Abstract

Strongly correlated electron systems show a rich variety of fascinating phenomena: magnetic and/or charge order, interaction-driven metal-to-insulator transitions, high-temperature superconductivity, the pseudogap phase, and strange-metal behavior. Yet, the strong correlation at play makes these physical phenomena exceptionally challenging to calculate, even with most advanced numerical methods. In particular, the research of the last decade has provided several hints that standard self-consistent many-body methods relying on the perturbative skeleton series fail to access these parameter regimes in principle. The underlying mathematical origin for this “breakdown” of the skeleton series is rooted in the surprising multivaluedness of the Luttinger-Ward functional, and the closely related divergence of the two-particle irreducible vertex. Both aspects have been observed in many fundamental models of strongly correlated electron systems.

This thesis tries to shed light on the underlying physical processes controlling various exciting phenomena, on the one- and the two-particle level, that can be directly related to the breakdown of the skeleton series. For this aim, first, the necessary mathematical one and two-particle formalism for quantum many-body physics is reviewed. Afterwards, the models and methods investigated in this thesis are presented. Then, focusing on the breakdown of the skeleton series, a short historical overview is given, followed by a discussion that motivates a specific functional form for the self-energy at strong coupling. The associated phenomena, the multivaluedness of the Luttinger-Ward functional, the divergence of the two-particle vertex, and arguably associated the violation of Luttinger’s theorem are then exemplified by an analytically solvable toy model. In the following, the divergences of the two-particle vertex are then reviewed in more detail. Thereafter, the emergence of Fermi arcs is discussed as a first exemplary non-perturbative phenomenon of this thesis, manifesting in the one-particle spectrum. In fact, this can be, with the help of an effective, analytically solvable model, unambiguously linked to the breakdown of the skeleton series. Then, negative eigenvalues of two-particle static generalized susceptibilities—originating from divergences of the two-particle vertex—are directly connected to thermodynamic phase instabilities in strongly correlated systems. These instabilities are then shown to be a topologically protected phenomenon due to intrinsic non-hermitian matrix symmetries of the associated two-particle quantities. In the following, the divergences of the two-particle vertex are studied in the antiferromagnetic Hubbard model solved by dynamical mean-field theory, thereby, extending previous studies to the symmetry-broken phase. Here, the enhancement of local charge fluctuations in the strong coupling Heisenberg regime can also be attributed to these negative eigenvalues, associating the Heisenberg physics with processes beyond the realm of the skeleton series. Finally, an outlook on future research directions, inspired by the investigations of this thesis, is presented.