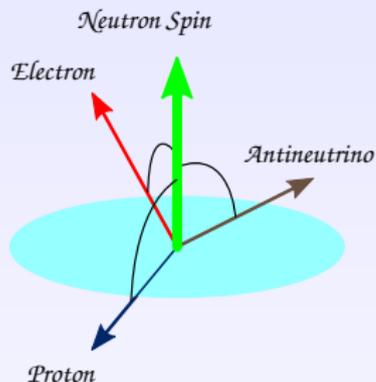
Bound State β^- - decay



Radiative β^- - decay

- $n \longrightarrow p + e + \bar{\nu}_e + \gamma$
- $n \longrightarrow H + \bar{\nu}_e + \gamma$

Kinematics of Continuum β^- - Decay



Conservation of four-momentum k^μ

$$m_n = E_p + E_e + E_\nu$$

$$\vec{0} = \vec{k}_p + \vec{k}_e + \vec{k}_\nu$$

Final state d.o.f.: 5

$$q = m_n - m_p - m_e = 0.782 \text{ MeV}$$

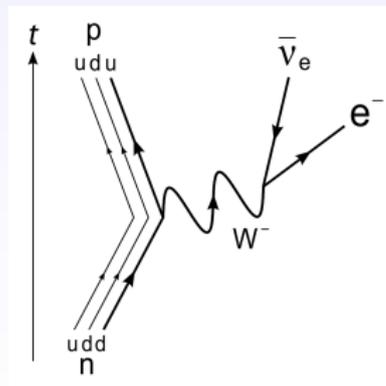
$$E_{max} = \frac{m_n^2 - m_p^2 + m_e^2}{2m_n} = 1.2927 \text{ MeV}$$

$$m_n = 939.5654 \text{ MeV}$$

$$m_p = 938.2720 \text{ MeV}$$

$$m_e = 0.5110 \text{ MeV}$$

$$m_{\bar{\nu}_e} = 0 \text{ MeV}$$



Standard $V - A$ Theory of Weak Interactions

"Hadronic" Lagrangian of $V - A$ Weak Interactions

$$\begin{aligned}\mathcal{L}_W(x) &= -\frac{G_F}{\sqrt{2}} V_{ud} [\bar{\psi}_p(x)\gamma_\mu(1 + \lambda\gamma^5)\psi_n(x)] [\bar{\psi}_e(x)\gamma^\mu(1 - \gamma^5)\psi_{\bar{\nu}_e}(x)] \\ &= -\frac{1}{\sqrt{2}} [\bar{\psi}_p(x)\gamma_\mu(g_V + g_A\gamma^5)\psi_n(x)] [\bar{\psi}_e(x)\gamma^\mu(1 - \gamma^5)\psi_{\bar{\nu}_e}(x)]\end{aligned}$$

with $g_V = G_F V_{ud}$ & $g_A = G_F V_{ud} \lambda$

Parameters of Weak Interactions

- Fermi Constant: $G_F = \frac{g_W^2}{8m_W^2} = 1.1664 \times 10^{-11} \text{ MeV}^{-2}$
- CKM Matrix Element: $V_{ud} = 0.97425(22)$
K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014)
- Axial-to-Vector Coupling Ratio: $\lambda = -1.2723(23)$
K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014)

"The Neutron Alphabet"

$$\frac{d^5\Gamma(\sigma_n)}{dE_e d^2\Omega_e d^2\Omega_\nu} = \xi(E_e) \left\{ 1 + a \frac{\vec{k}_e \cdot \vec{k}_\nu}{E_e E_\nu} + A \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} + B \frac{\vec{\xi}_n \cdot \vec{k}_\nu}{E_\nu} \right\}$$

$$\xi(E_e) = \frac{G_F^2 V_{ud}^2}{32\pi^5} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z=1) (1 + 3\lambda^2)$$

$$\frac{d^5\Gamma(\sigma_e)}{dE_e d^2\Omega_e d^2\Omega_\nu} = a, G, H, K$$

$$\frac{d^3\Gamma(\sigma_n, \sigma_e)}{dE_e d^2\Omega_e} = A, G, N, Q$$

- extensive literature (JTW '57, ...)
- many different notations

"The Neutron Alphabet"

The "Standard" Correlation Coefficients are

$$a = \frac{1 - \lambda^2}{1 + 3\lambda^2} \simeq -0.10$$

$$A = -2\lambda \frac{1 + \lambda}{1 + 3\lambda^2} \simeq -0.12$$

$$B = -2\lambda \frac{1 - \lambda}{1 + 3\lambda^2} \simeq 0.98$$

...

- Suppose we could "*switch off*" hadronic effects (perfect $V - A$ with " $\lambda = -1$ " - compare to quark level)
- Deviation of a , A from 0 and B from 1 is due to hadronization

The SM Leading Order Uncertainties

All *Correlation Coefficients* (a, A, B, G, H, K, N, Q) are only functions of λ (and m_e, E_e in the case H, N)

experimental uncertainty of λ ($\sim 10^{-3}$) leads to the only real "*theoretical*" uncertainty

A, N, Q have *highest sensitivity* with respect to λ

$\implies A$ used for experimental determination

\implies "*theoretical*" uncertainty matches experimental one for A, N, Q and is even *higher* for the other correlation coefficients

Standard V – A Theory of Weak Interactions

Inverse Lifetime of Free Neutron

$$\begin{aligned}\tau_n^{-1} &= \frac{1}{2} \sum_{\sigma_n = \pm} \int_{m_e}^{E_{max}} dE_e \int_{[4\pi]} d^2\Omega_e \int_{[4\pi]} d^2\Omega_{\bar{\nu}} \frac{d^5\Gamma(\sigma_n)}{dE_e d^2\Omega_e d^2\Omega_{\bar{\nu}}} \\ &= (1 + 3\lambda^2) \frac{G_F^2 V_{ud}^2}{2\pi^3} f(E_{max}, Z = 1)\end{aligned}$$

with the Fermi Integral

$$\begin{aligned}f(E_{max}, Z = 1) &= \int_{m_e}^{E_{max}} dE_e \sqrt{E_e^2 - m_e^2} (E_{max} - E_e)^2 E_e F(E_e, Z = 1) \\ &= 2\pi\alpha \int_{m_e}^{E_{max}} dE_e \frac{(E_{max} - E_e)^2 E_e^2}{1 - e^{-2\pi\alpha E_e / \sqrt{E_e^2 - m_e^2}}} \\ &= 0.0588 \text{ MeV}^5\end{aligned}$$

¹Some experimental values for the Lifetime and λ

Lifetime τ [s]	Document ID	Year
880.3 \pm 1.1	PDG Average Value	2014
887.7 \pm 1.2 \pm 1.9	YUE	2013
881.6 \pm 0.8 \pm 1.9	ARZUMANOV	2012
882.5 \pm 1.4 \pm 1.5	STEYERL	2012
880.7 \pm 1.3 \pm 1.2	PICHLMAIER	2010
878.5 \pm 0.7 \pm 0.3	SEREBROV	2005
889.2 \pm 3.0 \pm 3.8	BYRNE	1996
882.6 \pm 2.7	MAMPE	1993

¹taken from K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014)

²Some experimental values for the Lifetime and λ

$\lambda \equiv g_A/g_V$	Document ID	Year
-1.2723 ± 0.0023	PDG Average Value	2014
-1.2755 ± 0.0030	MENDENHALL	2013
$-1.2748 \pm 0.0008^{+0.0010}_{-0.0011}$	MUND	2013
$-1.275 \pm 0.006 \pm 0.015$	SCHUMANN	2008
$-1.2686 \pm 0.0046 \pm 0.0007$	MOSTOVOI	2001
-1.266 ± 0.004	LIAUD	1997
-1.2594 ± 0.0038	YEROZOLIMSKY	1997
-1.262 ± 0.005	BOPP	1986

The SM Next-to-Leading Order Contributions

2 independent Expansion Parameters

1 $q/M \sim 10^{-3}$

2 $\alpha/2\pi \sim 10^{-3}$

Neglecting SM terms: $(q/M)^2$, $(q/M)\alpha/2\pi$, $(\alpha/2\pi)^2 \sim 10^{-6}$
BUT numerical factors can vary quite strong

New SM Physics Effects appearing in NLO

- 1 Proton Recoil
- 2 Weak Magnetism
- 3 Radiative Corrections

Additional Correlation Coefficients

- SM next-to-leading order
- new physics?

1. Proton Recoil

SM leading-order

$$\psi_p(x) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \sigma_p$$

⇒ **no proton recoil** (i.e. neutron and proton are "static")

Next-to-leading-order

$$\psi_p(x) \sim \begin{pmatrix} 1 \\ \frac{\vec{\sigma} \cdot \vec{k}}{E+M} \end{pmatrix} \otimes \sigma_p$$

⇒ **proton recoil** (recoil of the proton gives $\mathcal{O}(q/M)$ contribution)

Theoretical Uncertainty due to PR is completely negligible

2. Weak Magnetism

Effective Lagrangian

$$\mathcal{L}_{WM}(x) = -\frac{G_F V_{ud}}{\sqrt{2}} \frac{\mu}{2M} \partial^\nu [\bar{\psi}_p(x) \sigma_{\mu\nu} \psi_n(x)] [\bar{\psi}_e(x) \gamma^\mu (1 - \gamma^5) \psi_{\bar{\nu}_e}(x)]$$

with additional parameter

$$\mu = \mu_p - \mu_n = 1.7928 + 1.9130 = 3.7058$$

- $\mathcal{O}(q/M)$ effect
- experimental uncertainty of $\mu_{n,p} \sim 10^{-7} - 10^{-8}$

Theoretical Uncertainty due to WM is completely negligible

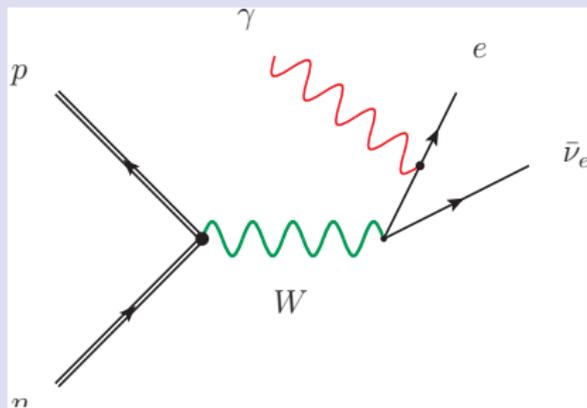
3. Radiative Corrections

All Observables (Lifetime, Correlation Coefficients) obtain Radiative Corrections due to the

- Emission of **Real Photons** (*Bremsstrahlung*)
- Exchange of **Virtual Photons** and **Z-bosons**

To leading order in α the principal graphs are

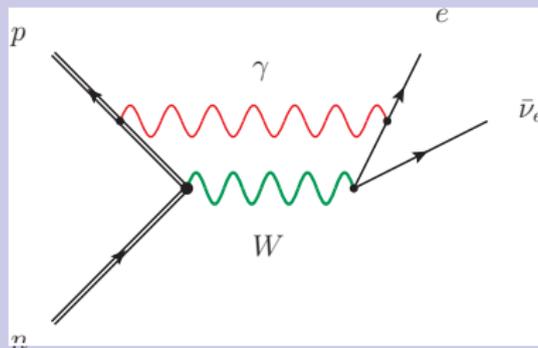
1. One γ *Bremsstrahlung* (mainly from the lepton)



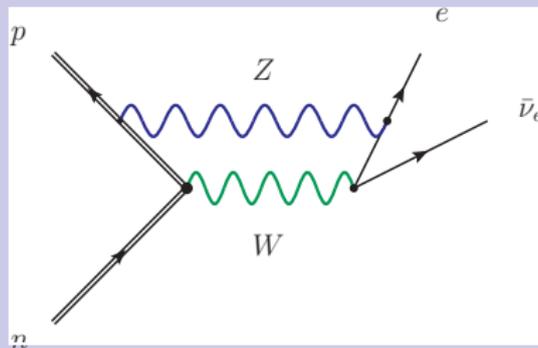
3. Radiative Corrections

To leading order in α the principal graphs are

2. γW -box



3. ZW -box graphs



3. Radiative Corrections

Following Marciano & Sirlin one splits ~ 1 GeV into

1. Low-energy part

- 1 *Bremsstrahlung*
- 2 Low-energy part of γW

IR-divergences of 1 + 2 cancel
requires *model calculation* of the *hadronic structure*

2. High-energy part

- 1 ZW
- 2 High-energy part of γW

Using *free-quark Lagrangian* neglecting the *hadronic structure*

QCD corrections have to be added

3. Radiative Corrections

- "bare values" $G_F^0 V_{ud}^0$ and λ^0 are *per se unobservable*,
- extraction hard due to the *"dirty" hadronic RC*
(important only for comparison with other physical systems $\sim G_F^\mu$)
- "bare" *Correlation Coefficients* can be extracted easily due to
well-know RC $\delta_\alpha^{(2)}$ (up to λ)
 \implies *Correlation Coefficients* well suited for search for **BSM**

$$\delta_\alpha^{(2)} = \frac{1 - \beta^2}{\beta} \ln \left(\frac{1 + \beta}{1 - \beta} \right) + \left(\frac{E_0 - E_e}{E_e} \right) \frac{4(1 - \beta^2)}{3\beta^2} \left[\frac{1}{2\beta} \ln \left(\frac{1 + \beta}{1 - \beta} \right) - 1 \right] \\ + \left(\frac{E_0 - E_e}{E_e} \right)^2 \frac{1}{6\beta^2} \left[\frac{1 - \beta^2}{2\beta} \ln \left(\frac{1 + \beta}{1 - \beta} \right) - 1 \right] \quad \text{with } \beta = \frac{|\vec{k}_e|}{E_e}$$

- 1 model-independent
- 2 depends only on the *kinematics* of the electron
- 3 **uncertainties are completely negligible**

"Part B: Matrix Element V_{ud} "

CKM Matrix

Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix is *central pillar* of the *Electroweak Standard Model*

Any deviation from *unitarity* would signal presence of "*new physics*" *beyond the Standard Model*, i.e. non *V – A interactions* or *fourth quark generation*

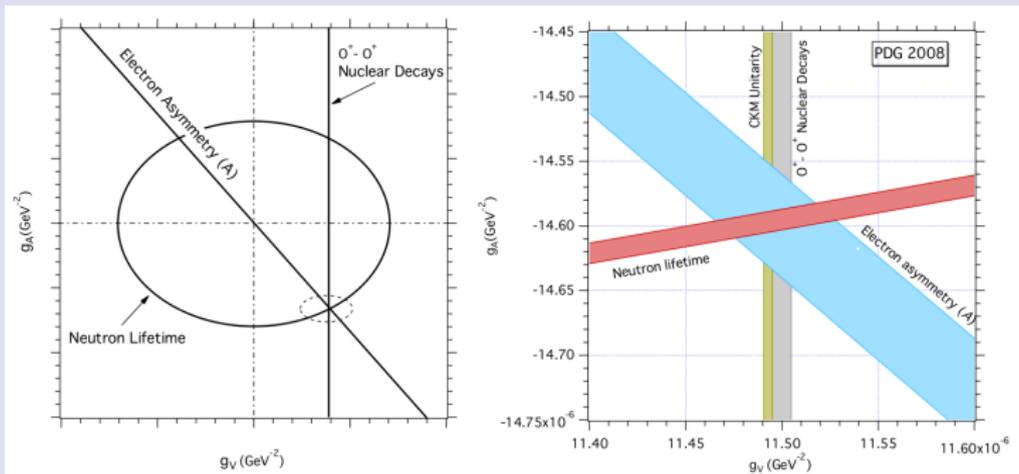
To date, best precision is obtained in the top-row test (with precision of 0.06%)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

based on measurements of *superallowed* $0^+ \rightarrow 0^+$ *nuclear β -decay* and of *kaon semileptonic and leptonic decays*

^aLifetime, λ and super allowed $0^+ \rightarrow 0^+$ nuclear decays as functions from g_V and g_A

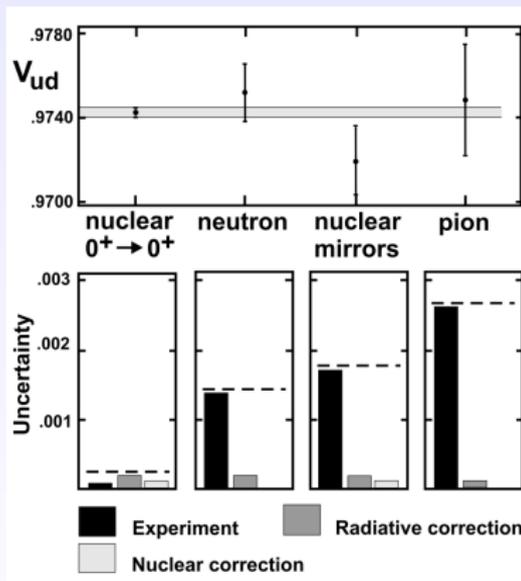
^ataken from J S Nico, J. Phys. G: Nucl. Part. Phys. **36** (2009) 104001



$$\tau_n^{-1} = \frac{f}{K} (g_V^2 + 3g_A^2) \quad \& \quad \lambda \equiv \frac{g_A}{g_V} \quad \text{with} \quad K = \frac{2\pi^3 \hbar^7}{m_e^5 c^4}$$

$$|V_{ud}|^2 = \frac{1}{f(1 + \delta_R)\tau_n} \frac{K}{G_F^2(1 + \Delta_R^V)(1 + 3\lambda^2)} = \frac{4908.7(1.9) s}{\tau_n(1 + 3\lambda^2)}$$

Four Values for V_{ud}



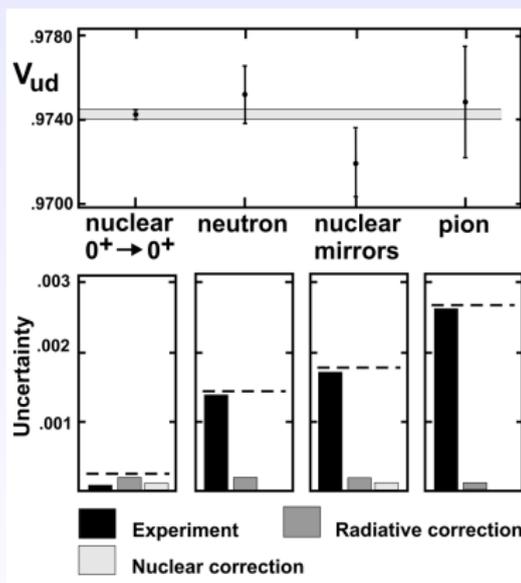
Nuclear $0^+ \rightarrow 0^+$

pro: only **vector current** involved - **CVC**, small exp. uncertainty

con: **isospin-symmetry breaking**, RC, nuclear structure

⇒ certainly the most precise current determination of V_{ud}

4 Four Values for V_{ud}



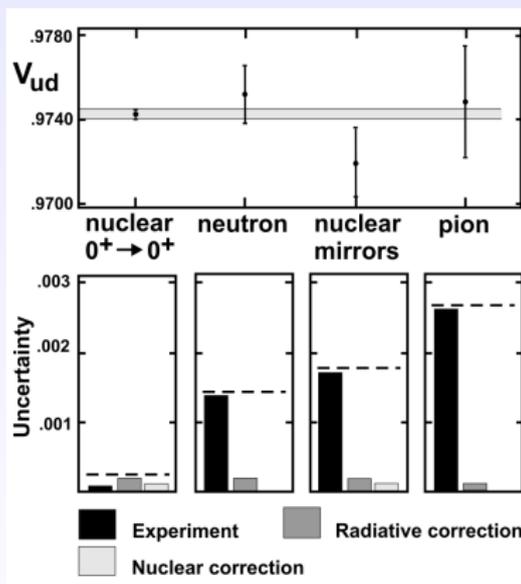
Neutron β -Decay

pro: no *isospin-symmetry breaking*
& nuclear structure

con: difficult to confine, also
axial-vector current involved
 \Rightarrow need λ which introduces
big *exp. uncertainties*

\Rightarrow *uncertainty* still more than
six times **nuclear** $0^+ \rightarrow 0^+$,
intrinsically *small corrections*
 \rightarrow promising future

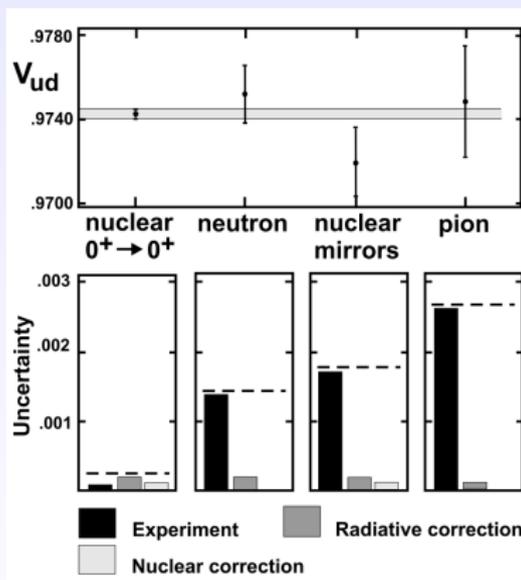
5 Four Values for V_{ud}



Nuclear $T = 1/2$ Mirror Decays

con: involves *axial vector current*
 \Rightarrow need λ , nuclear structure dependent corrections

⁶Four Values for V_{ud}



Pion β -Decay

pro: only **vector decay current**,
uncontaminated by
nuclear-structure uncertainties

con: **branching ratio** is very small
($\sim 10^{-8}$) and *difficult to measure*
with sufficient precision

\Rightarrow *in principle* the best way to
determine V_{ud} ,
but *insufficient precision*

Current Status

Taken from K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014)

Currently (2014) there is *2 sigma disagreement* for V_{ud} obtained by

1. Superaligned $0^+ \rightarrow 0^+$ Nuclear β -Decay

$$V_{ud} = 0.97425(8)_{exp.}(10)_{nucl.dep.}(18)_{RC}$$

2. Neutron β -Decay

$$V_{ud} = 0.9774(5)_{\tau_n}(16)_\lambda(2)_{RC}$$

$$\text{with } \tau_n^{ave} = 880.0(0.9) \quad \& \quad \lambda^{ave} = -1.2701(25)$$

This points towards *higher values* of λ and/or τ_n

"Part C: BSM"

Effective Hamiltonian

Most general (hadronic) effective low-energy weak Hamiltonian

$$\begin{aligned}\mathcal{H}_W(x) = & \frac{G_F}{\sqrt{2}} V_{ud} \left\{ [\bar{\psi}_p(x) \gamma_\mu \psi_n(x)] [\bar{\psi}_e(x) \gamma^\mu (C_V + \bar{C}_V \gamma^5) \psi_{\nu_e}(x)] \right. \\ & + [\bar{\psi}_p(x) \gamma_\mu \gamma^5 \psi_n(x)] [\bar{\psi}_e(x) \gamma^\mu (\bar{C}_A + C_A \gamma^5) \psi_{\nu_e}(x)] \\ & + [\bar{\psi}_p(x) \psi_n(x)] [\bar{\psi}_e(x) (C_S + \bar{C}_S \gamma^5) \psi_{\nu_e}(x)] \\ & \left. + \frac{1}{2} [\bar{\psi}_p(x) \sigma^{\mu\nu} \gamma^5 \psi_n(x)] [\bar{\psi}_e(x) \sigma_{\mu\nu} (\bar{C}_T + C_T \gamma^5) \psi_{\nu_e}(x)] \right\}\end{aligned}$$

Possible *pseudo-scalar interactions* are *suppressed* ($\sim 10^{-6}$) and hence *neglected*

with

$$C_V = 1 + \delta C_V$$

$$\bar{C}_V = -1 + \bar{\delta} C_V$$

$$C_A = -\lambda + \delta C_A$$

$$\bar{C}_A = \lambda + \bar{\delta} C_A$$

Correlation Coefficients

Differential Decay Rate

$$\begin{aligned} \frac{d^5\Gamma(\sigma_n)}{dE_e d^2\Omega_e d^2\Omega_\nu} &= \\ &= \xi(E_e) \left\{ 1 + b \frac{m_e}{E_e} + a \frac{\vec{k}_e \cdot \vec{k}}{E_e E} + A \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} + B \frac{\vec{\xi}_n \cdot \vec{k}}{E} + D \frac{\vec{\xi}_n \cdot (\vec{k}_e \times \vec{k})}{E_e E} \right\} \end{aligned}$$

$$\begin{aligned} \xi(E_e) &= \frac{G_F^2 V_{ud}^2}{32\pi^5} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z=1) (1 + 3\lambda^2) \\ &\quad \times \left\{ 1 + \frac{1}{1 + 3\lambda^2} \left(\text{Re}(\delta C_V - \delta \bar{C}_V) - 3\lambda \text{Re}(\delta C_A - \delta \bar{C}_A) \right) \right\} \end{aligned}$$

$$\begin{aligned} a &= a_{SM} + \frac{1}{1 + 3\lambda^2} \left\{ \text{Re}(\delta C_V - \delta \bar{C}_V) + \lambda \text{Re}(\delta C_A - \delta \bar{C}_A) \right. \\ &\quad \left. - a_0 \left(\text{Re}(\delta C_V - \delta \bar{C}_V) - 3\lambda \text{Re}(\delta C_A - \delta \bar{C}_A) \right) \right\} \end{aligned}$$

$$b = \frac{1}{1 + 3\lambda^2} \left\{ \text{Re}(C_S - \bar{C}_S) + 3\lambda \text{Re}(C_T - \bar{C}_T) \right\}$$

Correlation Coefficients

$$A = A_{SM} + \frac{1}{1 + 3\lambda^2} \left\{ -\lambda \operatorname{Re}(\delta C_V - \delta \bar{C}_V) + (1 + 2\lambda) \operatorname{Re}(\delta C_A - \delta \bar{C}_A) - A_0 \left(\operatorname{Re}(\delta C_V - \delta \bar{C}_V) - 3\lambda \operatorname{Re}(\delta C_A - \delta \bar{C}_A) \right) \right\}$$

$$B = B_{SM} + \frac{1}{1 + 3\lambda^2} \left\{ -\lambda \operatorname{Re}(\delta C_V - \delta \bar{C}_V) + (1 - 2\lambda) \operatorname{Re}(\delta C_A - \delta \bar{C}_A) - B_0 \left(\operatorname{Re}(\delta C_V - \delta \bar{C}_V) - 3\lambda \operatorname{Re}(\delta C_A - \delta \bar{C}_A) \right) - \left(\lambda \operatorname{Re}(C_S - \bar{C}_S) + (1 - 2\lambda) \operatorname{Re}(C_T - \bar{C}_T) \right) \frac{m_e}{E_e} \right\}$$

$$D = -\frac{1}{1 + 3\lambda^2} \left\{ \lambda \operatorname{Im}(\delta C_V - \delta \bar{C}_V) + \operatorname{Im}(\delta C_A - \delta \bar{C}_A) \right\}$$

Lifetime

$$\tau_n^{-1} = (\tau_n^{-1})_{SM} \times \left\{ 1 + \frac{1}{1 + 3\lambda^2} \left(\operatorname{Re}(\delta C_V - \delta \bar{C}_V) - 3\lambda \operatorname{Re}(\delta C_A - \delta \bar{C}_A) \right) + b \left\langle \frac{m_e}{E_e} \right\rangle \right\}$$

Experimental Access

To *linear order* one can express the Hamiltonian in terms of the *effective couplings*

$$\lambda_{\text{eff}} = \lambda \left(1 - \frac{1}{2} \text{Re}(\delta C_V - \delta \bar{C}_V) \right) - \frac{1}{2} \text{Re}(\delta C_A - \delta \bar{C}_A)$$
$$(V_{ud})_{\text{eff}} = V_{ud} \left(1 + \frac{1}{2} \text{Re}(\delta C_V - \delta \bar{C}_V) \right)$$

After this redefinition δC_V , $\delta \bar{C}_V$, δC_A , $\delta \bar{C}_A$ do **not appear** explicitly in the linearised Hamiltonian

This implies

- 1 $\text{Re}(\delta C_V - \delta \bar{C}_V)$ only accessible via **CKM unitarity tests**
- 2 $\text{Re}(\delta C_A - \delta \bar{C}_A)$ only accessible via **QCD calculation**

Neutron β -Decay

provides to *leading order*

- **partial access** to *scalar interactions* (only " $C_S - \bar{C}_S$ " for ν_L)
- **no access** to *pseudo-scalar interactions* (vs. pion decay...)
- **partial access** to *tensor interactions* (only " $C_T - \bar{C}_T$ " for ν_L)

All *experimental asymmetries* can be expressed in terms of $C_V, \bar{C}_V, C_A, \bar{C}_A, C_S, \bar{C}_S, C_T, \bar{C}_T$

for explicit expressions see

- A.N. Ivanov, M.P., N.I. Troitskaya: Phys. Rev. D **88**, 073002 (2013)
- A.N. Ivanov, M.P., N.I. Troitskaya: arXiv:1212.0332 [hep-ph]

Summary & Outlook

Summary

- The *SM Corrections* to leading order are well known (The theoretical uncertainties are *rather small*)
- **CKM unitarity** is crucial for the extraction of **BSM**

Outlook

- SM to *next-to-leading order* (proton recoil, weak magnetism, RC) for "*non-Standard*" *Correlation Coefficients* have still to be derived
- experimental improvements on λ and τ are most important to extract V_{ud} from Neutron β -Decay
- *hadronic couplings* $C_V, \bar{C}_V, C_A, \bar{C}_A, C_S, \bar{C}_S, C_T, \bar{C}_T$ have to be related to the corresponding *quark couplings (form factors)* in order to put bounds on *parameters of high-energy physics models* (MSSM, leptoquarks, ...)

The results, expounded in this talk, are obtained in
Collaboration with

- Hartmut Abele – TU Wien
- Andrei Ivanov – TU Wien

Thank You For Your Attention!