Almost All Products of Projections Converge

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Let $C_1, \ldots, C_K$ be closed, convex and quasi-symmetric subsets of H with a nonempty intersection $C = \bigcap_1^K C_j$. A sequence of indices $\alpha \in \{1, \ldots, K\}^{\mathbb{N}}$ and $x_0 \in H$ generate the sequence

$$x_{n+1} = P_{\alpha(n)} x_n, \qquad n = 0, 1, 2, \ldots; \tag{1}$$

here $P_{\alpha(n)}$ denotes the nearest point projection onto the convex set $C_{\alpha(n)}$.

We show that for almost all sequences α of indices the sequence of projections (1) converges to a point in C.

More precisely, we consider the finite set $[K] = \{1, \ldots, K\}$ equipped with the uniform distributed measure, that is, every point has measure $1/K$. The space $[K]^{\mathbb{N}}$ of all sequences of indices is equipped with the corresponding product measure. Similarly, we consider $[K]$ as a discrete set and the compact set $[K]^{\mathbb{N}}$ equipped with the corresponding product topology.

In [1] we show that there is a set $Q \subset [K]^{\mathbb{N}}$ of sequences of indices which is both of full measure and residual so that for each $x_0 \in H$ and each $\alpha \in Q$ the sequence of projections (1) converges to a point in C. For the special case when $C_1, \ldots, C_K$ are closed subspaces the measure-theoretic result was shown by de Brito, Melo, and da Cruz Neto in [2] and the topological one by Thimm in [3].

References

[1] P. A. Borodin and E. Kopecká, *Consecutive Projections and Greedy Approximation in Hilbert Space*, to appear in Journal of Mathematical Sciences.

[2] Í. D. L. Melo, J. X. da Cruz Neto, and J. M. M. de Brito, *Strong Convergence of Alternating Projections*, Journal of Optimization Theory and Applications 194, No. 1 (2022), 306–324.

[3] D. K. Thimm, *Most Iterations of Projections Converge*, Journal of Optimization Theory and Applications 203 (2024), 285–304.